A NEW

INTRODUCTION

TO THE

MATHEMATICKS,

BEING

ESSAYS

ON

VULGAR and DECIMAL ARITHMETICK.

CONTAINING,

Not only the practical RULES, but also the REASONS and DEMONSTRATIONS of them;

With so much of

The THEORY, and of universal ARITHMETICK or ALGEBRA, as are necessary for the better understanding the PRACTICE and DEMONSTRATIONS.

WITH

A GENERAL PREFACE, including a PANEGYRIC, on the Usefulness of Mathematical Learning.

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Nullius in Verba.

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TO

GEORGE BUCK, Esq;

Merchant in BIDDEFORD,

One of His Majesty's Justices of the Peace,

AND

Deputy Lieutenant, for the County of *Devon*;

As a proper Judge of the Subject, and in Gratitude for Favours received, this Volume is with due Respect dedicated by

His most

Obedient Servant,

BENJAMIN DONN.

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THE GENERAL PREFACE.

THE Design of a Preface, in its greatest Extent, is first to give the History of the Art treated of, then to shew that it is a useful Science, and, lastly, to give an Account of the Work. For the first of these, and the Usefulness of the particular Arts, the Reader is referred to the Preface to the several Essays; it being the Intention of this Preface only to say something on the Usefulness of Mathematical and Mathematico-philosophical Learning in general, and give some Account of the Design of the intended Work.

It being common to hear many Persons, and some who would be thought Men of Learning, demanding the Use of the Mathematics, calling the Study of them a dry Study, and affirming that it serves only for Amusement, it is, not only not improper, but in a Manner necessary, to spend a few Pages, in removing these Objections: In which, we shall endeavour to make evident, (not so much by Observations our own, as by * select Passages from esteemed Authors) that the Use of the Mathematics is very great; and, therefore, the above Assertions groundless, and consequently, founded either on Ignorance, or Malice.

It is an Observation of † M. *Fontenelle's*, "that People very readily call useless what they do not understand. It is a Sort of Revenge; and, as the Mathematics and Natural Philosophy are known but by few, they are generally looked upon as useless.— This is the Fate of Sciences which are studied and improved but by a few."

In this Panegyric, or Eulogium, we shall observe the following Order: 1 To shew the Dignity of those Sciences. 2. Their Use to all Men in general, in the Improvement of the Mind. 3. The Advantage of those Sciences in some particular Professions. 4. Lastly, to make some general Inferences by Way of Conclusion.

1. Of the Dignity of the Mathematical Sciences.

"‡ In all Ages and Countries, where Learning hath prevailed, "the Mathematical Sciences have been looked upon as the most "considerable Branch of it. The very Name Μάθησις implies

* We have chosen this Method, because it is natural to suppose, that the Authority of great Names will be much more persuasive, than any Assertions barely our own.

† In his Preface to the Memoirs of the Royal Academy of Sciences at *Paris*, in the Year 1699; and translated in *Miscellanea Curiosa*.

‡ Essay on the Usefulness of Mathematical Learning.

"no less; by which they were called either for their Excellency, "or because of all the Sciences they were first taught, or because "they were judged to comprehend πάντα τὰ Μαθήματα." In those Sciences * "the Art of Reasoning is allowed to reign in its "greatest Perfection. Hence it was that the Antients, who so "well understood the Manner of forming the Mind, always began "with Mathematics, as the Foundation of their Philosophical "Studies. Here the Understanding is by Degrees habituated to "Truth, contracts insensibly a certain Fondness for it, and learns "never to yield its Assent to any Proposition, but where the Evi- "dence is sufficient to produce full Conviction. For this Reason, "*Plato* has called Mathematical Demonstrations the Cathartics or "Purgatives of the Soul, as being the proper Means to cleanse it "from Error, and restore that natural Exercise of its Faculties, in "which just Thinking consists. And, indeed, I believe it will "be readily allowed, that no Science furnishes so many Instances "of a happy Choice of intermediate Ideas, and a dexterous Appli- "cation of them, for the Discovery of Truth, and Enlargement of "Knowledge." Hence *Mathesis* has been justly called (by the Reverend Mr. *Baker*) the Princess of all Sciences; and hence "† Kings and Princes heretofore have been so enamoured with "her Simplicity and Pleasantness, that (forsaking all the Delights "of their Kingdoms) they have made their Addresses to her "Shrines, paid Homage to her Altars; thus redeeming Science at "so great a Price. Should I mention *Anacharsis* the *Scythian*, and "*Heraclitus* the *Ephesian*, who preferred the Contemplation of "Philosophy before their hereditary Kingdoms, and chose rather "(leaving those) to sit at the Feet of Philosophers, than on their "Kingly Thrones: Should I recount *Atlas* King of *Mauritania*, "whom (for his Astronomic Skill, wherein he excelled) Antiquity "hath fabled to bear up the Heavens on his Shoulders; or *Aga- "thocles* King of *Sicily*, *Ptolomy* of *Philadelphia*, *Alphonsus* of *Castile*, "*Frederic* of *Denmark*, *William Landgrave* of *Hesse*, &c. Yea, but "should I mention Emperors, *viz.* *Cæsar*, *Adrian*, *Theodosius*, &c. "who (devoting themselves to these Studies, worthy indeed of "Emperors) rendered themselves more illustrious, by their Writings, "than by their warlike (though many and great) Atchievements; "I should but silently shame and reproach this our degenerate Age". In which, notwithstanding the Excellency of this Science is such, as to make it necessary to be studied as an Introduction to most other Arts; and that it is known from "‡ Experience that great Genius's "have surpassed themselves by cultivating it, and ordinary ones "have become great and sublime; and the meanest have thereby "acquired a Capacity and Enlargement of Judgment;" yet it is not so universally studied as some other Sciences, to the most convincing Arguments of which the Professors "§ cannot subfix *a Quod "erat demonstrandum*: And yet their Schools are so stuffed with

* *Duncan*'s Logic, p. 223. † Preface to the Rev. Mr. *Baker*'s Geometrical Key. ‡ *Stonehouse*'s Arithmetic. § *Baker*'s Geometrical Key.

"Profe-

"Proselytes, that they have scarce Room to breathe in; whilst the "Mathematic (School) only (in which, not some one Truth only is "expanded but even innumerable; and those not mean and obvious, but most high, admirable, and mysterious, are clearly demonstrated) lies orbate and neglected. From this they fly as from "a Pest-house; but to those they troop, as to a *Delphic* Oracle, "or as Doves to white Dove-houses.— Lastly, though her intrinsic "Worth and Beauty hath compelled others of the lowest Orb (who "(saluting her only at the Threshold) never entered, or had the "least Glimpse of her Arcana's or inner Rooms) to admire her; yet, "certain it is, very few are skilled in her Mysteries; by which "Means it comes to pass, that she is as little regarded, as her "Clients rewarded. For what Cause this beautiful Goddess should "thus suffer an Eclipse in her Glory and Esteem with the Vulgar, "now-a-days, I cannot divine; whether it be, she being a liberal "Science, and therefore (on that Account) unsuitable to the Humours of those close-fisted Misers (who are scarce to be reckoned "among the Number of Men) who love to have their Purses enriched rather than their Minds: Or, whether their Despondency "of ever arriving to any considerable Eminency of Height, (it being as good to be nothing, as not a none-such, or but a Spy, to "to an Art:) Or whether it be the fancied Difficulty and Knottiness "of the Study itself, (which I have most Cause to suspect.) Or, "what that supposed *Mormo* may be, that forestals and prejudiceth "some newly entered, and scares others, who have tasted some of "her Sweets, from farther Essays (which, in fine, would have crowned "their Sedulity and Diligence with Evidence and Certainty, I "shall not undertake to determine.—But this (Reader) is as much "absurd, as strange, *viz.* That what should recommend this "Study to thy Reason should discourage thee; that what should "animate thy Diligence, and quicken thee to a further Essay, "should decrest and dispirit thee. Real Difficulties (much less "conceived Prejudices) should be so far from blunting thy Edge, "that they should rather be the Whetstone of Virtue, and sharpen "thy Endeavours: Why may not the same Things, which (for "the Excellency of them) are the Objects of thy Admiration, be "(for their Possibility) as well the Object of thy Hope, and the "Encouragement of thy Industry? The Difficulties of this Art are "not so insuperable, but (as in War) may be overcome, either "by Industry, or Fortune, or both." But, if the Learner should meet with such Difficulties as he cannot easily surmount by himself, or would go through his Studies with more Pleasantness and Dispatch, if he is of Ability, he would do well to call in the Assistance of some able Professor; for * "there are few Persons of so penetrating a Genius and so just a Judgment, as to be capable of "learning the Arts and Sciences without the Assistance of Teachers. "There is scarce any Science so safely and so speedily learned, "even by the noblest Genius and the best Books, without a Tutor. "His Assistance is absolutely necessary for most Persons, and it is

* *Watts*'s Supplement to his Logic.

"very useful for all Beginners. Books are a Sort of dumb Teachers, "they point out the Way to Learning; but, if we labour under "any Doubt, or Mistake, they cannot answer sudden Questions, or "explain present Doubts and Difficulties; This is properly the "Work of a living Instructor." But, to return from this Digression, to shew, Secondly, the great Usefulness of those Sciences to all Persons in general, in the Improvement of the Mind.

The principal Advantages which the Mind receives from Mathematical Studies, are 1. The accustoming it to Attention. 2. The freeing it from Prejudice, Credulity, and Superstition. 3. The acquiring a Habit of close and demonstrative Reasoning.

"1. * The Mathematics make the Mind attentive to the Objects "it considers. This they do by entertaining it with a great Variety of Truths, which are delightful and evident, but not obvious. Truth is the same Thing to the Understanding as Music "to the Ear, and Beauty to the Eye. The Pursuit of it does really "as much gratify a natural Faculty implanted in us by our wise "Creator, as the Pleasing of our Senses: Only in the former Case, "as the Object and Faculty are more spiritual, the Delight is more "pure, free from the Regret, Turpitude, Lassitude, and Intemperance, that commonly attend sensual Pleasures. The most Part "of other Sciences consisting only of probable Reasonings, the "Mind has not where to fix; and, wanting sufficient Principles to "pursue its Searches upon, gives them over as impossible. Again, "as in Mathematical Investigations Truth may be found, so it is "not always obvious: This spurs the Mind, and makes it diligent "and attentive.—And *Plato* (in *Repub. Lib.* VII.) observes, that "the Youth, who are furnished with Mathematical Knowledge, "are prompt and quick at all other Sciences.—Youth is generally "so much more delighted with Mathematical Studies than with "the unpleasant Tasks that are sometimes imposed upon them, "that I have known some reclaimed by them from Idleness and "Neglect of Learning, and acquire in Time an Habit of Thinking, "Diligence, and Attention; (Qualities which we ought to study "by all Means to beget in their desultory and roving Minds.") And this is no Wonder, if we consider, that the Abstractedness of pure Mathematics is a proper Remedy to cure the Lightness of their Minds, acting as a Rein to curb the Impetuosity of their Passions. And that the Study of those Science inspires a Love for Truth, the Pursuit of which † "will give, the otherwise unemployed, a Distaste of those vain Occupations that hurry Men into "Libertinism and Debauchery."

"‡ Secondly, Mathematical Knowledge adds a manly Vigour "to the Mind, frees it from Prejudice, Credulity, and Superstition. "This it does two Ways, 1. By accustoming us to examine, and "not to take Things upon Trust. 2. By giving us a clear and "extensive Knowledge of the System of the World; which, as it "creates in us the most profound Reverence of the almighty and

* Essay on the Usefulness of Mathematical Learning. † *Stonehouse*'s Arithmetic.
‡ Essay on the Usefulness of Mathematical Learning.

"wise

"wise Creator, so it frees us from the mean and narrow Thoughts "which Ignorance and Superstition are apt to beget."

The third Advantage which the Mind receives from Mathematics is the Habit of clear, demonstrative, and methodical Reasoning.

Mathesis * is "a Study that tends not only to the Improvement "of Arts, but also to the Regulation of the Passions;—a Study that "will insensibly bring Men to think methodically, reason correctly, "and separate Truth from Falshood, and the Disguise of Words, "which it generally wears."

The Writings of the Mathematicians have been conducted by so perfect a Model, as to be † "an incontestable Proof of the Firmness and Stability of human Knowledge, when built upon so sure "a Foundation. For not only the Propositions of this Science "stood the Test of all Ages, but are found attended with that invincible Evidence, as forces the Assent of all, who duly consider "the Proofs upon which they are established.——The Mathematicians are universally allowed to have hit upon the right Method of arriving at Truths.—They have been the happiest in the "Choice, as well as Application of their Principles."

In a Word, some Knowledge in both pure and mixt Mathematics is by Experience found not only necessary in many particular Professions, but also of great Use to all Men in general, in the Improvement of the Mind; and, therefore, the Study of them is now deservedly thought, not only of the greatest Use, but also a necessary Part of the Education of Gentlemen; and are accordingly made a Part of it, in our two famous Universities. Not so much to make them Mathematicians, as, by engaging them to observe the Method of Reasoning made Use of in the Mathematical Sciences, they may acquire something of that Justness and Solidity of Reasoning, for which the Professors of these Sciences are so generally, and deservedly esteemed.

Perhaps what has been already said, may be sufficient to shew the great Usefulness of Mathematical Studies, for acquiring a just Method of Reasoning: However, that the Reader may himself be able in some Measure to judge of the Truth of the above Assertions, it may not be improper to lay before him a general Account of the Method made Use of by Mathematicians; which is this: They first begin with Definitions, (from *Definitio*, Lat.) in which the Meaning of their Words is so distinctly explained, as to prevent any Ambiguity, (or double Meaning): By which Means, every attentive Reader has the very same Ideas excited in his Mind, as the Writer has annexed to them.

By this Means, the Mathematicians have secured themselves, and the Sciences which they profess, from Wrangling and Controversy; and if the Writers of Natural Philosophy, and Morality, had used the same Accuracy and Care in adjusting the Definitions wheresoever necessary, ‡ "they had effectually secluded a Multitude of noisy and fruitless Debates out of their several Provinces. "Nor had that sacred Theme of Divinity been perplexed with so "many

* *Stonehouse*'s Arithmetic. † *Duncan*'s Logic, p. 181. ‡ *Watts*'s Logic.

" many intricate Disputes; nor the Church of *Christ* been torn to
" Pieces by so many Sects and Factions, if the Words *Grace, Faith,*
" *Righteousness, Repentance, Justification, Worship, Church, Bi-*
" *shop, Presbyter,* &c. had been well defined, and their Signifi-
" cation adjusted, as near as possible, by the Use of those Words
" in the New Testament; or at least, if every Writer had told us
" at first, in what Sense he would use those Words."

The second Step, in Mathematical Writings, is to lay down some self-evident Truths, which may serve as a Foundation on which to build the future Reasonings. These Propositions are divided into two Sorts, called Axioms and Postulates.

An Axiom (*Axioma*, Lat.) is a self-evident speculative Truth, as, " the Whole is greater than its Part." A Postulate (*Postulatum*, Lat.) is a self-evident practical Proposition, as, " grant that a finite " Right-line may be continued directly forward.

Having thus securely laid the Foundation, the Mathematicians begin in their next Step to build their Superstructure of demonstrable Propositions, i. e. Propositions which are not of themselves self-evident. Of demonstrable Propositions there are also two Kinds, speculative and practical; a speculative Proposition is called a Theorem (θεώρημα); and a practical one, a Problem (πρόβλημα). These they demonstrate in a Series of Reasoning, proceeding carefully Step by Step, assuming nothing for Truth, but the Axioms and Postulates, before laid down; or some Proposition already demonstrated; and hence it follows, that, as the Principles on which their Reasoning is founded is true, the Consequences (rightly deduced) must be true also.

Mathematicians also make Use of Lemma's, Corollaries, and Scholiums. A Lemma (λῆμμα) is a Proposition premised as introductory to the demonstrating a subsequent Proposition. Corollaries (from *Corollarium*, Lat. from *Corolla*) are subjoined either to Theorems, or Problems; and differ from them only in flowing so naturally from them, that the Truth of them appears almost instantaneously, from the preceding Proposition.

Scholiums (*Scholia*, Lat.) are Remarks made occasionally to explain whatever may appear intricate or obscure, in a continued Chain of Reasoning; or, to remove any Objection; or, to shew the Use and Application of the Subject; or, in short, to acquaint the Reader with any useful Thing, which could not be inserted in another Place, without interrupting the Series of Reasoning. These are annexed indifferently either to Definitions, Propositions, or Corollaries, answering the same Purposes as Annotations upon Classic Authors.

Thus we have taken a short View of the Method used by Mathematicians, and certainly it is no Wonder, if a System of Knowledge, so uniform and well connected, is recommended by the most celebrated Authors, as a Model, or universal Rule, for Reasoning, applicable in other Sciences. Thus Mr. *Duncan* says *;

* In his Logic, p. 213.

"I am apt to imagine, that if we were to employ the same Care "about all our other Ideas, as Mathematicians have done about "those of Number and Magnitude, by forming them into exact "Combinations, and distinguishing these Combinations by parti- "cular Names, in order to keep them steady and invariable, we "should soon have it in our Power to introduce Certainty and De- "monstration into other Parts of human Knowledge." And again, * "If we would form our Minds to a Habit of Reasoning "closely and in Train, we cannot take any more certain Method, "than the exercising ourselves in Mathematical Demonstrations, "so as to contract a kind of Familiarity with them." Not that we look upon it as necessary, (to use the Words of the great † Mr. *Lock*,) "that all Men should be deep Mathematicians, but that, "having got the Way of Reasoning, which that Study necessarily "brings the Mind to, they may be able to transfer it to other "Parts of Knowledge, as they shall have Occasion. For, in all sorts "of Reasoning, every single Argument should be managed as a "Mathematical Demonstration, the Connection and Dependence of "Ideas should be followed, till the Mind is brought to the Source "on which it bottoms, and can trace the Coherence through the "whole Train of Proofs. It is in the general observable, that the "Faculties of our Souls are improved and made useful to us, just "after the same Manner as our Bodies are. Would you have a "Man write or paint, dance or fence well, or perform any "other manual Operation, dexterously and with Ease? Let him "have ever so much Vigour and Activity, Suppleness and Address, "naturally, yet no-body expects this from him, unless he has been "used to it, and has employed Time and Pains in fashioning and "forming his Hand, or outward Parts, to these Motions. Just so "it is in the Mind; would you have a Man reason, you must "use him to it betimes, exercise his Mind in observing the Con- "nection of Ideas, and following them in Train. Nothing does "this better than Mathematics; which therefore I think should "be taught all those, who have Time and Opportunity; not so "much to make them Mathematicians, as to make them reasona- "ble Creatures; for though we all call ourselves so, because we "are born to it, if we please; yet we may truly say, Nature gives "us but the Seeds of it. We are born to be, if we please, ratio- "nal Creatures; but 'tis Use and Exercise only that makes us so, "and we are indeed so, no farther than Industry and Application "has carried us." And in another ‡ Place the same learned Gen- tleman says, "We must—if we will proceed as Reason advises, "adapt our Methods of Enquiry to the Nature of Ideas we exa- "mine, and the Truth we search after. General and certain "Truths are only founded in the Habitudes and Relations of "abstract Ideas. A sagacious and methodical Application of our "Thoughts, for the finding out these Relations, is the only Way

* *Duncan*'s Logic, p. 224. † *Lock*'s Conduct of human Understanding.
‡ In his Essay concerning human Understanding, Vol. 2. p. 262.

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"to discover all that can be put, with Truth and Certainty concerning them, into general Propositions. By what Steps we are to proceed in these, is to be learned in the Schools of the Mathematicians, who from very plain and easy Beginnings, by gentle Degrees, and a continued Chain of Reasonings, proceed to the Discovery and Demonstration of Truths, that appear at first Sight beyond human Capacity. The Art of finding Proofs, and the admirable Methods they have invented for the singling out, and laying in Order those intermediate Ideas, that demonstratively shew the Equality or Inequality of unapplicable Quantities, is that which has carried them so far, and produced such wonderful and unexpected Discoveries."

Having thus shewn the great Usefulness of Mathematical Learning, in improving the Mind by giving it an Habit of close and demonstrative Reasoning, we might now proceed to shew, that the Understanding is by it greatly enlarged, yea enlarged so vastly, that an ingenious Author makes no Scruple of asserting, that whoever is ignorant of these Sciences, "though they may have had the Honour of the Title, are yet only nominal Masters of Arts;" but this will in a great Measure appear under the next Head, in which we are to shew, Thirdly, the Advantages of these Studies in some particular Professions, *&c. viz.*

1. That it is a useful Study for Children.
2. For Tradesmen.
3. For young Gentlemen.
4. For Physicians.
5. For Divines.

1. We are to shew that it is a useful Study for Children. There are, says * Dr. *Watts*, "several of the Sciences that will more agreeably employ our younger Years, and the general Parts of them may be easily taken in by Boys. The first Principles and easier Practices of Arithmetic, Geometry, plain Trigonometry, Measuring Heights, Depths, Lengths, Distances, *&c.* the Rudiments of Geography and Astronomy, together with something of Mechanics, may be easily conveyed into the Minds of acute young Persons from 9 or 10 Years old or upward. These Studies may be entertaining and useful to young Ladies, as well as young Gentlemen; and all who are bred up to the learned Professions. The fair Sex may intermingle these with the Operations of the Needle, and the Knowledge of domestic Life. Boys may be taught to join them with their Rudiments of Grammar, and their Labour in the Languages. And even those who never learn any Language but their Mother Tongue, may be taught these Sciences with lasting Benefit in early Days.

"That this may be done with Ease and Advantage, take these three Reasons.

"1. Because they depend so much upon Schemes and Numbers, Images, Lines, and Figures, and sensible Things, that the Imagination

* Supplement to his Logic.

"gination or Fancy will greatly assist the Understanding, and render the Knowledge of them much more easy.

"2. These Studies are so pleasant that they will make the dry "Labour of learning Words, Phrases, and Languages, more tolerable to Boys in a *Latin* School by this most agreeable Mixture, "The Employment of Youth in these Studies will tempt them to "neglect many of the foolish Plays of Childhood, and they will "find sweeter Entertainment for themselves and their leisure Hours, "by a Cultivation of these pretty Pieces of alluring Knowledge.

"3. The Knowledge of these Parts of Science are both easy "and worthy to be retained in Memory by all Children, when they "come to manly Years, for they are useful through all the Parts "of Human Life: They tend to enlarge the Understanding early, "and to give a various Acquaintance with useful Subjects betimes. "And surely it is best as far as possible to train up Children in the "Knowledge of those Things which they should never forget, "rather than to let them waste Years of Life in Trifles, or in "hard Words which are not worth remembering."

Secondly, That the Mathematical Sciences, are useful to Tradesmen, or of the greatest Service in the Conveniencies of Life and Commerce; will appear by only enumerating a few of them; such are, the regular Keeping of Accounts, Measuring and Gauging of Solids and Vessels, &c. for giving every Man his just Property; the Regulation of Time by Sun-dials, Clocks, Watches, &c. the Feasts of the Church by the Motion of the Sun and Moon; and Chronology for the better Understanding of History. The Construction of Houses for the Conveniency of Life, and of Fortifications for our better Defence from the Ravages of our Enemies. By these Sciences it is, that we, after the best Manner, construct "* all sorts "of Instruments to work with, all Engines of War, Ships, "Bridges, Mills, curious Roofs and Arches, stately Theatres, "Columns, Pendent Galleries, and all other grand Works in "Building. Also Jacks, Chariots, Carts and Carriages, and even "the Wheel-barrow.———and whatever hath artificial Motion "by Air, Water, Wind, or Cords." By these Sciences we ascertain the Changes or Alterations in the Air, either as to Weight or Moisture, Heat and Cold; by Barometers, Thermometers, Hygroscopes, &c. By these Sciences we construct Globes, Spheres, Orreries, &c. for the readier conveying Ideas of the Motion of the Sun, Stars, and Planetary Bodies. By these Sciences we are furnished with Telescopes, Microscopes, &c. by which the Organ of Sight is surprizingly extended and augmented. † "How surprized had the Antients been, if they had been told that their "Posterity, by the Help of some Instruments, should one Day see "a Heaven that was unknown to them, and Plants and Animals "they did even suspect it was possible to exist."

* *Emerson*'s Mechanics in the Preface. † *Fontenelle*'s Speech.

Mathesis

Mathesis * "now explains all the Phænomena of Motion; "which involve almost every Branch of Natural Philosophy. She "pursues the Rays of Light in the Immensity of ætherial Space; "pervades all Substances through which they penetrate, detects "them in all the Mazes of Reflection and Refraction; and compels them to discover those Objects which either Distance or "Minuteness had so long concealed. She has stretched her Line "round the World on which we live, has determined its Magnitude, and described its Figure; she has pointed out to Geography the true Situation of every Sea, and every Shore, and "and taught the Mariner to pursue his Way without Deviation "wherever bound, though the Wave that closes after him leaves "no Track, and the Land which sinks below the Horizon affords "no Direction."

In short, the Usefulness of these Sciences even in Mechanical Professions is very great: For what could the Architect, Shipwright, Optician, Painter, &c. perform without some Knowledge in the Mathematics? A complete Architect and Shipwright must have considerable Knowledge in Geometry and Mathematics, for drawing their Designs, and judging of the Strength, &c. of their Works. The Optician ought to understand the Laws of Refraction and Reflection, the Foci of Glasses, Mirrors, &c. and the Painter, the Principles of Perspective. And to the Mathematics it must be owned the principal Improvements in these Arts have been owing, though the generality of Workmen are ignorant thereof. In a Word, how could we perform the necessary Helps of Society? without these inestimable! these admirable Arts!

3. That Mathematical Sciences are proper Studies for young Gentlemen.

Mr. Ray was so fully persuaded of the Usefulness of these Studies to young Gentlemen, that, he says, † "I——do earnestly "exhort those that are young, especially Gentlemen, to set upon "these Studies, and take some Pains in them." For he adds, he does not "see what more ingenious and manly Employment they "can pursue; tending more to the Satisfaction of their own "Minds, and the Illustration of the Glory of God. For He is "wonderful in all his Works." Mr. Ray is also of Opinion, that, "did but young Men fill up that Time with these Studies, "which lies upon their Hands, which they are incumbered with, "and troubled how to pass away, much might be done, even "so."

Dr. Watts makes this Observation ‡ "That where Students, or indeed any young Gentlemen, have in their early Years "made themselves Masters of a Variety of elegant Problems in the "Mathematic Circle of Knowledge, and gained the most easy, "neat, and entertaining Experiments in Natural Philosophy, with "some short and agreeable Speculations or Practices in any other

* Mess. *Maupertuis*'s Speech to the *Royal Academy* at *Berlin*; see *Gentleman's Magazine*, Vol. 21. † In his Wisdom of God in the Creation, p. 203. ‡ In the Spplement to his Logic.

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"of the Arts and Sciences, they have hereby laid a Foundation "for the Esteem and Love of Mankind among those with whom "they converse, in higher or lower Rank of Life; they have "been often guarded by this Means from the Temptation of no-"cent Pleasures, and have secured both their own Hours, and the "Hours of their Companions, from runing to Waste in Santering, "and Trifles, and from a thousand Impertinences in silly Dialogues. "Gaming and Drinking, and many criminal and foolish Scenes "of Talk and Action, have been prevented by these innocent and "improving Elegancies of Knowledge."

This learned Gentleman in another * Place says, that, "besides "the common Skill in Accounts which is needful for a Trader, "there is a Variety of pretty and useful Rules and Practices in "Arithmetic, to which a Gentleman should be no Stranger; and, "if his Genius lie that Way, a little Insight into Algebra would "be no Disadvantage to him. It is fit that young People of any "Figure in the World should see some of the Springs and Clues "whereby skilful Men, by plain Rules of Reason, trace out the "most deep, distant, and hidden Questions; and whereby they "find certain Answers to those Enquiries, which, at first View, "seem to lie without the Ken of Mankind, and beyond the Reach "of human Knowledge. It was for Want of a little more general "Acquaintance with Mathematical Learning in the World, that a "good Algebraist and Geometer were accounted Conjurers a Cen-"tury ago, and People applied to them, to seek for lost Horses "and stolen Goods."

Young Gentlemen "should know something of Geometry, so "far at least as to understand the Names of the various Lines and "Angles, Surfaces and Solids; to know what is meant by a right "Line or a Curve, a right Angle, &c. The World is now grown "so learned in Mathematical Science, that this sort of Language is "often used in common Writings and Conversations, far beyond "what it was in the Days of our Fathers. And besides, without "some Knowledge of this Kind, we cannot make any farther "Progress toward an Acquaintance with the Arts of Surveying, "Measuring, Geography, and Astronomy, which are so enter-"taining and so useful an Accomplishment to Persons of a polite "Education. Geography, and Astronomy, are exceeding de-"lightful Studies. The Knowledge of the Lines and Circles of "the Globes of Heaven and Earth is counted so necessary in our "Age, that no Person of either Sex is now esteemed to have had "an elegant Education without it. Even Tradesmen and the "Actors in common Life should, in my Opinion, in their younger "Years, learn something of these Sciences, instead of vainly "wearing out seven Years of Drudgery in *Greek* and *Latin*.

"It is of considerable Advantage, as well as Delight, for Man-"kind, to know a little of the Earth on which they dwell, and of "the Stars and Skies that surround them on all Sides. It is almost "necessary for young Persons, who pretend to any thing of In-

* *Watts* on Education.

"struction and Schooling above the lowest Rank of People, to "get a little Acquaintance with the several Parts of the Land and "Sea, that they may know in what Quarter of the World the "chief Cities and Countries are situated. —— Without this Know-"ledge we cannot read any History with Profit; nor so much as "understand the common News-Papers. It is necessary also to "know something of the Heavenly Bodies, their various Motions "and Period of Revolution, that we may understand the Ac-"counts of Time in past Ages, and the Histories of antient Na-"tions; as well as know the Reasons of Day and Night, Sum-"mer and Winter, and the various Appearances and Places of the "Moon, and other Planets. Then we shall not be terrified at "every Eclipse *, nor presage and foretel public Desolations at the "Sight of a Comet: We shall see the Sun covered with Darkness, "and the Full Moon deprived of her Light, without foreboding "Imaginations that the Government is in Danger, or that the "World is come to an End. This will only increase rational "Knowledge, and guard us against foolish and ridiculous Fears, "but it will amuse the Mind most agreeably; and it has a most "happy Tendency to raise in our Thoughts the noblest and most "magnificent Ideas of God by the Survey of his Works, in their "surprising Grandeur and Divine Artifice. Natural Philosophy, "at least in the more general Principles and Foundation of it, "should be infused into the Minds of Youth. This is a very "bright Ornament of our rational Natures, which are inclined to "be inquisitive into the Causes and Reasons of Things. A Course "of Philosophical Experiments is now frequently attended by the "Ladies as well as Gentlemen, with no small Pleasure and Im-"provement. God and Religion may be better known, and "clearer Ideas may be obtained of the amazing Wisdom of our "Creator, and of the Glories of the Life to come, as well as of "the Things of this Life, by the rational Learning, and the "Knowledge of Nature, that is now so much in Vogue—— "These Things will enlarge and refine the Understanding, im-"prove the Judgment, and bring the Faculty of Reasoning into "a juster Exercise, even upon all manner of Subjects." Other Passages might have been added from the learned Mr. LOCK's Essay on Education and Mr. CLARK's Essay on Study; but what is already given is sufficient to shew that Mathematical Learning is very useful to young Gentlemen: We therefore proceed to shew,

* The *Chinese* have such an odd Notion of an Eclipse of the Sun or Moon, that they, at the Beginning of an Eclipse, beat their Drums and Kettles, in Hopes to frighten away a prodigious great Monster whom they fancy to be in Heaven, and going to devour the Sun or Moon; and hence, while the Astronomers are making their Observations, the *Mandarins* belonging to the Court of *Lipou* fall on their Knees in the Palace, and, looking towards the Sun, express their Concern for him, and implore the Dragon to have Compassion on the World, and not deprive them of the Light of this glorious Planet.

Salmon's Modern History.

Fourthly,

Fourthly, That an Acquaintance with the Mathematical Sciences is of great Service in the Study of Medicine, and other Arts relating thereto. Before the Discovery of true Philosophy, the Art of Chymistry and Medicine were made up of unintelligible Terms, false Hypotheses, and Metaphysical Jargon: Hence the more a Person read, the more likely he was to be misled and confounded, unless he confined himself to read barely the experimental Knowledge of a few celebrated Names. But now, since the great Discoveries of the most learned NEWTON, we are enabled to enquire into the Principles of Chymistry and Medicine in a rational Manner, from the Knowledge of the Laws of Motion and Action of Bodies. Now we are enabled to make more accurate and deeper Enquiries into the Nature of the Animal Oeconomy, so necessary to the Improvement of the Art of Healing. For, since it is confirmed by the modern Observations and Improvements in Anatomy, * "That the Animal Body is a pure Machine, and that "all its Operations and Phænomena, with the several Changes "which happen to it, are the necessary Result of its Organization "and Structure;" it follows, that such as are acquainted with Mathematical Philosophy are best able to study the Animal Oeconomy, and consequently, *cæteris paribus*, are better qualified for curing Diseases.

This may be supported by the Authority of several Men of the greatest Eminence in their Profession; such as Dr. *Boerhaave*, Dr. *Mead*, Dr. *Keil*, Dr. *Morgan*, &c.

† "Some Things," says Dr. *Boerhaave*, "the Knowledge of "which a few Years ago was despaired of, are now, from simple "and indisputable Experiments of the Senses, demonstrated in a "Geometrical Way by Mechanics. Consult for this Purpose — those "Problems which *Pitcairne* proposed to the learned World, and "demonstrated. Examine what *Scheiner*, *Des Cartes*, and *Huygens* "have written on the Eye; and what *Kircher*, *Schelhammer*, and "*Moreland* have taught us concerning the Ear and Hearing. All "these prove, beyond Contradiction, the Usefulness of Mechanical Knowledge in Medicine; and shew what might be expected, "were the Use of it introduced into the salutary Art by some "skilful Physicians, and persisted in for so long a Time as human "Patience has been able to endure the idle Systems of some Sects "in Medicine.

"All these Things will be allowed to be true, and the Usefulness "of Mechanic Learning in Medicine is acknowledged, with respect "to the Theory: But it is very commonly said, that Mechanic Knowledge is of no Service at all to a practical Physician. This plausible "Distinction, made with so much Confidence, does not appear to "me to be consistent; for I do not suppose that by Theory they mean "any other than what clearly shews, from proximate Causes, what is "the Life of a Man in Health. If this be admitted, as it ought to be,

* Preface to *Morgan*'s Philosophical Principles of Medicine. † In his *Oratio de Usu Ratiocinii Mechanici in Medicina*.

"it will follow, that this Science affords us the best Assistances for the "Knowledge and Cure of Diseases. For he who knows the Causes "of perfect Health, must, whenever they are deficient, be very "well qualified to comprehend the Origin and Nature of such De-"fect, that is, the Disease; and certainly he who has the clearest "Notion of the immediate Cause of Sickness, is the fittest Person "to encounter with it: Just as it is in a Clock, where every one "observes where the Hand deviates, but none knows how to cor-"rect it according to Art, but he who, knowing the exquisite Struc-"ture of the Machine, can both find out the Defects of the Parts, "and Remedies for the same. So that there is not a Truth in the "Theory of Medicine, which a skilful Artist does not know how "to apply to his own Advantage in Practice; and, consequently, "to confess the Excellency of the Mechanic Science in Theory is "to grant its Usefulness in Practice."

Dr. *Mead*, having given a Mechanical Account of Poisons, concludes in his Preface, "that, if so abstruse Phænomena as these "come under the known Laws of Motion, it might very well be "taken for granted, that the more obvious Appearances in the "same Fabric are owing to such Causes as are within the Reach of "Geometrical Reasoning; and that therefore, as the first Step to-"wards the Removal of a Disease is to know its Origin, so he is "likely to be the best Physician, who, having the same Assistance "of Observations and Histories with others, best understands the "human Oeconomy, the Texture of the Parts, Motions of the "Fluids, and the Power which other Bodies have to make Altera-"tions in any of these.

"Nor indeed ought any one to doubt of this, who con-"siders that the Animal Compages is not an irregular Mass, and "disorderly Jumble of Atoms, but the Contrivance of infinite "Wisdom, and the Master-piece of that creating Power, who "has been pleased to do all Things by established Laws and "Rules, and that Harmony and Proportion should be the Beauty "of all his Works.

"It were therefore heartily to be wished, that those Gentlemen, "who are so much afraid of introducing Mathematical Studies, "that is, Demonstration and Truth, into the Practice of Physic, "were so far at least instructed in the necessary Disciplines, as to "be able to pass a true Judgment, what Progress and Advances "may be made this Way. They would not then perhaps decry an "Attempt of so much Moment to the Welfare of Mankind, as "vain and impossible; because it is difficult, and requires Appli-"cation and Pains.

"It is very evident, that all other Methods of improving Me-"dicine have been found ineffectual, by the Stand it was at for "above two thousand Years; and that, since of late Mathematici-"ans have set themselves to the Study of it, Men already begin to "talk so intelligibly and comprehensibly, even about abstruse Mat-"ters, that it may be hoped in a short Time, if those, who are "designed for this Profession, are early, while their Minds and

"Bodies

" Bodies are patient of Labour and Toil, initiated in the Know-
" ledge of Numbers and Geometry, that Mathematical Learning
" will be the distinguishing Mark of a Physician from a Quack;
" and that he who wants this necessary Qualification, will be as ri-
" diculous as one without *Greek* or *Latin*.

"——The Dissertations of Dr. *Pitcairne*, the Honour of his
" Profession in *Scotland*, are a convincing Proof of the Advantage
" of such a Mechanical Way of reasoning: Nor could Malice itself
" deny this, were not Ignorance in Confederacy with it, which
" will secure any one from being benefited by the most useful De-
" monstrations."

* Dr. *Keil* says, " As all Diseases whatsoever, which are inci-
" dent to human Bodies, are in reality nothing else than Disorders
" of the Animal Oeconomy; the Quantity and Quality of which
" are more or less clearly understood, in Proportion to the Know-
" ledge of the Oeconomy itself; whatsoever can explain it, must
" also add Light to the occult Natures of Diseases, establish the Prac-
" tice of Physic upon a surer Foundation, and enable Physicians
" to make truer and more certain Judgments, in most Cases.

——" There are many Phænomena of the Animal Oeconomy,
" which the Ages past thought inexplicable, which have now by
" several been made the Subjects of Geometrical Demonstration;
" and if there were sufficient Data, as they are called, I do not
" doubt but those Phænomena, which now torture the Brains of
" Philosophers, would be clear to all. For, if some Things,
" which to former Ages have appeared unaccountable, are clear to
" the present Age, why should not Posterity, happier than our-
" selves, and studious of the Good of Mankind, and their own
" Reputation, discover the Things which have been long earnestly
" sought after by the Learned, and are still involved in Dark-
" ness? This is by no Means to be despaired of, if we consider the
" Progress that has already been made, notwithstanding the Me-
" chanical Philosophy, as applied to Physic, is still in its Infancy.

—— " There were formerly some Physicians, nor are there
" wanting, even since the Improvement of Physic by Philosophy,
" some who think that the Art of curing Diseases is only to be
" promoted by Experiments, by observing what Things are hurt-
" ful, what beneficial in each Disease; and that the Study of the
" hidden Natures of Things is altogether superfluous, and of as
" little Use to a Physician, as it would be to a Sailor to know the
" Reason of the Flux and Reflux of the Sea, or the wonderful
" Theory of the Loadstone. But, if we diligently consider the
" Number of Diseases, their different Species, different Appear-
" ances, according to the almost infinite Variety of the Constitu-
" tions of our Bodies, and the Air in which we live: If we reflect
" likewise on their various Complications, on the almost infinite
" Variety of Medicines, and the critical Times of using sometimes
" one, and sometimes another, and even sometimes of abstaining
" from them altogether, we may as well expect that a blind Man
" should shoot flying, or one that is deaf tune an Organ, as that

* In his Essay on the Animal Oeconomy (in the Preface.)

 " a Phy-

" a Physician, led only by that blind Guide Experience, should " cure Diseases; and whosoever judges otherwise, must either not " have considered these Things, or not sufficiently have attended " to them.

—— " But, tho' I would fain persuade the Students of Physic, " that the Knowledge of the Animal Œconomy is highly necessary " to be acquired, yet I do not deny but that Experiments have " their Use."

In short, " Natural Philosophy and Diseases must go Hand in " Hand, in the improving the Art of Curing; it is not possible to " make any Use of the last without the Knowledge of the first. " And I may venture to say, that there is no Man that practises, but " who does it upon some Knowledge of the Animal Œconomy, or " some Notions of his own, which are more or less clear, accord- " ing to his Skill in Natural Philosophy. And, for the Truth of " this, I appeal to Dr. *Sydenham's* own Writings, who, by his Phi- " losophizing, has evidently shewn us the Necessity of that Science " he so much decried, and so little understood. He was undoubt- " edly a great Man, and the World will always be obliged to " him for his accurate Histories of Diseases; but there is no Man " without Errors, and, where one of his deserved Character falls " into a Mistake, it does a great deal more Hurt, than if Hun- " dreds of others of lesser Note had been guilty of the same."

Dr. *Morgan* says *, " since the animal Body is a Machine, and " Diseases are nothing else but its particular Irregularities, De- " fects, and Disorders, a blind Man might as well pretend to regu- " late a Piece of Clock-work, or a deaf Man to tune an Organ, " as a Person ignorant of Mathematics and Mechanism to cure " Diseases, without understanding the natural Organization, " Structure, and Operations of the Machine which he undertakes " to regulate.

" As there are two Things necessary to constitute a good Philo- " sopher, namely, a just Acquaintance with the Phænomena of " Nature, grounded upon accurate Observations and Experiments; " and a competent Skill in Arithmetic, Geometry, and Algebra, " to enable him to reduce the Forces and Operations of Bodies to " a Calculus, in Order to find out the Adequation and Proportion " between the natural Causes and their Effects: So the like Ma- " thematical and Mechanical Reasoning, joined with the History " of Diseases, their Symptoms and Cure, drawn from Experience, " are both necessary in Physicians, and one without the other is " altogether insufficient. It is a little surprising therefore to hear " some Gentlemen of the Faculty declaim against Mathematical " and Mechanical Theories in *Re Medica*, since this is in Effect to " maintain, that Medicine is grounded upon no Principle at all; " that, if Diseases are cured, it must be by Chance; and that con- " sequently there is no Difference, but that of a Diploma, between " a Physician and a Quack. 'Tis evident to all Experience, that

* In the Preface to his Philosophical Principles of Medicine.

" new

" new Species of Diseases, or new Symptoms attending the same " Diseases, daily arise and offer themselves in the Practice of every " Physician, in which the Histories of Diseases can be of little " Use: And in this Case, where Experience fails, as it will in a " thousand Instances, every one, how much soever he may declaim " against Theories, presently recurs to his own Theory, such as it " is, true or false, right or wrong, and accordingly attempts the " Cure at least for Experiment's Sake; and so the Patient often " pays dear for what the Doctor decries, only because he does not " understand. But I must do our Physicians the Justice to own " that they now seem pretty generally disposed to abandon My- " stery for plain Sense, and to substitute demonstrative and ex- " perimental Truths, in the Room of unintelligible Terms, occult " Qualities, precarious Hypotheses, and that infinite Jumble of " Chymical and Metaphysical Jargon, which had a long Time passed " for the Rationale of Medicine. A moderate Skill in the Mathe- " matics, and a tolerable Acquaintance with the Mechanical Pow- " ers, begin to be reckoned a necessary Qualification for one who " would make a Figure in his Profession; and 'tis to be hoped, " that this, in Time, will come to be allowed as the true Cha- " racteristic of a rational Physician, as distinguished from an Em- " pyric. And indeed, since it is the Business of a Physician to " assist Nature in its Operations under the most nice and difficult " Circumstances, it is impossible he should acquit himself herein " with Satisfaction and Success, or act otherwise than at blind " Random, if he has not the Skill of applying, as Occasion serves, " Mathematical Quantities and Proportions to the Mechanical " Powers; upon which all the Laws of the Animal Œconomy, and " the Effects and Consequences of Motion in the mutual Action and " Re-action of Bodies, entirely depend.

—— " Tis from the Knowledge of the Animal Œconomy only, " or the Laws and Principles of Motion in the Animal Machine, " that the Disease can be found out by a rational Deduction from " the Symptoms, and from hence alone can the general Indications " of Cure be taken. For he who is ignorant of the Disease, or the " real internal State of the Organs and Fluids in which the Disease " consists, can never form any rational Judgment of the most pro- " per Methods of Cure.

" Any one, by a little Reading, may easily inform himself of the " real or reputed Powers, Virtues, and Properties of Medicines, " so far as the Experiences of others have been committed to Wri- " ting, and reduced to general Rules; but this is the least Part of " a Physician, and he who only knows thus much, knows only " how to act at Random and to do Mischief. Every Apothecary " (or even his Man at a Year's Standing) may be acquainted with " the several Classes of Medicines made up by him, or sold in his " Shop; he may be well versed in the several Tribes of Simples " and Compounds, and be sufficiently acquainted with the several " Classes of Cathartics, Emetics, Sudorifics, Diuretics, &c. and " yet be no better a Physician than his Horse. 'Tis one thing

"to know how to bleed, purge, vomit, &c. and a quite different "thing to know when, and under what particular Circumstances, "either the one or the other of these is to be chosen; how far to "be allowed, and when or by what Means to be moderated and "restrained. The former may be got by Reading, or Learning "by Rote; but the latter can only be obtained by a just Acquaint-"ance with that Part of Natural Philosophy which respects the "Animal Œconomy."

Instances of the great Usefulness of Mathematical Learning, in the Art of curing Diseases, might be given from several Authors; but one from Dr. *Mead*'s Medical Precepts will be sufficient. In that truly valuable Work, the learned Author, discoursing on the Cure of the *Gutta Serena*, says, "That he had found by "the Laws of Optics that certain Corpuscles, floating in the aqueous "Humour of the Eye, could not be the Cause of this Disease, ac-"cording to the common Opinion; because they must be too near "the Bottom of the Eye to be able to depict their Image there. "Wherefore there was a Necessity of seeking some other * Cause; "and whether I have found the true one is submitted to the Ma-"thematicians. For my Part, I cannot help thinking, that this "Invention is a remarkable Instance of the great Use of true Ma-"thematical Knowledge towards establishing a right Method of "Practice."

Such great Discoveries having already been made by Mechanical Reasoning, though so lately applied to the Art of curing Diseases, Mr. *Brown* might well conclude his Encomium on Dr. *Morgan*'s Principles of Medicine with these poetic Lines:

"My raptur'd Muse sees with prophetic Eyes
"New Ages roll along, new Nations rise:
"Sees Physic on Mechanic Reasoning climb,
"And raise a Structure to the Skies sublime:
"Sees Sickness fled, Health bloom in ev'ry Face,
"And Age creep on with slow reluctant Pace.
"Experience with her Torch shall guide our Youth,
"Scatter the Mists, and light the Way to Truth.

* According to Dr. *Mead*, "it proceeds from various Causes, of which the "most common is an Obstruction gradually formed in the Arteries of the *Retina* "by a sizy Blood. For the Consequence of this Obstruction is, that the Rays "of Light, which should depict the Images of Objects on the Bottom of the "Eye, falling on these dilated Blood-vessels, produce no Effect; whence the "Sight is either diminished, or entirely lost, according to the Degree of the Ob-"struction. Again, this Disease is sometimes owing to a Palsy of the Nerves of "this Membrane; as in some Measure destroys their Sensibility; whereby the "Impulse of the Corpuscles of Light on them is not sufficient to make them "transmit Objects to the Brain. In fine, I have observed that this Species of "Blindness is also occasioned by a Pressure on the Optic Nerves, either by the Ex-"travasation of a glutinous Humour, or by a hard Tumour formed upon the Place, "where they pass from their *Thalami* into the Eyes; whereby the Passage of the "Animal Spirits to the Brain is totally intercepted."——Grounding his Method of Practice on these Causes, he has cured several of a Disease which was generally reckoned incurable.

"While dark Hypothesis no more prevails,
"Nor Pupils listen to romantic Tales;
"Nor proud Authority, with bugbear Rules,
"Controuls the Church, nor dictates in the Schools,
"But Liberty sits Goddess of our Isle,
"And peaceful Blessings all around her smile;
"Darkness and Bigotry before her fly,
"And Truth and Virtue grow beneath her Eye."

As to the Usefulness of Mathematical Learning to Divines, it may be sufficient to give what Dr. *Watts* says on this Subject, in the Dedication of his Astronomy and Geography, *viz.* "I shall be "told perhaps, that these Sciences are not my special Province. "It is the Knowledge of God, the Advancement of Religion, and "Converse with the Scriptures, are the peculiar Studies which "Providence has assigned me. I know it, and I adore the Divine "Favour. But I am free and zealous to declare, that, without "commencing some Acquaintance with these Mathematical Sci-"ences, I could never arrive at so clear a Conception of many "Things delivered in the Scriptures; nor could I raise my Ideas "of God the Creator to so high a Pitch: And I am well assured "that many of the Sacred Function will join with me, and support "this Assertion from their own Experience.

—"If we look down on the Earth, it is the Theatre on which "all grand Affairs recorded in the Bible have been transacted. "How is it possible that we should trace the Wanderings of *Abra-"ham*, that great Patriarch, and the various Toils and Travels of "*Jacob*, and the Seed of *Israel* in successive Ages, without some "Geographical Knowledge of those Countries? How can our "Meditations follow the blessed Apostles in their laborious Jour-"neys through *Europe* and *Asia*, their Voyages, their Perils, their "Shipwrecks, and the Fatigues they endured for the Sake of the "Gospel; unless we are instructed by Maps and Tables, wherein "those Regions are copied out in a narrow Compass, and exhi-"bited in one View to the Eye? If we look upward with *David* "to the Worlds above us, we consider the Heavens as the Work "of the Finger of God, and the Moon and the Stars which he "hath ordained: What amazing Glories discover themselves to "our Sight? What Wonders of Wisdom are seen in the exact "Regularity of their Revolutions? Nor was there ever any thing "that has contributed to enlarge my Apprehensions of the im-"mense Power of God, the Magnificence of his Creation, and his "own transcendent Grandeur, so much as that little Portion of "Astronomy which I have been able to attain. And I would not "only recommend it to young Students for the same Purposes, but "I would persuade all Mankind, if it were possible, to gain some "Degrees of Acquaintance with the Vastness, the Distances, and "and the Motions of the Planetary Worlds, on the same Account. "It gives an unknown Enlargement to the Understanding, and "affords a divine Entertainment to the Soul and its better Pow-

" ers. With what Pleasure and rich Profit would Men survey " those astonishing Spaces in which the Planets revolve, the Hugeness of their Bulk, and almost incredible Swiftness of their Motions? And yet all these governed and adjusted by such unerring " Rules, that they never mistake their Way, nor lose a Minute " of their Time, nor change their appointed Circuits in several " Thousands of Years. When we muse on these Things, we may " lose ourselves in holy Wonder, and cry out with the Psalmist, " *Lord, what is Man that thou art mindful of him, and the Son of Man* " *that thou shouldst visit him?*"

Lastly, upon the Whole, are the Mathematics so truly noble, useful, and excellent, as has been just now represented? And shall we affect the "vain Trappings of Words, and the Delusions of a " painted Speech, while the Nature of Things lies unregarded, " and the Use of plain Reason is set aside?" No! but rather "let " us take a View of these Sciences in all their Splendor, dignified " in the Robes of Nobles, glorying in the Titles of Princes, and " sitting upon the Thrones of Kings themselves." For, to sum up the Whole in the Words of Dr. *Barrow* *, let us study "the " Mathematics, which effectually exercises, not vainly deludes, " nor vexatiously torments studious Minds with obscure Subtilties, perplexed Difficulties, or contentious Disquisitions; which " conquers without Opposition, triumphs without Pomp, compels without Force, and rules absolutely without the Loss of " Liberty; which does not privately over-reach a weak Faith, but " openly assaults an armed Reason, obtains a total Victory, " and puts on inevitable Chains; whose Words are so many Oracles, and Works as many Miracles; which blabs out nothing " rashly, nor designs any thing from the Purpose: But plainly " demonstrates, and readily performs all Things within its Compass; which obtrudes no false Shadows of Science, but the very " Science itself: The Mind firmly adhering to it, as soon as possessed of it, and can never afterwards, of its own Accord, desert " it, or be deprived of it by any Force of others. Lastly, (says " he) the Mathematics, which depends upon Principles clear to " the Mind, and agreeable to Experience; which draws certain " Conclusions, instructs by profitable Rules, unfolds pleasant " Questions, and produces wonderful Effects; which is the fruitful Parent of, I had almost said, all Arts, the unshaken Foundation of Sciences, and the plentiful Fountain of Advantage to " human Affairs. In which last Respect, we may be said to receive from Mathematics the principal Delights of Life, Securities of Health, Increase of Fortune, and Conveniencies of Labour. That we dwell elegantly and commodiously, build decent Houses for ourselves, erect stately Temples to God, and leave " wonderful Monuments to Posterity: That we are protected by " those Ramparts from the Incursions of the Enemy, rightly use

* In his Inaugural Oration on his Admittance to the Professorship at *Cambridge*, annexed to his Mathematical Lectures.

" Arms,

"Arms, skilfully range an Army, and manage War by Art, and "not by the Madness of wild Beasts: That we have safe Traffic "through the deceitful Billows, pass in a direct Road through the "trackless Ways of the Sea, and arrive at the designed Ports, by "the uncertain Impulse of the Winds: That we rightly cast up "our Accounts, do Business expeditiously, dispose, tabulate, and "calculate scattered Ranks of Numbers, and easily compute them, "though expressive of huge Heaps of Sand; nay, immense Hills "of Atoms: That we make pacific Separations of the Bounds of "Lands, examine the Momentums of Weights in an equal Ba-"lance, and distribute every one his own by a just Measure; that "with a light Touch we thrust Bodies forward, which Way we "will, and stop a huge Resistance with a very small Force; that "we accurately delineate the Face of this earthly Orb, and sub-"ject the Œconomy of the Universe to our Sight: That we aptly "digest the flowing Series of Time, distinguish what is acted by "due Intervals, rightly account and discern the various Returns "of the Seasons, the stated Periods of the Years and Months, the "alternate Increasements of Days and Nights, the doubtful Limits "of Light and Shadow, and the exact Difference of Hours and "Minutes; that we derive the Solar Virtue of the Sun's Rays to "our Uses, indefinitely extend the Sphere of Sight, enlarge the "near Appearances of Things, bring remote Things near, disco-"ver hidden Things, trace Nature out of her Concealments, and "unfold her dark Mysteries: That we delight our Eyes with "beautiful Images, cunningly imitate the Devices and pourtray "the Works of Nature. Imitate did I say? Nay excel; while "we form to ourselves Things not in Being, exhibit Things ab-"sent, and represent Things past; that we recreate our Minds, "and delight our Ears, with melodious Sounds, attemperate the "inconstant Undulations of the Air to Musical Tunes, add a plea-"sant Voice to a sapless Log, and draw a sweet Eloquence from "a rigid Metal; celebrate our Maker with harmonious Praise, "and not unaptly imitate the blessed Choirs in Heaven: That we "approach and examine the inaccessible Seats of the Clouds, di-"stant Tracts of Land, and unfrequented Paths of the Sea; lofty "Tops of Mountains, low Bottoms of Valleys, and deep Gulphs of "the Ocean; that we scale the ætherial Towers, freely range "thro' the celestial Fields, measure the Magnitudes and deter-"mine the Interstices of the Stars, prescribe inviolable Laws to "the Heavens themselves, and contain the wandering Circuits of "the Stars within strict Bounds: Lastly, that we may comprehend "the huge Fabric of the Universe, admire and contemplate the "the wonderful Beauty of the Divine Workmanship, and so learn "the incredible Force and Sagacity of our own Minds by certain "Experiments, as to acknowledge the Blessings of Heaven with a "pious Affection.

"I omit the advantageous Spur to our Reason, which accrues "from this Mathematical Exercise, both effectually to turn aside "the Strokes of true Arguments, and warily decline the Blows of "false

THE

PREFACE

TO THE

ARITHMETIC.

IT is highly reaſonable to ſuppoſe, that ſome Method of Numbering was uſed by *Adam* and *Eve* in *Paradiſe*, for communicating their Ideas to each other, of ſo many, or ſo much, *&c.* but in the Beginning, whilſt the Manner of Men's Living was ſimple, and Things were in a manner common, as there was not then any great Skill requiſite or neceſſary in Numbers, an Enquiry into the Nature or Properties of them was certainly much neglected.

Though Hiſtory neither acquaints us with the Author, or Time of the Invention of Arithmetic; yet is it natural to ſuppoſe, that, when Commerce firſt began in the World, then, as ſome kind of Computation was abſolutely neceſſary, Men began, in good Earneſt, to apply themſelves to ſtudy the Properties of Numbers, and to reduce them into a kind of Art.

All Arts and Sciences have had their happy Ages, in which they have appeared with greater Splendor, and caſt a ſtronger Light; but this Splendor, this Light, and thoſe Times of Knowledge, have been, many Times, not only of ſhort Continuance, but ſucceeded by long, very long, Ages of Ignorance and Obſcurity.

For the more regular Shewing the Progreſs of Arithmetic, we ſhall endeavour to trace it through the ſeveral Ages of Learning. And, as we are in the Dark concerning the Affairs of Mankind before the Deluge, the firſt Age of Arithmetic, and many other Arts and Sciences, may be computed from thence, (about 2 or 3000 Years before *Chriſt*, for Chronologiſts are not agreed in this Point), to the Time that the *Greeks* travelled into *Egypt* and *Babylon*, (a-

bout

bout 660 Years before *Christ*) for the Improvement of themselves in Literature.

As * "the *Phœnicians* were the first Navigators, having no Rivals for many Ages," it is natural to suppose, that Arithmetic must either have been invented by them, or have received great Improvements from them. For, in Order to carry on Mercantile Business, there is a Necessity of Computations, and Necessity is the Mother of Invention: And this Supposition is consonant with History, which acquaints us, that, if it did not receive its Origin from them, it was greatly improved, and applied by them to Mercantile, Nautical, Architectonical, and Fabrical Uses. From hence, Arithmetic passed into *Greece*; for the *Greeks* had no Letters, until *Cadmus*, King of *Bœotia*, brought them, and Arithmetic, and Navigation, and Commerce from *Phœnicia*; and set up Schools, and taught Arithmetic, Trade, and Navigation (about 1440 Years before *Christ*.) From *Phœnicia* also † Trade and Arithmetic were carried into *Egypt*.——*Josephus* says that, when *Abraham* was in *Egypt*, he taught Arithmetic to the *Egyptians*.

Though Arithmetic, like other Arts, was no Doubt at first very rude, and improved by Degrees, and in this first Age fell far short of the modern Systems; yet they had laid a very good Foundation for an Art of Computation; for they were not ignorant of that Method of Notation still used by us, and called the *Arabian*. For which excellent Notation, the *Arabians* acknowledged themselves beholden to the Genius of the *Indians*. This is all we know concerning the Progress of Arithmetic in the first Age of Learning, and, therefore, we pass on to the second Age of Arts and Sciences; which is for the most Part included amongst the *Greeks*, beginning with *Pericles* (about 460 Years before *Christ*) and ending with the Death of *Alexander*'s first Successors, (*viz.* about 300 Years before *Christ*.)

The second Age of Arts.

In this Age flourished a Number of learned Men, *viz. Thales*, *Plato*, *Aristotle*, *Pythagoras*, the Inventor of the Multiplication Table, *&c.* who travelled into *Egypt* and various Parts of *Asia*, in Order to acquire a greater Degree of Knowledge in Arithmetic, Geometry, Astronomy, *&c.* which they brought into *Greece*, and taught their Countrymen. The *Greeks* made Use of two Methods of Notation; the First of which was of the same Nature as the *Roman* Notation, (which we shall explain presently;) the other and best Method was thus: The first nine Letters of their Alphabet, Α, Β, Γ, Δ, Ε, Ζ, Η, Θ, Ι, expressed the first Numbers from 1 to 9; the next nine Letters Κ, Λ, Μ, Ν, Ξ, Ο, Π, Ρ, Σ, represented any Number of Tens, from one to nine, *viz.* 10, 20, 30, 40, 50, 60, 70, 80, 90, respectively. Any Number of Hundreds they expressed by other Letters, supplying what was wanting by other Characters; and thus they

* Universal History.

† *Malcolm*'s History of Arithmetic, in his Arithmetic.

they proceeded, using the same Letters with some other Marks to represent Thousands, Tens of Thousands, &c.

* From the *Greeks*, Arithmetic was handed to the *Romans*. Which Observation naturally brings us to the third Age of Arts and Sciences, which commenced at the Birth of our Saviour, and continued to the Destruction of the *Roman* Empire by the *Goths*, (about the Year 410.)

The third Age of Sciences.

† The *Roman* Method of Notation, as it is still in Use amongst us, we shall just describe: The Letters made Use of by the *Romans* for this Purpose were, I. V. X. L. C. D. M. The Letter I was to denote a Unit; V=5, X=10, L=50, C=100, D=500, M=1000. As they had no different Value for their Places, the intermediate Numbers were expressed by a Repetition of some of these Letters; the Letter, denoting the greatest Value, being generally placed to the left Hand: Thus II=2, III=3, &c. VI=5+1=6, VII=5+1+1=7, &c. XI=10+1=11, XII=10+1+1=12, &c. XX=10+10=20, XXX=10+10+10=30, &c. LX=50+10=60, LXVIII=50+10+5+1+1+1=68, &c. DX=500+10=510, DC=500+100=600, DCCLXXXVII= 500 + 100+100+50+10+10+10+5+1+1=787, &c.

But, to express themselves more compendiously, they sometimes wrote a Letter denoting a lesser Value before a greater, and in such Case their Difference is to be understood: Thus, IV=5—1=4, IX=10—1=9, IIX=10—1—1=8, XL=50—10=40, CD=500—100=400, &c. When a Number is expressed by more than two Letters, and Part of it is expressed by this last Method, it is proper to distinguish it from the Letters on the left Hand by a Comma, or Point; thus, for Instance, 149 may be thus expressed, C,XL,IX, instead of CXXXXVIIII. Again, 449 may be thus expressed, CD,XL,IX, instead of CCCCXXXXVIIII. They had also other Peculiarities in their Notation as follows. For D=500, they sometimes write IƆ; and the Addition of every Ɔ made a Number 10 Times as much; thus, IƆƆ=5000, IƆƆƆ=50000, &c. Also, for M=1000, they sometimes wrote CIƆ; and, by placing C and Ɔ on each Hand, they expressed 10 times as much as before. Thus CCIƆƆ=10000; CCCIƆƆƆ=100000. But, for expressing any Number of Thousands, they had yet a more compendious Way; *viz.* by making a Dash (—) over any Letter denoting a Number less than a Thousand; thus, $\overline{\mathrm{V}}$=5000, $\overline{\mathrm{VI}}$=6000, $\overline{\mathrm{X}}$=10000, $\overline{\mathrm{LXX}}$=70000, $\overline{\mathrm{C}}$=100000, and $\overline{\mathrm{M}}$= a thousand Thousands = a Million $\overline{\mathrm{MM}}$=2000000, &c. Hence it appears, that though some Numbers are more compendiously expressed, according to this (*Roman*) Method of Notation, than by the common Method; yet, they are but few, if compared with what are otherwise. And if we consider that there is not such a regular Progression in the Value of the simple

* Supplement to *Harris*'s Lexicon. † *Malcolm*'s Arithmetic.

simple Characters, &c. in the *Roman* Method as there is in the *Arabian*, it will plainly follow, that the *Roman* Method is not so convenient for Computations.

* A Sexagesimal Notation was invented in the second Century of *Christianity*, as is supposed, by *Claudius Ptolomeus*. In this Notation, every Unit was supposed to be divided into 60 equal Parts, and each of these Parts subdivided into 60 Parts, &c. and hence the Parts were called Sexagesimal Fractions; and, to make the Computation easier, the Progression in the whole Numbers was also Sexagesimal. From 1 to 59 was expressed in the common Way; then 60 was called a *Sexagena prima*, and was denoted by 1 with a Dash over it; thus, $\acute{1}=60$; $\acute{I}\acute{I}=$twice $60=120$, &c. Sixty-times 60 or a *Sexagena secunda* was thus expressed, $\overset{\prime\prime}{I}$, &c. In Sexagesimal Fractions, Numerators less than 60 were expressed by the proper Letters, and their Denominators by one or more Dashes, set either over the Numerator on the left Hand, or under it on the right Hand; thus, the Fraction $\frac{10}{60}$ was wrote thus $\acute{X}$ or $X_{,}$. For the readier Performing of Multiplication and Division, the following Tables were made Use of:

$$1\times\text{by}\left\{\begin{array}{c}1\\2\\3\\4\\\text{\&c.}\end{array}\right\}=\left\{\begin{array}{c}0\cdot1\\0\cdot2\\0\cdot3\\0\cdot4\\\text{\&c.}\end{array}\right\}\quad 5\times\left\{\begin{array}{c}5\\6\\7\\8\\\text{\&c.}\end{array}\right\}=\left\{\begin{array}{c}0\cdot25\\0\cdot30\\0\cdot35\\0\cdot40\\\text{\&c.}\end{array}\right\}$$

$$10\times\text{by}\left\{\begin{array}{c}10\\11\\12\\13\\\text{\&c.}\end{array}\right\}=\left\{\begin{array}{c}1\cdot40\\1\cdot50\\2\cdot00\\2\cdot10\\\text{\&c.}\end{array}\right\}\quad 11\times\left\{\begin{array}{c}11\\12\\13\\14\\\text{\&c.}\end{array}\right\}=\left\{\begin{array}{c}2\cdot1\\2\cdot12\\2\cdot23\\2\cdot34\\\text{\&c.}\end{array}\right\}$$

$$30\times\text{by}\left\{\begin{array}{c}30\\31\\32\\33\\\text{\&c.}\end{array}\right\}=\left\{\begin{array}{c}15\cdot00\\15\cdot30\\16\cdot00\\16\cdot30\\\text{\&c.}\end{array}\right\}\quad 50\times\left\{\begin{array}{c}50\\51\\52\\53\\\text{\&c.}\end{array}\right\}=\left\{\begin{array}{c}41\cdot40\\42\cdot30\\43\cdot20\\44\cdot10\\\text{\&c.}\end{array}\right\}$$

And continued after this Manner so far as 60 by 60 =60.00. Then, for knowing the Names of the Product, the following Table was used:

$$\text{Integers}\times\text{by}\left\{\begin{array}{l}\text{Primes}\\\text{Seconds}\\\text{Thirds}\\\text{Fourths}\\\text{\&c.}\end{array}\right\}\text{give}\left\{\begin{array}{l}\text{Primes}\\\text{Seconds}\\\text{Thirds}\\\text{Fourths}\\\text{\&c.}\end{array}\right\}$$

$$\text{Primes}\times\left\{\begin{array}{l}\text{Primes}\\\text{Seconds}\\\text{Thirds}\\\text{Fourths}\\\text{\&c.}\end{array}\right\}\text{give}\left\{\begin{array}{l}\text{Seconds}\\\text{Thirds}\\\text{Fourths}\\\text{Fifths}\\\text{\&c.}\end{array}\right\}$$

Seconds

* *Harris*'s Lexicon Technicum.

Seconds into	Seconds		give	Fourths
	Thirds			Fifths
	Fourths			Sixths
	Fifths			Sevenths
	&c.			*&c.*

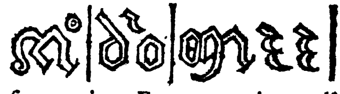
But this Method of Multiplying, *&c.* even if we have such Tables at hand, is so very troublesome and perplexed, that it is no Wonder to find, that, as the *Arabian* Notation gained, the Sexagesimal lost Ground; and is now entirely out of Use. The *Sexagenæ Integrorum* went first out of Use; but the Sexagesimal Fractions were continued until the Invention of Decimals.

* We cannot find the *Romans* made any great Improvement in Arithmetic.

† By the *Moors* Arithmetic was brought into *Spain*; from *Spain* it was carried into several other Parts of *Europe* by several learned Men, who went there to study the *Arabic* Learning, (and even the *Greek* Learning from the *Arabic* Versions, before they got the Originals themselves) imported by the *Saracens*.

During the long Period of about 1100 Years, from the End of the third Age of Arts and Sciences to the Commencement of the fourth, almost all *Europe* lay in a State of Slavery, attended with the Ravages of Superstition and Ignorance. In such a State, the very Principles of Science must be almost buried in Oblivion; however, this is the most ancient State, in which we can trace Arithmetic in *Europe*. Dr. *Wallis* has shewn by good Authorities, that it was known in *Europe* before the Year 1000. He has particularly shewn, that *Gilbertus* a Monk, afterwards known by the Name of Pope *Silvester* the Second, (who died in the Year 1003) was acquainted with this Art, and carried it from *Spain* into *France* long before his Death. He has also shewn, that it was known in *Britain* before the Year 1150, and far advanced in common Use before the Year 1250; as is evident by the Arithmetic of *Joannes de Sacro Bosco*, who died about the Year 1256.

As to the Antiquity of Numeral Figures, the said Dr. *Wallis* thinks ‡ their Use in *Europe* was as old at least as the Time of *Hermanus Contractus*, (who lived about the Middle of the eleventh Century) and, if not frequent in ordinary Affairs, yet at least in Mathematical Things, and especially in Astronomical Tables. He gives us an Account of a Mantle-tree of the Parlour-chimney at the Dwelling-House of Mr. *William Richards*, Rector of *Helmdon* in *Northamptonshire*, with this Date, (*viz. Ao. Doi. Mo.* 133.)



Both the Letters and Figures are of an antique Form, agreeing well enough with that Age; hence the Dr. concludes, that the Use of such Figures here, (in *England*) even on ordinary Occasions, is at least

* Supplement to *Harris*'s Lexicon. † *Malcolm*'s History of Arithmetic.
‡ Philosophical Transactions, No. 154.

least as ancient as the Year 1133; and judges it to have been somewhat more ancient, because the Shape of the Figures, though not come just to the Shape we now use, was even then considerably varied from the *Arabic*; which argues, that they had then been some Time in Use; such Change of Shape in Figures and Letters coming on gradually with Time.

That the *Arabic* Numerals were in common Use in *England*, about this Time, may be also supported by the Date on a Chalice in the Church of *Welch Bicknor* in *Monmouthshire*, which, Mr. *Green* says *, is 1176; which, by the Make of the Vessel and Mode of the Figures, seems to be genuine.

The fourth Age of Arts and Sciences begins with the Art of Printing; that is, about the Middle of the fifteenth Century; and continues to the present Time.

Now, we are come to that happy Age of Learning, (and may it long continue through the Blessing of God, and Encouragement of great Men!) in which not a Century, nor a single Year, passes, without a Discovery of something of Moment, some great Improvement in the Sciences.

To particularize the Discoveries and Improvements of this Age would require a Volume; we shall, therefore, only take Notice, in this Place, that Decimal Arithmetic was invented about the Year 1550. The first that used Decimals, in extracting the Square and Cube Roots, was (if we are not mistaken) our Countryman *Bucklæus*; but the first who writ an express Treatise of Decimals was *Simon Stevinus*, about the Year 1585. As to Circulating Decimals and Loggarithims, we shall give their History hereafter; and shall only here add, for the Sake of the Curious, a List of Arithmetical Writers:

Psellus wrote in the Year 1008. In the Year 1503. *Nemorarius* —1504. *Carolus*—1513. *Blasius*—1514. *Boethius, Siliceus*—1515. *Lax*—1520. *Suisset*—1522. *Tonstal*—1526. *Cirvello*—1530. *Bradwarden*—1532. *Aventinus*—1536. *Morsianus, Peurbachius*—1537. *Andreas*—1539. *Bronchorst, Cardanus, Glareanus, Ringelbergius*—1540. *Scheubelius, Willichius*—1542. *Finæus*—1544. *Vulpius, Welpius, Caius, Elias*—1549. *Vicar*—1550. *Alberti, Anatolius, Flicker*—1552. *Postellus*—1553. *Faber*—1554. *Camerarius, Gerasenus, Stifelius*—1555. *Cuno, Ramus*—1556. *Nabod*—1557. *Archimedes*—1558. *Peletarius, Rhæticus*—1559. *Monson, Scalichius*—1560. *Barres*—1563. *Beda*—1564. *Thierfelders, Ulmans*—1565. *Euclid, Kaltenbrunners, Nesius, Stenius, Strigelius* ——— 1566. *Messen, Munnos, Pencerus, Segura*—1568. *Steinmet*—1573. *Beausardus, Kopffers*—1575. *Diophantus*—1576. *Lagne*—1577. *Hammelius, Hobelius, Saligniacus*—1580. *Riesens (Jacobus)*—1581. *Lonicerus*—1585. *Schrectenbergers, Stevinus*—1587. *Poppins*—1591. *Kundlers, Meurerus (Joan.) Xylander*—1592. *Piscator*—1593. *Suevius*—1595. *Helmreich*—1596. *Fiffeld, Snellius*—1598. *Schleupnerus*—1599. *Buteo*—1600. *Gletsmannus, Reinhards, Reymers, Schey, Sculken, Taf*—1601. *Goffens, Ursi*——1602. *Johnson*——1603. *Caraldus,*

* *Gentleman's Magazine* for *May*, 1756.

Clichtovæus, Daviden, Richters, Riesens (Isaac.) Romanus, Waserus, Wurstisius——1604. *Cortes, Frisius, Landus, Sectgerwick*——1605. *Dibvadius*—1606. *Chauvet, Hofflins, Ruinellus*——1607. *Clavius, Meurerus (Christ.)*——1609. *Barlaamus, Engelbertus, Henischius, Peckoldts, Wildgovels, Zaichetta*—1610. *Lossius, Noviomagus, Sotters*—1611. *Campanella, Herwart, Longomontanus, Metius, Resenius, Severinus*—1612. *Brandts, Kauffungers, Laurembergius, Wittekind*—1613. *Alstedius, Brunus, Jacobs, Latomus, Neudorffers, Wilhelmus*—1614. *Kraffton, Monhemius, Mullerus (Christ.) Neper*——1615. *a-Culen, Finckius, Grunowald, Lucius*——1616. *Daetrus, Rudolffs*——1617. *Faulhabern, Hainkelmans, Heern, Langius, Vitalis, Zonsen*——1618. *Bangus, Cappaus, Geigers, Lucas, Stephanus, Ursinus, Wirth*—1619 *Lantz, Olaus, Remmelinus, Strubius* — 1620. *Bevern, Cassini, Hennings, Kandler, Malapertuis, Micylus, Rossels, Van-Zesen*——1621. *Bechmanus, Lavus, Mulichs, Spikers*—1622. *Follinus*—1623 *Riesens (Adam.) Wagner*—1624. *Brigs, Buscherus, Cæsar, Kepler, Neufville, Katen*—1625, *Servin*—1627. *Veronensis*—1628. *Taccius, Ulacq*—1629. *Malleolus*—1630. *Mullinghausen*—1631. *Mullerus (Jacob.) Oughtred*—1633. *Row* or *Roe*—1635. *Bartschius, Kruger*—1636. *Crugerus*—1638. *Van-Schoten*—1640. *Chavin, Salmasius, Johnson*—1641. *Curtius* — 1644. *Bullialdus, Launay, Smirnæus*——1646. *Micrælius*——1647. *Schmidts, Trenchant*—— 1648. *Middendorpius, Renaldus*—1650. *Frommius, Meierus, Mengolus, Nottragelius, Weberus*—1652. *Pierantonius*—1653. *Meenenaer, Moller*——1655. *Tacquet*—1656. *Wallis, Willisford*——1658. *Dee, Gendre, Hoffman, Mellis, Record*——1660. *Behm, Leotaudus, Wingate*——1661. *Reyherus*—1662. *Strauchius*——1663. *Duke, Levera, Schottus*—1664. *Biermans*——1665. *Kircher* —— 1666. *Branker, Pajottus, Pell*——1667. *Clavel, Voigt* ——— 1668. *Baker, Philips, Tylkawski*———1669. *Beverege, Graffenriedt, Hodder, Jamblichus, Kersey, Zaragosa*—1670. *Brown, Jackson, Newton,*—1671. *Clark, Fontaine*—1673. *Morland, Severius, Tabing, Tassius*——1674. *Brasser, Gottignus, Mercator*—1676. *Forbes, Hugerus*—1680. *Ozanam, Tartaglia*———1687. *Jordaine, Moore*———1690. *Hawkins, Mandey, Pickering*———1693. *Leybourn*——— 1694. *Prestet* ——— 1696. *Chamberlain, Jeake* —1697. *Mose*——1700. *Ayres, Cocker, Cole, Heinlin, Sturmius, Treu*—1710. *Harris, Lydal, Royer, Ward, Wolfius* —— 1714. *Cunn*———1720. *Fuller, Gordon, Hatton, Jones, Sharpe*——1423. *Wells*———1725. *Claircombe* ——— 1728. *Chambers, Hayes, Hill*———1730. *Malcolm* — 1731. *Leadbetter, Hodgkin*— 1732. *Grey, Wilson*———1735. *Kirby, Martin, Shelley, Stephens*——1736. *Barrenie, Weston*——1737. *Gore, Stonehouse*——1740. *Fletcher, Webster* ——1742. *Mark* ——1743. *Holmes*———1744. *Fisher, Worley*———1745. *Chapple, Markham, Pardon*——1746. *Dilworth, Holliday*——1747. *Richards, Reward* —— 1749. *Lowe* —— 1751. *Smith*——1753. *Potter, Thorpe, Fenning*——1755. *Martin.*

Having thus given some Account of the Antiquity and Progress of Arithmetic, we shall now proceed to say something concerning its Usefulness; which at first Glance appears to be so great, as to be inestimable; for, without it, we could not so much as have a compleat

pleat Idea of Number, Weight, and Measure; and, without it, Traffic, *&c.* must immediately cease.

Plato held it in such Estimation, that he said, "Take away Arithmetic, which is the Art by which we come to the Knowledge of Weight and Measure, and all that remains is base and of no Estimation." It is an Introduction to all other Arts and Sciences, and, without some Knowledge first obtained of this, it would be impossible to make a Progress in any other.

Numbers are so much the Measure of every Thing that is valuable, that Sir *Richard Steele* thinks * "it is not possible to demonstrate the Success of any Action, or the Prudence of any Undertaking, without them."

The most ancient Method of Numbering was by the Fingers; to which *Solomon* is supposed to allude, when he says, *Wisdom cometh with Length of Days in her right Hand*; and *Solomon* speaks greatly in its Praise in another Place, where he breaks out, *Thou O Lord, hast disposed all Things, in Measure, Number, and Weight.*

Arithmetic will shew us the great Advantages that may redound to Science in General, by a happy Notation or Expression of our Thoughts; for it is owing to the *Arabian* Method of Notation (see Chap. II. Essay 1.) that the most complicated Operations are managed with so much Ease and Dispatch.

This Art will shew us the Conduct and Manner of the Mind, when employed in the Exercise of Invention; and the great Advantages derived from an artful Method of classing our Perceptions. For by considering Numbers as divided into Parts, (by the Method of Notation) we are able, easily and readily, to perform that by considering their respective Parts, which would perplex and confound the Mind, to consider their Wholes, without considering their Parts separately. This manifestly appears in the Operations of Addition, Subtraction, Multiplication, Division, *&c.* for, though it goes beyond the Limits of the Human Mind to find the Sum or Product of two very large Numbers, without considering their Parts separately, yet, by finding the Sum or Product of their several Parts, we, with very little Trouble to the Mind, easily and readily, find the Sum or Product of their Wholes: Since the Sum or Product of all their Parts must be equal to the Sum or Product of their Wholes.

It is now high Time, and necessary, to give some Account of the present Performance; for, as Arithmetic has been treated of by such a Multitude of Authors, perhaps some may think there is no Need of this, or any other new Treatise on this Subject; and, therefore, to send a Book into the World on a Subject that has gone through so many Hands as this hath, without some Introductory Account, is but little better than exposing it. I shall therefore first observe, that, supposing I could not make any Improvement in Arithmetic, yet, as my Intention is to publish a Course of the Mathematics,

* *Spectator*, N°. 174.

thematics, it would be neceſſary to have ſome Treatiſe of Arithmetic to render that Courſe compleat; and therefore this alone would be a ſufficient Reaſon for Writing on this Subject: But this it is hoped will be found to be the leaſt of my Reaſons in Vindication of theſe Eſſays.

Long before I had any Thoughts of publiſhing a Treatiſe of Arithmetic, I looked into ſeveral Arithmetical Writers for the Demonſtrations and Reaſons of all the Rules; but did not meet with any Book that contained them. The greateſt Part of the Treatiſes that are moſt eſteemed, are ſo very defective that they do not ſo much as contain the Demonſtration of a ſingle Rule. The beſt that came to my Sight were *Ward*'s Introduction to the Mathematics and *Kerſey* upon *Wingate*; but the Rules are few in Number that theſe have demonſtrated. In ſhort, it is notorious, that moſt Writers, * Teachers of Arithmetic, have too much neglected this valuable Part, without which a Perſon can only work by Rote, juſt as a Parrot is taught to prate; not having ſeen the Reaſon or Inveſtigation of the Rules he makes Uſe of. Hence ſuch a Perſon, being aſked how he knows the Rules to be true, can only anſwer ſuch an Author, or ſuch a Maſter told me ſo: How liable then is ſuch a one to be miſled and to make Uſe of falſe Rules? Farther, though the Method of teaching the Rules without Demonſtrations may ſerve the Purpoſe of the Idle, and the Worthleſs, who, if they can be taught ſome practical Rules, by which they may ſcramble through ſome Buſineſs or Poſt, have as much as they deſire, and more than they deſerve; yet how can the inquiſitive and rational Mind be ſatisfied, without ſeeing the Nature and Reaſon of Things? Certainly no Perſon, who has not taſted it, can be ſenſible of that exquiſite Pleaſure, (Pleaſure much more delightful than the greateſt Enjoyment of the Senſes) which ſuch Minds poſſeſs on any Diſcovery of the Nature and Reaſon of Things. It is this Taſte, this Curioſity, and Ability of enquiring into the Nature of Things, which is the principal Characteriſtic that diſtinguiſhes Man ſo much ſuperior to the Brute; for neither our Form or ſenſual Pleaſures are a ſufficient Diſtinction; for who can affirm that our ſenſual Pleaſures are greater than thoſe enjoyed by ſome of the brute Creati-

* Having, in the General Preface, ſaid ſomething on the Advantage of having a good Maſter, it may be of ſome Uſe to caution Parents to be careful in the Choice of a Preceptor for their Children; for there are Quacks in Mathematics as well as in Phyſic, and, as Mr. *Webſter* juſtly obſerves, in his Eſſay on Education, when a Man has tried all Shifts and ſtill failed, if he can but---compute the Minutes in a Year, or the Inches in a Mile, he ſets up for a Teacher of Arithmetic, and, by the Bait of low Prices, perhaps gathers a Number of Scholars, and, thus impoſing on the inconſiderate Parents, both robs them of their Money, and the more unhappy Children of their Time.

The Ignorance of ſo many Maſters is really the Fault of Parents, for what is ſaid in N°. 313 of Mr. *Addiſon*'s *Spectator* is ſtrictly true, *viz.* "We often ſee "20 Parents, who, though each expects his Son ſhould be made a Scholar, "are not contented all together to make it worth While for any Man of liberal "Education to take upon him the Care of their Inſtruction." The Conſequence of which is, that, "for Want of Encouragement in the Country, we have many "a promiſing Genius ſpoiled and abuſed in thoſe little Seminaries."

tion?

tion? And, as to the Form of our Bodies, how nearly do some of the Ape-kind approach unto it, particularly the *Oran-Otan*, which is very little different in Shape, &c. from the Human Species?

Upon these Considerations, I was determined, for my own Use and that of my Pupils, to attempt to demonstrate all the Rules of Practical Arithmetic; not having then any Thoughts of making them public, but now, for the Reasons mentioned in the General Preface, I have yielded to the Publicationof them.

The rough Draught of these Essays was almost finished, before I heard any Thing of Mr. *Malcolm*'s Arithmetic; in which, when I first heard its Character, I thought that ingenious Gentleman might have done what was my Intention; but, upon Perusing it, found it could not be any material Objection to the publishing these Essays, as may appear from the following Reasons.

1. Because this is designed as the first Volume of an intended Treatise of the Mathematics.

2. Though Mr. *Malcolm*'s is both an ingenious and laborious Treatise, and abounds with many Things not useful in Practical Arithmetic, and which more properly belong to Treatises of Algebra, yet there are several useful Rules omitted, particularly Single and Double Position; Rules, without which, many curious and useful Questions could not be easily solved by those Persons who do not understand Algebra; and I may venture to affirm, that Double Position may be of Use even to *Algebraists, in solving some affected and exponential Equations, &c. more expeditiously than the Method of Series.

3. Lastly, The Bulk will deter many from Perusing it; for very few Learners would be willing to study 526 Quarto Pages, before they arrive at the Golden Rule.

Hence, after all that hath been hitherto done by others, I flatter myself with Hopes, that a compendious Treatise of Practical Arithmetic, with so much of the Theory as is useful in Demonstrating the Practice, will meet with a favourable Reception; having shewn to all impartial Judges, that the great Number of Writers on this Subject cannot be any great Objection to the Publishing of these Essays.

As to the Method made Use of in treating these Essays, the Reader will first mind the necessary Definitions and Axioms; then the Rule with its Illustration by Examples, and its Demonstration generally in a Note. And, being of Opinion that it is best to treat every Subject distinct and entire by itself, to prevent embarrassing the Learner, he will find the Doctrine of Vulgar Fractions at the End of Common Arithmetic; and, for the same Reason, Decimal Arithmetic is treated of in a separate Essay. The Demonstrations

* Notwithstanding the Assertion of some ingenious Authors, who, having acquired some Knowledge in the Algebraic Art, entirely contemn and slight this Rule, and say it is only useful to such as have no Acquaintance with Algebra, and fit only for the Ignorant and Indolent; yet, had they been well acquainted with the Application of the Method of Trial and Errors (as some call this Rule) they would have been convinced, that it may be of great Use even to the most expert Algebraists.

are Algebraical, because that Method is both the easiest, and shortest; and, without it, many Things could not be well demonstrated at all; and the Algebra, here given, will not only serve for this Purpose, but also be an useful Introduction to the Algebraic Art; which will be more fully treated of in its proper Place. The Algebraical Demonstrations are given by Way of Notes, because, if the Learner hath not already acquired some Knowledge in the Practical Part of Arithmetic, his best Way of Studying will be to get first some Acquaintance with that Part, omitting the Algebraical Part in his first Reading; and then, in his second Reading, to take in the Algebraic Part as it occurs.

In the Course of this Work there are several Questions in Verse, taken from the Annual Diaries, &c. Not on Account of the Accurateness of the Versification, (for in that many are deficient) but for these two Reasons, 1. That no Solutions, at least by Arithmetic only, have been published of them; 2. That they have been found by Experience to contain a kind of Charm, to entice young Students to solve them.

For a more particular Account of these Essays, the Reader is desired to look into the Book itself; for I shall not detain him here, with any Thing more in its Favour, well knowing that, if it has not Merit sufficient to recommend itself, all that I could here say would be of no Service. Therefore, such as it is, I submit it to the Examination of all impartial Judges and Lovers of Truth; desiring them either to pass by, or candidly acquaint me with, what they find amiss; for I shall not be ashamed to retract, or alter, whatever shall appear to me to be not consistent with the Nature and Reason of Things.

BENJAMIN DONN.

☞ As the Author at first had an Inclination to publish these Essays by Subscription, he thinks himself obligated to return Thanks to the Gentlemen who offered to encourage that Design.

ADVERTISEMENT.

IN the General Preface, we have hinted, that, if these Essays meet with a favourable Reception from the Public, we intend to attempt the other Parts of the Mathematics, as a new and compendious Course of Mathematical Learning is, in our Opinion, a Thing much wanted: And of the same Opinion is a learned Correspondent, as will appear by the following Extract of a Letter to the Author:

" WHITBY, *Feb.* 5, 1757.

" SIR,

" I——sincerely wish that what you are now printing may " answer your Expectation, and meet with the Approbation " of the Public. As soon as I see your Essays advertised, I intend " to be one of the Purchasers. I have long thought a compleat " System of Mathematical Learning to be a Thing much wanted, " and which, if well executed, could not fail of meeting with " proper Encouragement——As for *Oughtred, Ozanam, Leybourn*, " and some others, which were looked upon in their Age as good " Things, they are now grown quite obsolete, and are hardly " worth the Notice of our present Mathematicians, any more than " the voluminous Performances we meet with in *Harris, Chambers*, " and other Dictionary Writers.

" *I am, with Esteem,*

" *SIR,*

" *Your most humble Servant,*

" LIONEL CHARLTON."

Of the *ERRATA*.

In Works of this Nature it is impossible but Errata will happen: but, if they are more in Number than some might expect, the candid Reader will attribute them to the Author's being, at the Time of Printing this Volume, about 200 Miles distant from the Press. Note, The Letter *b*. added after the Number of the Line signifies to count from the Bottom.

Page 8 l. 11 *b*. for *having* r. *have*; p. 27, in the Operation in Art. 69. the Line between the Numbers 115 and 23 should be between 23 and 34500000; p. 32 in the Margin, for 8350768, r. 88350768; p. 34 l. 11, for *unlikely* r. *likely*; p. 40 l. 21, for $b \times = n$, r. $b \times c = n$; p. 60 l. 20, for 1 *remaining* r. 41 *remaining*; p. 61 l. 20, for *in* r. *into*; p. 64 l. 8, for *Duantities* r. *Quantities*; ibid. l. 9, for *a*, *b*, *c*, *&c.* = *the Divisor* r. $a \times b \times c$ *&c.* = *the Divisor*; ibid. l. 11, for *Qnext uote* r. *next Quote*; ibid. l. 12, after *by* r. *dividing*; p. 66 in the Note l. 5 *b*. for *bt* r. *b*; p. 96 l. 4 *b*, for *answered* 4 *l*. r. *answered* 2 *l*; p. 98, l. last, for 167 r. 163; p. 104, in the two last Lines, for 10, *oz* r. 1, *cz*; p. 112 in Question second, for 8 r. 5; p. 122 l. 20 *b*. for *here* r. *there*; p. 133. Art. 226, for *less than* r. *less*; p. 153, the marginal References should be * 306 and † 305; p. 156 l. 7, for 19 r. 9; p. 157 l. 8 *b*. for 293 r. 312; p. 158 l. 12, for *of a Bill* r. *off a Bill*; p. 159, l. 11, for 299 r. 318; ibid. in the marginal References, for 293 r. 312, and for † 296 r. † 315; p. 161 l. 12, for : *Men* r. : 40 *Men*; p. 164, for *Month* and *Months* r. *Week* and *Weeks* respectively; p. 174 l. 2, for 589 r. 598; p. 177 l. 10, for 10*s*. × 10*s*. r. 10*s*. + 10*s*.; p. 179 l. 23 for 349 r. 87, and for $\frac{69}{42}$ r. $5\frac{1}{28}$; p. 185 l. 20 *b*. for *Rule* r. *Rate*; p. 187 l. 17, for *in that* r. *in* $\frac{1}{2}$ *that*; ibid. l. 18, for *in that* r. *in* $\frac{1}{3}$ *that*; p. 191 l. 19. between *Flemish* and 19 place :; p. 197 l. 2. of Art. 405, blot out *not*; p. 201, Example 2, for 6 *d. per lb* r. 10 *d. per lb*; p. 203. Art. 418, the Numbers 48 and 72 should be linked, and not 40 and 48; p. 204 l. 10 *b*. for *intended* r. *intend*; p. 205 in the first Stating, for — r. :; ibid. l. 4 *b*. for *Alligat Aionlternate* r. *Alligation Alternate*; p. 207 l. 14, for *last* r. *least*; p. 233 l. 21, for *either* 1 *or* r. *neither* 1 *nor*; p. 235 l. 10 *b*. for b r. b^2; p. 237 l. 9, for 8 r. b^2; ibid. l. last, for $\overline{2rx + x} \times x$ r. $\overline{2r + x} \times x$; p. 240, for $\overline{a + b^2}$ r. $\overline{a + b}^2$; ibid. l. 3 *b*. for $n - r$ r. $n - r^2$; p. 240, at the Beginning of l. 12 *b*. prefix the Asterism *; p. 247, at the End of Art. 485, write *and the lesser Root*; ibid. at the End of the Note for *Squares* r. *Cubes*; p. 265 l. 14 *b*. for *Profit* r. *Proof*; p. 276 l. 5 for *supposed* r. *suppose*. p. 294 l. 18, for *Eyes* r. *Ears*; p. 300 l. 5, for $\frac{1}{3}$ r. $\frac{3}{1}$; p. 303 l. 8. of the Note, after *Quote* write *e*; p. 308 l. 7, for *Abbreviation* r. *abbreviated*; ibid. l. 15 and 26, for × r. +; p. 324 l. 4, for 1 *D*, o's × 1 *with d*, o's r. 1 *with D*, o's + 1 *with d*, o's; p. 348 in the last Line of the Note, for $\frac{bc}{a} = \frac{b}{a} \times c$ r. $\frac{bc}{a} = \frac{b}{a} \times c$; p. 355 in Note l. 2 *b*. for *c* r. *d*; p. 356 at the End of the Note, for *b being* = r. *being* = *b*.

The

The Tables may be thus corrected.

In Table 11. p. 108, for 525940 r. 525948. In the Table of Coins, *Seville* Piece of Eight 43 *d.* 11; Ducatoon of *Holland* Weight 20.21; Gulden of *Zell* 27 *d.* 7; old *Italian* Pistole, Standard Weight 4.6.11;

The Table of Squares.

R.	Square	R.	Square	R.	Square
181	32761	394	155236	663	439569
194	37636	519	269361	899	808201
391	152881	570	324900	923	851929

Table of Cubes.

R.	Cube	R.	Cube.	R.	Cube
97	912673	352	43614208	651	275894451
205	8615125	476	107850176	685	321419125
208	8998912	522	142236648	711	359425431
293	25153757	586	201230056	840	592704000
				844	601211584

Table of Primes.

For 9069 r. 9067; for 9541, r. 9547.

Direction to the Binder.

The Table of Coins to face Page 192.

A

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ESSAY I.

On Vulgar Arithmetick.

CHAP. I.

Of Definitions, &c.

1. MATHEMATICKS (μαθηματικὴ) originally ſignified any Diſcipline, or Learning (μάθησις); but now is the Science, or rather thoſe Sciences, which enable us to conſider the various Relations and Properties of Quantity.

2. *Mathematicks* may be conſidered as conſiſting of two Parts, *viz.*

1. *Pure*, *ſimple*, or *abſtracted*, which treats of Quantity conſidered in the Abſtract, *i. e.* without any Relation to Bodies, or ſenſible Objects.

2. *Mixt Mathematicks* conſider Quantity as ſubſiſting in material Beings. This is a very extenſive Part of the Mathematicks.

3. *Mathematicks* are alſo divided (by ſome Authors) into Speculative and Practical.

Speculative treats only of the ſimple Knowledge of the Thing propoſed, contenting itſelf with the bare Diſcovery of Falſhood, or the Inveſtigation of Truth.

Practical ſhews ſomething uſeful, or advantageous to Mankind.

4. *Quantity*, (*Quantité*, Fr. *Quantitas*, Lat.) may be defined to be whatever admits of any Kind of Computation, or Menſuration; or which, by Compariſon, may be ſaid to be greater, or leſs, than another Thing of the ſame Nature.

5. *Unity*, (*Unitas*, Lat.) is the Idea we have of any Thing conſidered as alone and undivided; or as ſeveral Things collected into one Whole; (not taking Notice of any Differences that may happen to be amongſt them in other Reſpects;) as one Pen, one Ship, one Flock, &c. in Contradiſtinction to a Multitude, or many, as Pens, Ships, Flocks, &c.

6. A * *Number* (*Nombre*, Fr.) is either an Unit, or two or more Units collected together.

It is by Numbers that we explain how many the Objects of our Knowledge are, as how many Men, Houſes, Ships, &c.

Numbers are divided into three Kinds, Integers, Fractions, and Surds. An Integer is that which an Unit meaſures; a Fraction is meaſured by a Submultiple of the Integer; a Surd is incommenſurable by Unity. (In writing Numbers we make Uſe of theſe *Arabian* Characters, 0, nothing; 1, one; 2, two; 3, three; 4, four; 5, five; 6, ſix; 7, ſeven; 8, eight; 9, nine.) Thus 3, when conſidered as ſo many diſtinct Things, as 3 Farthings, (3) is an Integer, or whole Number, and is meaſurable by an Unit; for 1 is contained exactly 3 Times in 3: But when 3 Farthings are conſidered as Part of a Penny, as $\frac{3}{4}$, (three

* It has been diſputed, whether an Unit be a Number, by ſome ingenious Writers; however, it is evident, that, the Definitions of Terms being arbitrary, it may be defined, ſo that an Unit may, or may not be a Number.

(three Fourths, or 3 Parts of four) then it (*viz.* $\frac{3}{4}$) is a fractional Number; and is measurable by a Sub-multiple or Part of the Integer, *viz.* by $\frac{1}{4}$, one Fourth of the Integer; for one Fourth is contained in three Fourths, exactly three Times. As to a surd Number, it cannot be well explained in this Place; and therefore we shall refer the Reader to EVOLUTION in this Essay; and for the Definitions of Multiple and Sub-multiple, to MULTIPLICATION and DIVISION respectively.

7. *Arithmetick*, (ἀριθμὸς and μετρέω) is that Part of the Mathematicks which teaches some of the chief Properties of Numbers, with the Methods of applying them to Computation in the common Purposes of Life; such, for Instance, is computing the Value or Price of any Parcel of Goods, either bought or sold. There are several Kinds of Arithmetick, as Vulgar, Decimal, &c. but in this Essay we shall only illustrate and demonstrate the Elements of Vulgar Arithmetick; so called from its being that which is most commonly used and best understood by the Vulgar, or common People.

CHAP. II.

Of NOTATION.

8. BY *Notation*, (*Notatio*, Lat.) or *Numeration*, (*Numeration*, Fr.) we understand the Art of reading and writing Numbers. That this may be regularly treated of, the first Thing to be done is to explain the Characters we intend to make Use of; which are as follows:

∴, *Therefore.*

+, is the Sign, or Mark for (*More*, or) *Plus*, or *And*.

−, the Sign for (*Less*, or) *Minus*.

 ×, the

×, the Sign of *Multiplication*, as 2×3, is, 2 multiplied by 3.

÷, the Sign of *Division*, as 4÷2, is, 4 divided by 2.

=, *equal* to, or the Sign of *Equality*.

We have already taken Notice, that 1, is the Mark for 1 or an Unit; 2, two; 3, three; 4, four; 5, five; 6, six; 7, seven; 8, eight; 9, nine; and 0, for nothing: Now, the better to explain what we mean by these Names, it may be proper to observe, that

$1+1=2$, is called, in our Language, Two.
$1+1+1=3$, Three.
$1+1+1+1=4$, Four.
$1+1+1+1+1=5$, Five.
$1+1+1+1+1+1=6$, Six.
$1+1+1+1+1+1+1=7$, Seven.
$1+1+1+1+1+1+1+1=8$, Eight.
$1+1+1+1+1+1+1+1+1=9$, Nine.

Or thus, $1+1=2$, $2+1=3$, $3+1=4$, $4+1=5$, $5+1=6$, $6+1=7$, $7+1=8$, $8+1=9$, and $9+1$ is called Ten; and ten Tens a Hundred, ten Hundreds a Thousand, a thousand Thousands a Million, a Million of Millions a Billion, a Million of Billions a Trillion, a Million of Trillions a Quadrillion, *&c.* The intermediate Numbers are thus named: Ten and one are called Eleven; ten and two, Twelve; ten and three, Thirteen; ten and four, Fourteen; ten and five, Fifteen; ten and six, Sixteen; ten and seven, Seventeen; ten and eight, Eighteen; ten and nine, Nineteen; ten and ten, Twenty: Then twenty and one we call Twenty-one; twenty and two, Twenty-two, *&c.* till we come to twenty and ten, which we call Thirty; the next ten, Forty; the next, Fifty; the next, Sixty; seven tens, Seventy; eight tens, Eighty; nine tens, Ninety; ten tens, a Hundred: Then we say on a hundred and one, a hundred and two, *&c.*

9. For

9. For the readier Writing and Reading Numbers, it is now, through Cuſtom, an eſtabliſhed Rule, that, when a Number conſiſts of ſeveral Places, the Figure, which is in the firſt Place, on the Right-hand, ſhall denote (as alſo when it ſtands alone) only its ſimple Value: But the Figure in the ſecond Place, counting from the Right-hand towards the Left, ſhall be ſo many Tens as it would be Units in the firſt Place; and ſo onwards to the Left, always ten Times as much as the ſame Figure would denote in the preceding Place. Thus, for Example Sake, in this Sum 3333, three in the firſt Place on the Right-hand is only 3; 3 in the ſecond Place towards the Left-hand is 3 Tens, or Thirty; the 3 in the third Place is ten Thirties, or three Hundred; the 3 in the fourth Place is ten three Hundreds, or 3 Thouſands: Hence that Sum may be read thus: Three Thouſand, 3 Hundred, and Thirty-three.

10. But having, many Times, Occaſion to write down ſo many Hundreds, Thouſands, &c. without any intermediate Terms, it is neceſſary, to prevent Confuſion, to fill up the vacant Places with ſome Character, which, in itſelf, ſhould ſignify nothing; and hence appears the Uſe of the Cipher (0) nothing; thus *E. G.* we may write three Hundred thus 300; for, by Virtue of the two Ciphers being placed to the Right-hand of the 3, it ſtands in the third Place; and therefore, by the laſt Article, denotes three Hundred. Thus alſo 30, is Thirty, and 30000, thirty Thouſand; and hence it will appear that 35071 is thirty-five Thouſand and Seventy one; becauſe the 1 ſtanding in the firſt or Unit's Place is ſimply 1; the 7 ſtanding in the ſecond Place is Seventy; and the 5 being in the fourth Place is five Thouſand: Laſtly, the 3 being in the fifth Place is thirty Thouſand, and conſequently the whole Number is thirty-five Thonſand and Seventy-one. Hence it will be no difficult Matter to underſtand the following Table.

1	Units	1
12	Tens	10
123	Hundreds	100
1234	Thousands	1000
12345	X Thousands	10000
123456	C Thousands	100000
1234567	Millions	1000000
12345678	X Millions	10000000
123456789	C Millions	100000000
	&c.	

11. The Learner being supposed to understand what has been already said, we shall now shew him how to read a very large Number, *e. g.* 614.321631. 543261. 701810. 718432. 171816. 743215. 407184. 321718.765671. The Method is thus: Over the seventh Figure, counting from the Right-hand toward the Left, put 1; from which count six, and over it put 2, *&c.* as in the above Number: Then the Figure over which 1 stands is Millions, that over which 2 is placed is Millions of Millions or Billions, that over which 3 stands is Millions of Millions of Millions, or Trillions, *&c.* Hence the above Number may be read thus, 614 Nonillions, 321631 Octillions, 543261 Septillions, 701810 Sexillions, 718432 Quinquillions, 171816 Quadrillions, 743215 Trillions, 407184 Billions, 321718 Millions, 765671. After this Manner we may numerate any Number, though the Number of Places be very great.

12. The Writing down of Numbers being only the Reverse of Reading them, when written, we shall not here expatiate on; (especially as we suppose our Readers to have some Acquaintance with more Rules than this;) but shall only take Notice, that Youths are sometimes nonplussed when such Questions as the following are proposed to them, *viz.* To write down in Figures 11 Thousand, 11 Hundred, and 11; for

Want

Want of considering that 11 Hundred is 1 Thousand and 1 Hundred: But give them this Hint, and they will then readily know, that the above-mentioned Number is equal to 12 Thousand, 1 Hundred, and 11; and then they will presently write 12111.

13. When we are to demonstrate the Truth of a Theorem that is not limited to particular Numbers, but only, that, on such Conditions as are therein expressed, it will be as in the Theorem; it will not be a sufficient Proof of the Truth of the Rule, to shew that it holds good in some particular Numbers; and therefore we must make Use of some Contrivance, by the Help of which we may be able to argue abstractedly from particular Numbers. In order to which, it is many Times the best Method to put some Letters, as a, b, c, &c. to represent the Numbers; and then, as these Letters may stand for any particular Numbers, whatever is deduced from Reasoning on these Letters, with the Properties common to all Numbers, must be true in all possible Cases; and hence the Name of *Literal* or *Universal Notation*. This Method of putting Letters for Numbers is also called *Algebraical Notation*. We have already said, that $+$ is the Mark for Plus; thus $2+3$ is 2 Plus 3; and $a+b$ is, a Plus b; also that — is Minus, and therefore $a-b$ is a Minus b; and $=$ is equal to, as $a=b$ is a equal to b. The Signs of Comparison are $\sqsubset$ and $\sqsupset$; the Mark for *less than* being $\sqsubset$, and for *greater than* $\sqsupset$; thus $2 \sqsubset 3$ is to be read, 2 is *less than* 3; and $3 \sqsupset 2$ is, 3 is *greater than* 2. The Sign of the Difference of two Quantities is $\backsim$, which is made Use of when we do not know which is the greatest of the Quantities; thus $a \backsim b$ is the Difference between a and b; that is, if a be the greater Quantity, it stands for $a-b$; but, if b be the greater Quantity, it is $b-a$. What other Characters we may occasionally make Use of, will be explained in their proper Places.

14. *Quantities* may be considered either as affirmative, that is, greater than nothing, or negative,

viz. leſs than nothing: Thus the Caſh and Goods of a Merchant, and the Debts owing to him, may be conſidered as affirmative, or greater than nothing; becauſe they affirm he has ſo much. As to negative Quantities, though to ſay that any Quantity is leſs than nothing is a Contradiction, yet a Quantity may be ſo circumſtantiated with reſpect to ſome Thing, or Perſon, as to be in effect (with reſpect to that Thing, or Perſon) as leſs than nothing: Thus *e. g.* the Debts a Merchant owes, are, with reſpect to him, Negative Quantities; for they are, with reſpect to him, really ſo much leſs than nothing; denying him to be worth by ſo much as the Debts are, what his Goods, Caſh, and Debts owing to him would make him to be.

15. In *Algebra* + is the affirmative Sign, and therefore all Quantities which have this Sign before them, are to be underſtood as affirmative Quantities: When there are many Quantities written, the firſt, or leading Quantity, has frequently no Sign ſet before it; but then it is ſuppoſed to have this Sign before it, or to ſtand for an affirmative Quantity.

16. In the *Algebraick*, or *Analytick Art*, — before any Quantity denotes, that that Quantity is to be conſidered as a negative Quantity.

17. Before we put an End to this Chapter, it will not be improper to lay down the following *Corollaries*.

Corollary 1. If two Numbers (written in Figures) having the ſame Number of Places, that is of the greateſt Value, which has the greateſt Figure in the higheſt Place. As, for Example, $830 \sqsupset 590$; and $2360 \sqsupset 2359$.

18. *Coroll.* 2. Of two Numbers conſiſting of an unequal Number of Places, that is the greateſt which has the greateſt Number of Places, *e.g.* $101 \sqsupset 97$.

19. *Coroll.* 3. If to the Right-hand of any two unequal Numbers be placed an equal Number of Figures, that will remain the greateſt which was ſo before.

CHAP.

CHAP. III.

Of ADDITION.

20. ADDITION, (from *add*; from the Lat. *addo*) teaches to find a Number equal to two or more given Numbers, taken together; that is, to find one Number called the *Sum*, (*Summa*, Lat.) or *Total Sum*, which ſhall contain ſo many Units as are contained in all the given Numbers taken together.

21. *Poſtulate.* Grant that any Number may be increaſed by adding of another Number to it. See Art. 25.

22. *Axiom* 1. If equal Things be added to equal Things, the Sums will be equal.

23. *Axiom* 2. Such Quantities as are equal to one and the ſame, or equal Things, are equal to each other.

24. *Axiom* 3. All the Parts, taken together, are equal to the Whole.

25. It is evident that the Sum of any two Numbers may be found, by adding to one of the Numbers (the greateſt is the beſt) ſeparately, one by one, the Number of Units contained in the other Number.

Thus, for Example, the Sum of the two Digits 9 and 5 is equal to $9+1+1+1+1+1$, which may be collected together, by ſaying, $9+1=10$, $+1=11$, $+1=12$, $+1=13$, $+1=14$. But, ſince this would be a very tedious Method of adding large Numbers, we muſt ſeek out for a better; but firſt it will be neceſſary to make, by this Method, the following Table, expreſſing the Sum of any two Digits.

Index

Top Column.

Index Column.

	1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9	10
2	3	4	5	6	7	8	9	10	11
3	4	5	6	7	8	9	10	11	12
4	5	6	7	8	9	10	11	12	13
5	6	7	8	9	10	11	12	13	14
6	7	8	9	10	11	12	13	14	15
7	8	9	10	11	12	13	14	15	16
8	9	10	11	12	13	14	15	16	17
9	10	11	12	13	14	15	16	17	18
10	11	12	13	14	15	16	17	18	19

26. The Sum of any two Digits is found after this Manner by the above Table, *viz.* Always find one of the Numbers on the Left-hand of the Table, and the other on the Top, and the Number ſtanding in the Place, where the Rows meet, will be the Sum required. *E. g.* Suppoſe we wanted to know the Sum of 8+6; find 8 on the Left, and then againſt it, in the ſixth Column, we ſhall have 14, the required Number.

27. By committing the above-mentioned Table to Memory, we ſhall readily know the Sum of any two Digits; and then be qualified to make Uſe of a much better Way of adding large Numbers, *viz.* Firſt, take Care to place the Numbers to be added, one under another; ſo, that Units may ſtand under Units, Tens under Tens, Hundreds under Hundreds, &c. Then add up the Row of Units; and if the Sum be more than Ten, or two Tens, &c. write down the Overplus, and carry the Tens to the next Row, as ſo many Units; which (Row) add up as you did the firſt, and carry the Tens of this Row, as ſo many Units, to the third, or Row of Hundreds; and thus proceed 'till all the Rows are added up; and then the Tens of the laſt Row muſt be placed

placed to the Left-hand of all the other Figures of the Sum, and then the Whole will be the *Total Sum.*

28. *Example.* What is the Sum of 5768+123 +879?

The Numbers being rightly placed will ſtand thus:

$$\left\{\begin{array}{r} 57\,6\,8. \\ 1.2.3. \\ 8\,7\,9 \\ \hline 67\,7\,0 \end{array}\right. = \text{Sum of the Whole.}$$

Operation. Add up the firſt Row, thus: Say (mentally) 9+3=12=(by Art. 9.) 2 above 10; make a Dot (.) for the 10, and ſay 2+8=10, for which make a Dot (.): Put down 0 under the firſt Row; and then, looking on the Sum, we ſee two Dots, ſignifying two Tens, to be carried to the ſecond Row: Therefore we ſay, 2 (we carry) +7=9, +2=11, = 10+1; for the 10 make a Dot; then ſay 1+6=7, which put down; then carrying 1 for the Dot, to the third Column, we ſay, 1 (we carry) +8=9, +1=10, for which make a Dot; then 7 is 7, which place down: Laſtly, in the fourth Row we ſay, 1 (the Carriage from the third Column) +5=6; which, being put down, compleats the whole Sum.

29. After the young Student has been ſome Time converſant in Addition, he will be able to add up, without dotting, more expeditiouſly, thus: In the Example, in the laſt Article, ſay 9+3=12, +8= 20; under the firſt Row put 0; and becauſe the 2 in 20 is 2 Tens, (or, which is the ſame, the 2 ſtands in the Place of Tens, by Art. 9.) the ſaid 2 muſt be carried to the ſecond Row, or Row of Tens; ſaying 2 (you carry) +7=9, +2=11, +6=17; (but this 17, being 17 Tens, is really 170) ∵ under the ſecond Row we muſt put 7, and carry the 1 (*viz.* 100) to the Row of Hundreds, ſaying 1 (you carry) +8 =9, +1=10, +7=17; (but this being the Sum of the Row of Hundreds, is really 1700) ∵ under the third Row we muſt write 7, and carry the 1 (*viz.* 1000) to the Row of Thouſands, ſaying 1 (you carry)

carry) $+5=6$; which being the Sum of the Row of Thousands, the 6 must be written in the fourth Place; which compleats the Sum, *viz.* 6770. From what has been said in this Article, the Reason of carrying the Tens of one Row, as Units to the next higher Row, must appear plain. Or the Reason of this Method of performing Addition may be shewn in a more general Manner, thus: It is plain, that, in an Operation worked by Article 27, we first add up the Row of Units, which Sum we put down in the Place of Units, if it doth not amount to 10; but if it be 10, or more, we only put down the Excess above 10, or a compleat Number of Tens, in the Place of Units, and carry the Tens to the next Row, and add up that, &c. Now it is evident, that as we put down the Excesses above any Number of Tens, and carry the Tens as so many Units to the next Left-hand Row, &c. to the End, that the Sum so found must be that required; because, that by the Nature of our Notation, 10 of 1 Row is 1 in the next Left-hand Row; and therefore, by that Method, we have taken all the Parts together, which must * be equal to the Whole.

* 24.

30. When the Columns, or Rows, to be added up, are very long, it will be proper to part them into several Parcels, so that the Sum of each Row of a Parcel may not exceed an Hundred, (or there be not more than 10 Figures in any Row or Parcel;) then, having found the Sum of each Parcel, add these Sums together for the Total Sum.

The Reason of this is grounded on Art. 24. The Example in the Margin will make this plain.

31. The best Way of proving Addition is by adding the Rows downward; by which, if they bring out the same as when added upward, we may reasonably conclude we are right.

231	
531	
718	
172	
232	
121	
317	
213	
———	25 34
710	
310	
728	
631	
841	
231	
978	
891	
701	
301	
———	6322
	———
Total Sum	8856
	———

32. As to *Literal*, or *Algebraical Addition*, all that is necessary to be observed here, is, that the Quantities are added by collecting them together with their proper Signs: Thus, x added to y is $x+y$; x and $-y$ is $x-y$; also $-x$ and $-y$ is $-x-y$; all this is plain from the bare Definition of Addition. We shall only further observe, that when the Quantities are alike, *i. e.* the Letters are all of one Sort, the Expression may be shortened, by adding them together after the same Manner, as in common Additions; so, $a+a=2a$; $3a$ and $5a=3a+5a=8a$; also $3x$ and $-2x=3x-2x=x$.

CHAP. IV.

Of SUBTRACTION.

33. SUBTRACTION, (*Subtractio*, from the Verb *subtraho*, Lat.) shews to take one Number from another, and is only the Reverse of Addition.

34. Hence,

34. Hence, the Number to be ſubtracted may be equal, but cannot be greater than, the Number from which it is to be taken.

35. Grant, that any Number may be leſſened, by taking a leſſer, or equal Number, from it. (See Art. 38.)

36. If from equal Things equal Things be taken away, the Remainders will be equal.

37. * The Number from which another is to be taken, we call the *Subducend* (from the *Latin* Verb *ſubduco*); the Number to be ſubtracted the *Minorand* (*Minor*, Lat.); and their Difference, the *Remainder* (from the Verb *remaneo*, Lat.)

38. Before Children can be taught to ſubtract one Number from another, they muſt be able to tell readily, without ſtudying, the Difference between any two Digits; for which Purpoſe the Table in Addition will be uſeful; for by it the Difference between any Digit and Number, leſs than 20, is given by Inſpection only; thus, find the Minorand-Digit in the Left-hand vertical Column, and look againſt it in the Table for the Subducend; then, directly over it, on the Top of the Table, will be the required Difference or Remainder. *Example.* Let it be required to find the Difference between 4 and 9. Here againſt 4 (in the Index Column) we find 9 in the Table, and directly over it, in the Top Column, is 5, their Difference.

39. To ſubtract one Number from another.

Having placed Units under Units, and Tens under Tens, &c. as we did in Addition; take each Digit (beginning with the Units Place) of the Minorand out of its correſpondent Digit of the Subducend, and put the Difference under; but if any Digit of the Minorand is greater than its correſpondent Digit of the Subducend, you may ſubtract that Digit

* What we here call the *Subducend,* ſome Authors call the *Compound Number*, others the *Subtrahend*; and what we call the *Minorand,* ſome call the *Subtrahend,* others the *Subtractor.*

Digit of the Minorand from 10, and to the Remainder add the correspondent Digit of the Subducend; but, in this Case, you must remember to add an Unit to the next Digit of the Minorand. *Note*, It is customary to put the Subducend over the Minorand, but this is entirely arbitrary.

40. *Example.* From 7540 subtract 2172.

Place the Numbers thus:	Subducend	7540
	Minorand	2172
	Remainder	5368

Then, since you cannot take 2 (the first Figure of the Minorand) from 0, (in the first, or Unit's Place of the Subducend;) say, 2 from 10, or $10-2=8$, which put down; then, adding 1 (you carry) to 7, say, 8 (from 4 you cannot, and $\therefore$) from 10, or $10-8=2$, $+4=6$, which write down; and proceed, saying, 1 (you carry) $+1=2$, and 2 from 5, or $5-2=3$, which put down: Lastly, 2 from 7, or $7-2=5$, which, being placed down, gives the Difference or Remainder $=5368$.

41. When the Tyro has been some Time exercised in the above Method, he may learn the following more commodious Way of performing Subtraction, *viz.* by adding 10 to any Digit of the Subducend, when the correspondent Digit of the Minorand is greater than that of the Subducend; and then, from that Sum, subtracting the correspondent Digit of the Minorand; thus, in the above Example, we may say, $10-2=8$, which set down; then 1 (we carry) $+7=8$, and $14-8=6$, which write down; then 1 (we carry) $+1=2$, and $5-2=3$, which place down: Lastly, $7-2=5$. Hence the Remainder is 5368 as before.

42. Mr. *Lowe* thinks Subtraction is better performed by Addition; thus, in the above Example, say 2 and as much as will make the Amount to the next Row is 8, which 8 put down; then 1 (we carry)

carry) $+7=8$, and $8+6=14$, put down 6; and say, 1 (we carry) $+1=2$, $+3=5$, put down 3; then $2+5=7$, put down the 5; which makes the Remainder 5368, as above.

43. A *Demonstration* of Art. 39. or Rule for finding the Difference of two Numbers; which may be conveniently parted into two Cases.

1. When all the Figures of the Minorand are less than (or equal to) their Correspondents in the Subducend, it is manifest, that the Difference of the Figures in the several Places, being put in the same Place as the Figures stand whose Differences they are, must, taken all together as one Number, be equal to the Difference sought; for, as all the Parts of any Number taken together are equal to the Whole, so must the Differences of all the like Parts of any two Numbers make up the Difference of the Wholes. * *Q. E. D.*

2. When any Figure of the Minorand is greater than its correspondent Figure in the Subducend, by the Rule in Art. 39. we subtract that Figure of the Minorand from 10, and add the Subducend Digit to that Remainder; or, which is the same, add 10 to the Figure in the Subducend, and take the correspondent Figure of the Minorand from that Sum; and then add 1 to the next higher Figure in the Minorand. Now, since by * Notation 10 in any Place * 9.
is $=1$ in the next higher Place, it is evident, we have increased both the Subducend and Minorand with an equal Number; and therefore the Difference of the Subducend and Minorand, so increased, will be the * 47. same * as if they were not increased. *Q. E. D.*

The only Thing that remains here to be taken Notice of, is, that, any Digit being taken from the Sum of any lesser Digit and 10, the Difference will never exceed 9; the Reason of which will plainly appear, by considering, that as the lesser Digit wants at

* Note, *Q. E. D.* amongst Mathematicians, signifies, *Quod erat demonstrandum* (which was to be demonstrated); and *Q. E. I.* is, *Quod erat inveniendum* (which was to be found).

at least 1 to make it = the greater; when we have taken all we can out of the lesser Digit, there must, at least, remain 1 (of the greater Digit) to be taken from the 10, and therefore the Remainder can never exceed 9.

44. When we are to subtract one Number from several, or several from one, or several from several, it is best, before we subtract, to shorten the Work, as much as may be, by Addition: Thus, if it be required to take 30+15+16, from 8+23+16+50, the best Method of Operation will be as under:

Add { 30		8	Subducend	97
15		23	Minorand	61
16		16		—
—		50	Remainder	36
Sum 61		—		—
—	Sum	97		
		—		

45. *Subtraction* may be proved by adding the Minorand and Remainder together; for their Sum must (by Art. 24.) be equal to the Subducend.

46. In *Literal* or *Algebraical Subtraction*, all that is necessary to be observed is, that, Subtraction being directly contrary to Addition, to subtract an Affirmative must be the same as to add a Negative; and to subtract a Negative, the same as to add an Affirmative: Hence, Subtraction of Algebra may be performed by changing (or supposing in your Mind) all the Signs of the Minorand (to be changed); and then adding them as in Addition. Thus, $2x$ from $5x$, remains $5x-2x=3x$; and $2x$ from $-5x$ remains $-2x$ and $-5x=-7x$; also $-2x$ from $+5x$ leaves $+2x+5x=+7x$; and $-2x$ from $-5x$ leaves $+2x-5x=-3x$.

47. Before we put an End to this Chapter, it may not be improper here to remark, that, in Art. 43, we took it for granted, that, when equal Quantities be added to both the Subducend and Minorand, the

Difference of these Sums will be equal to the Difference of the (former) Subducend and Minorand; which may be thus demonstrated:

Let $s=$ the Subducend, $m=$ the Minorand, $d=$ their Difference, or $s-m=d$; $a=$ the Number to be added; then we have, for a new Subducend, $s+a$, and for a new Minorand $m+a$. Now m from $s=s-m$, and a from $a=o$; $\because \overline{s+a}$ Minus $\overline{m+a}=s-m=d$, *per* above. *Q.E.D.*

Note, When a Dash (—) is drawn over any Quantities, it denotes that those Quantities are to be taken together; thus, in the above, by $\overline{s+a}$ Minus $\overline{m+a}$, is to be understood, that the Sum of m and a is to be taken from the Sum of s and a.

48. We will put an End to this Chapter, with the following *Axioms*, because they may, perhaps, be of some Use hereafter.

First, If from the Sum of any two Quantities be taken either of them, the Remainder will be equal to the other Quantity: Thus, if from the Sum of any two Quantities, $a+b$, be taken one of them, b, there must remain the other, a.

49. *Axiom* 2. If from the greater of any two Quantities their Difference be taken, the Remainder must be the lesser Quantity: Thus, if a denote any Quantity, and $a+d$ another; if from the greater $a+d$ be taken their Difference d, there will remain a, the lesser Quantity.

50. *Axiom* 3. If to the lesser of any two Quantities be added their Difference, the Sum will be the greater Quantity; Thus, if a denote any Quantity, and $a+d$ a greater Quantity, their Difference is d; and if to a, the lesser Quantity, we add d, their Difference, the Sum will be $a+d$, the greater Quantity.

CHAP. V.

Of MULTIPLICATION.

51. *MULTIPLICATION* (*Multiplicatio*, Lat.) is the Method of adding a Number, a given Number of Times, to itſelf; or, repeating it ſo many Times as the Number, by which you are to multiply, contains Units. Or, the finding a Number which ſhall contain any given Number, a given Number of Times.

52. The Number to be multiplied (or added a given Number of Times to itſelf) is called the *Multiplicand* (*Multiplicandus*, Lat.)

53. The Number by which we multiply, (*viz.* the Number which denotes how many Times the Number to be multiplied muſt be taken) is called the *Multiplier* (from *Multiply*, from *Multiplico*, Lat.)

54. The Sum of all theſe Additions, or the Number ſo repeated, is called by Arithmeticians the *Product* (*Productus*, Lat.) and by Geometricians the *Rectangle* (*Rectangle*, Fr. *Rectangulus*, Lat.) The Reaſon of which laſt Appellation will be explained in its proper Place.

55. A *Multiple* (*Multiplex*, Lat.) is a Number produced by the Multiplication of two other Numbers (each greater than an Unit.) Thus 6 is a Multiple of 2 and 3, for 3 twice repeated is $=6$. Or, in *Euclid*'s Words, a Multiple is a greater Number compared with a leſſer, when the leſſer meaſures the greater.

56. An *Axiom*. If equal Things be multiplied by equal Things, the Products will be equal.

57. Any two Digits may be multiplied together by adding the Multiplicand to itſelf, ſo many Times as there are Units in the Multiplier; for Inſtance, the Product of 5 by 4 may be found by repeating 5 four Times, *viz.* $5+5+5+5=20$. By this Method the following TABLE (called the *Multiplication*,

or *Pythagorean* Table, from the Inventor *Pythagoras*) may be made; and, when it is committed to Memory, we ſhall have a much ſhorter Method to perform Multiplication by. To find the Product of any two Digits by this Table, find one of the Digits in the Side of the Table, and the other on its Top, and in the Point of Meeting will be the Product, which was required.

The Pythagorean TABLE.

1	2	3	4	5	6	7	8	9	12
2	4	6	8	10	12	14	16	18	24
3	6	9	12	15	18	21	24	27	36
4	8	12	16	20	24	28	32	36	48
5	10	15	20	25	30	35	40	45	60
6	12	18	24	30	36	42	48	54	72
7	14	21	28	35	42	49	56	63	84
8	16	24	32	40	48	56	64	72	96
9	18	27	36	45	54	63	72	81	108

58. Having learnt the above Table perfectly by Heart, it will be eaſy to multiply any two Numbers together by this Rule, *viz.* Multiply the Figure in the Units Place of the Multiplicand by the firſt Figure (on the Right-hand) of the Multiplier, and put the Product, if it be leſs than 10, directly under; but, if it exceed 10, write down the Exceſs above any Number of Tens; and remember, after you have multiplied the ſecond Figure of the Multiplicand by the ſame Figure of the Multiplier, to add to the Product the Number of Tens which was to be carried from the laſt Multiplication, &c. till you have multiplied all the Figures of the Multiplicand by the firſt Figure of the Multiplier; then, (if the Multiplier conſiſts of more than one Figure) in like Manner multiply every Figure of the Multiplicand by the ſecond Figure of the Multiplier, and ſo proceed, till all the Figures in the Multiplicand

have

have been multiplied by each Figure of the Multiplier; then collecting all these Products together by Addition, we shall have the required Product.

59. *Example.* Multiply 23416 by 23.

The Work will stand thus; and was thus performed: First, we say 3 Times 6=18, put down the 8, and carry the 1 to the next Figure of the Multiplicand, saying 3 Times 1 is 3, + 1 (we carry) = 4, which write down; then 3 Times 4=12, put down the 2, and reserve the 1: Again, 3 Times 3=9, + 1 (we carry) = 10, put down the 0; and then say 3 Times 2 is 6, + 1 (we carry) = 7, which, being wrote down, compleats the Multiplication of the Multiplicand by the first Figure of the Multiplier. Now begin to multiply by the second Figure of the Multiplier, saying 2 Times 6 is 12; put down the 2 directly under that Figure of the Multiplier which we are multiplying by, and reserve the 1; say then, 2 Times 1 is 2, +1 (we carry) = 3, which put down; and thus proceed 'till each Figure in the Multiplicand has been multiplied by the last Figure of the Multiplier. Lastly, add the several Products together for the required Product.—As to the Reason of this Method of Multiplication, it may be easily shewn by Help of the following *Lemma*, viz. If either the Multiplicand, or Multiplier, or both, be divided into two or more Parts, and these Parts are multiplied by each other, the Products of all these Parts, when added together, must be equal to the Products of the Whole. (Thus, for Example Sake, if the Multiplicand be 23=20+3, and the Multiplier 14=10+4; then 20x10=200, and 3x10=30; also 20x4=80, and 3x4=12; now 200+30+80+12=322=23x14.) The Reason of which is evident, for all the Parts of any Number must make up that Number; ∴ when any Number, or all the Parts of any Number, are multiplied by all the Parts of any other Number, those two Numbers are multiplied together.

$$\begin{array}{r} 23416 \\ 23 \\ \hline 70248 \\ 46832 \\ \hline 538568 \\ \hline \end{array}$$

 The

The *Demonstration* may be conveniently parted into two Cases.

1. When the Multiplier is but one Figure, it is plain, that, by the Method delivered in Art. 58, we find the Product required; for, we multiply each Part of the Multiplicand by the Multiplier, and put the Product down in its proper Place, if less than 10, *viz.* in that Place which is directly under the Place we are multiplying of; but, if that Product is more than 10, we only put the Excess (above 10) in that Place, and carry the 10's to the next superior Place; for, since 10 in any Place is by Notation *=1 in the next higher Place, our carrying the 10's of any Place as Units to the next higher Place does in Effect collect together the similar Parts of the Products, and all these Products, collected together, are by the *Lemma* equal to the whole required Product. *Q.E.D.*

* 9.

2. But, if the Multiplier consists of more than one Figure, after we have found the Product of the Multiplicand by the first Figure of the Multiplier as above demonstrated, we suppose the Multiplier parted into Parts, and therefore find after the same Manner the Product of the Multiplicand by the second Figure of the Multiplier; but, as the Figure we are multiplying by stands in the Place of Tens, the Product must be ten Times its simple Value; and, therefore, the first Figure of this Product must be placed in the Place of Tens, or, which is the same Thing, directly under the Figure we are multiplying by; and, proceeding in this Manner separately with all the Figures of the Multiplier, it is evident we shall multiply the several Parts of the Multiplicand by those of the Multiplier, and therefore, by the *Lemma*, these several Products, being added together, will be equal to the Product of the Whole. *Q.E.D.*

60. In long Calculations the making a *Tariffa* or small Table of the Multiplicand, after the following Method, will be very useful; because it is not subject to Error, and performs the Whole by common Ad-

Addition; but in ſhort Multiplications the common Method is beſt.

Example. Multiply 25476 by 3468.

The Method of *Tabulating* or making a *Tariffa*, is thus: Make a Ladder of 10 Steps; then againſt the firſt Step ſet the Multiplicand; and againſt the ſecond put its Double, found by adding it to itſelf. The ſecond Step added to the firſt gives the third; and the third added to the firſt gives the fourth Step, &c. till you get 10 Steps, which laſt Step, if it be = 10 Times the firſt Step, proves the Table to be made right.

1	25476
2	50952
3	76428
4	101904
5	127380
6	152856
7	178332
8	203808
9	229284
10	254760

Then the Operation will be thus;

	25476	
	3468	
In the eighth Step is	203808	= 25476x8
In the ſixth	152856	= 25476x60
In the fourth	101904	= 25476x400
In the third	76428	= 25476x3000.
	88350768	

61. Sometimes, when it is not neceſſary to have a Tariffa for all the Digits, we may ſave a little Trouble, by making a Table for only a few more Figures than are in the Multiplier. Thus, in the laſt Example, firſt put down for 1; then for the third multiply the firſt by 3; then add the firſt and third for the fourth Step; and for the ſixth double the third; for the eighth double the fourth; and, for Proof of the Table, add the ſixth and fourth for the tenth; which, bringing out the ſame Figures as the firſt with an 0 on the Right-hand, proves the Table to be right.

1	25476
3	76428
4	101904
6	152856
8	203808
10	254760

 62. If

62. If we make a Tariffa, or Table, for 1, 2, 3, and 5 Steps only, by doubling for 2; and adding 2 and 1 for 3, and 3 and 2 for 5; and, for Proof, adding 5 to itself for 10; we may from such a Table have the Product by any Digit, by Addition only: for 1, 2, 3, and 5, are in the Table; and $3+1=4$; $5+1=6$; (or $3+3=6$.) $5+3=8$; and $5+3+1=9$, (or, by Subtraction, $10-1=9$.)

63. Lord *Neper*, observing, that Tabulating the the Multiplicand was in many Cases of great Use, contrived Tables, or Rulers, for this Purpose; which are now commonly known by the Name of *Neper's-Bones*, or *Neper's-Rods*. The Contrivance is this: Each Column of *Pythagoras*'s Table is made on a separate Piece of Box or Ivory, &c. (whence the Appellation of Bones). The following Representation will convey a better Idea of them than a Multitude of Words.

NEPER's RODS.

Index Rod.										
1	1	2	3	4	5	6	7	8	9	0
2	/2	/4	/6	/8	1/0	1/2	1/4	1/6	1/8	/0
3	/3	/6	/9	1/2	1/5	1/8	2/1	2/4	2/7	/0
4	/4	/8	1/2	1/6	2/0	2/4	2/8	3/2	3/6	/0
5	/5	1/0	1/5	2/0	2/5	3/0	3/5	4/0	4/5	/0
6	/6	1/2	1/8	2/4	3/0	3/6	4/2	4/8	5/4	/0
7	/7	1/4	2/1	2/8	3/5	4/2	4/9	5/6	6/3	/0
8	/8	1/6	2/4	3/2	4/0	4/8	5/6	6/4	7/2	/0
9	/9	1/8	2/7	3/6	4/5	5/4	6/3	7/2	8/1	/0

64. For performing of Multiplication by these Rods, one Index-Rod is sufficient; by the Index-

Rod

Rod we would be understood to mean the Rod of Digits which is not diagonally divided, and which is the first to the Left-hand in the above Figure. But of each of the other Sort we must have so many as any one Figure (or 0) is repeated in the Multiplicand. Six or eight Rods of each Species will be sufficient for common Practice.

65. In using these Rods, first take the Index-rod, and, next to it on the Right-hand, place a Rod which has on its Top the Figure which stands in the highest Place of the Multiplicand; and next to this the Rod, on the Top of which is the Figure which stands in the next inferior Place of the Multiplicand: And thus proceed 'till all the Rods belonging to the Multiplicand are taken out; then, against each Digit of the Index-rod, will stand the Product of the Multiplicand by that Figure; but, since by our Notation, beginning at the Right-hand, the Figure on the higher Part of any Square is of the same Height as the lowest in the next (Left-hand) Square, in taking out the Products, first take out the Figure in the lowest Part (or Units Place) of the Square of that Rod which is on the Right-hand of all the others; then add the Figure in the upper Part of this Square to that in the lowest Part of the Square of the next Rod; and so proceed till all the Product of the Multiplicand by that Figure is taken out.

66. For an Example, take that in Art. 60. *viz.* Multiply 25476 by 3468.

The proper Rods, being taken out and properly placed, will appear thus: Which being done, againſt 8 in the Index-rod, we have the Product by 8, to be taken out after this Manner, beginning at the Right-hand; ſay 8 is 8, which place down; (as in Art. 60.) then 4+6=10, put down the 0, and carry 1; ſaying 1 (we carry) +5+2=8, which write down; and go on, ſaying 3 (+0)=3, which write down; and then 4+6=10: Laſtly, 0 being written down, we have 1 (we carry) +1=2; ſo that the Product by 8 is 203808: After this Manner take out the other Products, and, due Regard being had in placing the ſeveral Products, (the Operation will appear as in Art. 60, which ſee; and) their Sum will be the required Product.

1	2	5	4	7	6
2	/4	1/0	/8	1/4	1/2
3	/6	1/5	1/2	2/1	1/8
4	/8	2/0	1/6	2/8	2/4
5	1/0	2/5	2/0	3/5	3/0
6	1/2	3/0	2/4	4/2	3/6
7	1/4	3/5	2/8	4/9	4/2
8	1/6	4/0	3/2	5/6	4/8
9	1/8	4/5	3/6	6/3	5/4

67. In ſome Caſes, the Work of Multiplication may be ſhortened, as we ſhall now illuſtrate in the moſt uſeful Contractions, and in the moſt uſeful only; becauſe the giving a great Number of Contractions, not in frequent Uſe, would be both waſting our own Time, and alſo that of our Readers.

Caſe 1. When any Number is to be multiplied by 10, 100, or 1000, *&c.* the Product will be found, by only annexing the Ciphers to the Right-hand of the Figures; thus 25×10=250; 25×100=2500; 25×1000=25000, *&c.*

68. *Caſe* 2. When either the Multiplicand, or Multiplier, or both, have Ciphers to the Right-hand of the Figures, multiply by the Figures, taking no Notice

of

of the Ciphers, till the ſeveral Products are added together; but then to the Sum annex ſo many Ciphers as are to the Right-hand of the Figures both in the Multiplicand and Multiplier.

69. *Example*. Multiply 2300 by 1500.

```
        2300
       15000
    --------
     115
    --------
     23
The Work will appear thus:   34500000
    --------
```

70. When all the Figures in the Multiplier except that in the Units Place (which may be any Figure) are One's, the Work may be performed in one Line, as will be ſhewn in the following Examples: Firſt, multiply 381 by 15.

```
 381
  15
----
5715
----
```

Here (and in multiplying by 12, 13, 14, 15, 16, 17, 18, or 19) we muſt remember, as we multiply by the Digit in the Units Place of the Multiplier, to add that Figure of the Multiplicand which ſtands next on the Right-hand of the Figure we are multiplying of; thus, in this Example, ſay, 5x1=5, which put down; then 5x8, =40, +1=41, put down 1, and carry the 4; then 5x3=15, +4 (we carry)=19, +8=27, put down 7, and carry 2: Now as we have multiplied all the Figures of the Multiplicand by the firſt Figure (5) of the Multiplier, we omit taking any further Notice of that Figure, and therefore have only now to ſay 1x3=3, +2 (we carry)=5, which, being placed down, compleats the Product.

71. *Example* 2. Multiply 381 by 115.

```
  381
  115
-----
43815
-----
```

Here, as we have two One's in the Multiplier, we multiply by the Digit in the Units Place, remembering, as we multiply, to add thoſe two Figures of the Multiplicand which ſtand next

on

on the Right-hand of the Figure we are multiplying of. Thus, in this Example, say $5\times1=5$, which put down; then say $5\times8=40$, $+1$ (the Figure on the Right-hand of 8) $=41$, put down the 1, and carry 4; saying $5\times3=15$, $+4$ (we carry) $=19$, $+8=27$, $+1=28$; (here we add in the two Figures to the Right-hand of the 3) put down the 8, and reserve the 2; then omitting the 5 in the Multiplier, because all the Figures have been multiplied by it, we say 2 (we carry) $+3=5$, $+8=13$, (here we add in two Figures of the Multiplicand) put down the 3, and carry 1; saying 1 (we carry) $+3$ (the remaining Figure of the Multiplicand $=4$, which, being placed down, compleats the Product.

72. To multiply any Number by 102, 103, 104, &c. at one Operation; multiply by 2, 3, 4, &c. and when you have found two Figures of the Product, as you multiply, each Time add one Figure (beginning with that in the Units Place) of the Multiplicand.

73. *Example.* Multiply 437 by 105.

$$\begin{array}{r} 437 \\ \underline{105} \\ \underline{45885} \end{array}$$

First say, $5\times7=35$, put down 5, and carry 3; then $5\times3=15$, $+3$ (you carry) $=18$, put down 8, and carry 1; again $5\times4=20$, $+1$ (you carry) $=21$, $+7=28$, put down the 8, and carry 2; then having done with the 5, we say, 2 we carry $+3$ $=5$, which place down: Lastly 4 is 4, which, being put down, compleats the Product.

74. *Corollary.* After the same Manner we may multiply by 1001, 1002, 1003, &c. by 10001, 10002, 10003, &c. &c. by only observing, if the 1 on the Left-hand of the Multiplier be in the Place of Thousands, to have three Figures in the Product before you add a Figure of the Multiplicand: And, if the 1 stands in the Place of Tens of Thousands, to have four Figures in the Product, before you begin to add.

Whosoever will compare these Abbreviations with the Operations at large, will easily see the Reasons of these Contractions.

75. When we are to multiply by any Number, which is nearly equal to a certain Number of Tens, Hundreds, Thousands, &c. it will be a short Method to multiply by the Number of Tens, Hundreds, &c. and then to add, or subtract, for what the given Multiplier exceeds, or is short of, the Number of Tens, Hundreds, &c.

Example. Multiply 2378 by 999.

Here the Multiplier 999 wants but 1 of 1000; ∴ multiply 2378 by 1000, and the Product by

Art. 17. is — — —	2378000
Subtract	2378
Then 1000 Times — 1 Time = 999 Times =	2375622 = the Product.

76. *Example* 2. Multiply 2762 by 4002.

First, 2762 × 4000 =	11048000
And 2762 × 2 =	5524
But 4000 Times + 2 Times = 4002 Times; ∴ the Product required will be found, by Addition,	11053524 = the Product.

77. *Example* 3. Multiply 27620 by 29998.

Here 30000 — 29998 = 2.

27620 × 30000 =	828600000
And 27620 × 2 =	55240
But 30000 Times — 2 Times = 29998 Times; ∴ the Product, by Subtraction, is — — — —	828544760

78. When the Multiplier can be parted into Periods which are Multiples of one another, the Operation may be contracted, after the Manner explained in

in the following Examples; which are the same the ingenious Mr. *Robert Robinson* has given in *Miscellanea Curiosa Mathematica*, Pag. 1. Vol. 2.

Example 1. Multiply 5697487 by 96488.

First multiply A by 8, and the Product call B. Now because $6\times8=48$, (or $60\times80=480$) multiply the Number B by 6, and, the 8 in 48 standing in the Place of Tens, put the first Figure of this Product in the Place of Tens,

Let	5697487	$= A$.
	96488	
	45579896	$= B$
	273479376	$= C$
	546958752	$= D$
	549739125656	

and call this Product C: Again, since $2\times48=96$, as C taken in its simple Value is 48 Times A, $C\times2$ must be $=96$ Times A, ∴ multiply C by 2, and, since the 96 is really 96000, the first Figure of the Product must be placed in the Place of Thousands; this Product call D. Now B being $=8$ Times A, $C=480$ Times A, and $D=96000$ Times A, the Sum of
* 24. $B+C+D$* $=$ the Product required.

79. *Example* 2. Multiply 5742135 by 52575.

Here, because $75=25\times3$, and $25=5\times5$, we multiply in a reverse Order to the first Example; first multiplying the Number A by the 5 which is on the Left-hand of the Multiplier; but, this 5 being really 50000, we must put the first Figure of this Pro-

Let	5742135	$= A$
	52575	
	28710675....	$= B$
	143553375..	$= C$
	430660125	$= D$
	301892747625	

duct in the Place of Tens of Thousands, or, which is the same Thing, under the Figure (5) which we multiplied by; call this Product B: Then, 5×5 being $=25$, and B taken in its simple Value being $=5$ Times A, if we multiply B by 5, the Product will be $=25$ Times A, but, by placing the first Figure of this Product in the Place of Hundreds, we shall have the Product of A by 2500; call this second Product

duct C: Again, 25×3 being $= 75$, and C taken in its simple Value being $= 25$ Times A, it is plain that by multiplying of C (taken in its simple Value) by 3 we shall have the Product of A by 75; which being wrote down under C, so that the first Figure of this last Product may stand in the Place of Units, it is manifest, that, calling this D, we have $B = 50000$ Times A, $C = 2500$ Times A, and $D = 75$ Times A; and, consequently, $B + C + D$* $= 50000 + 2500 + 75$ Times $= 52575$ Times $A =$ the required Product. * 24.

80. When the Figures of which the Multiplier consist, are all of one Sort, *viz.* all Two's, Three's, or Four's, *&c.* the Operation may be much shortened, as will be illustrated by this

Example, viz. Multiply 34018 by 2222.

First, multiply by 2, (one of the Figures of the Multiplier) the Product is 68036; which, being placed down, we call

34018	
2	
68036	Number to be added.
75587996	Product.

the Number to be added; because, by adding the Figures of which it is composed together, after the following Manner, we shall get the required Product. To proceed then, say, 6 (the Figure in the Units Place) is 6, which put down; then $6+3=9$, which put down; then $6+3+0=9$, which put down: Again, $6+3+0+8=17$, put down the 7, and reserve the 1; then, because we have obtained as many Places of the Product as the Multiplier has Figures, omitting the 6, (in the Units Place of the Number to be added) say, 1 (we carry) $+3+0+8+6=18$, put down 8, and reserve the 1; then say, (leaving out 36, the two first Figures) 1 we carry $+0+8+6=15$, put down 5, and carry 1; saying (omitting the three first Figures 036 of the Number to be added) 1 (we carry) $+8+6=15$, put down 5, and carry 1: Lastly, (omitting all the Figures of the Number to be added, except that in the highest Place,

Place, *viz.* 6,) say 1 (we carry) + 6=7, which, being wrote down, compleats the Product.

The Reason of this Operation will easily appear by comparing it with the Work at large.

81. Several Figures may be multiplied by several in one Line, as is shewn in the following Example.

1. Multiply 25476 by 3468; (this is the same Example as that in Art. 60.)

$$\begin{array}{r} 25476 \\ 3468 \\ \hline 88350768 \\ \hline \end{array}$$

Say first 8x6=48, put down 8, and carry 4; then 8x7+4 (we carry) +6x6 =96, put down 6, and carry 9: Again, 8x4+9 (we carry) +6x7+4x6=107; put down 7, and carry 10; then 8x5+10 (we carry) +6x4+4x7+3x6=120; write 0, and carry 12: Again, 8x2+12 (we carry) + 6x5+4x4+3x7=95, put down 5, and carry 9: Now, since we have no more Figures remaining in the Multiplicand to be multiplied by 8 the Figure in the Units Place of the Multiplier, we omit that Figure, and proceed saying, 9 (we carry) +6x2+4x5+3x4=53, put down 3, and carry 5. Having now multiplied all the Figures of the Multiplicand by 6, the Figure in the second Place of the Multiplier, we omit that Figure, and then go on, saying, 5 (we carry) +4x2+3x5=28, put down 8, and carry 2. Lastly, having now multiplied all the Figures of the Multiplicand by 4, the Figure in the third Place of the Multiplier, we omit that Figure, and then there only remains to say, 2 (we carry) +2x3=8, which, being writ on the Left of the other Figures, compleats the Product. See Art. 122.

If the Reader compares this with the Operation at large, it will be no difficult Matter for him to see the Reason of this Method.

82. This Method of multiplying several Figures, by several, in one Line, lately made a great Noise: Authors boasting of its Novelty, and their first Publishing it; though there is nothing new in the Invention, it being, in Fact, nothing more than the common Method performed in one Line, which

makes

makes it burdensome to the Memory, and consequently not so proper for Business. Neither is it a new Curiosity, for a Gentleman (in the *Gentleman's Magazine*, Vol. 20. P. 25.) declares, that, when he went to School, which is upwards of 30 Years ago, this Method of multiplying was common amongst the Boys.

83. Multiplication may be proved by multiplying the Multiplier by the Multiplicand; that is, making the Multiplier a Multiplicand, and the Multiplicand a Multiplier; which must produce the same Product, as before: For, when two Numbers are to be multiplied together, it matters not which you make the Multiplier; (for (by Art. 95.) $a \times b = b \times a$.

84. Multiplication may also be proved by casting out the Nines; the Method of doing which is better explained by an Example, than by formal Precepts. Let it then be proposed to prove the Sum in Art. 60. *viz.* that $25476 \times 3468 = 88350768$. First cast the Nines out of the Multiplicand after this Manner: Say $2+5=7$, $+4=11$, which is 2 more than 9: Then say (omitting the 9) 2 (the Excess above 9) $+7=9$, exceeding 9 by 0; ∴ say 6 is 6, which reserve. Secondly, casting the Nines out of the Multiplier by the same Method, by saying $3+4=7$, $+6=13$, (which is 4 above 9) and $4+8=12$, we get 3 (above 9) to be reserved. Now (by Art. 100.) the Product cannot be right, unless the Product of these reserved Numbers (remembering, when this Product is more than 9, to cast out the Nines, and then the remaining Number must) be equal to the Number remaining after all the Nines are cast out of the Product (of the Sum): Thus, in this Example, the Nines cast out of the Product, leaves 0; and the Product of the reserved Number, $3 \times 6 = 18$, which, when the Nines are cast out, leaves 0, the same as before, for Proof.

85. But against the Proof of Multiplication by casting out the Nines it is objected, that a false Product may, by this Method, prove right; for, if the Product had been brought out (in the above)

 88530768,

88530768, the Remainder, after all the Nines are cast out, will be o as before: But then, if it is considered, that,* to make a false Product appear to be right, there must be at least two Figures wrong, and one of them must be exactly as much greater as the other is less than it ought to be; and, if there are more than two Figures wrong, the Sum of the Errors which are too much, must balance, or be equal to the Sum of the Errors, of those which are less than they ought to be; we may (I think) reasonably trust to this Proof. For, how unlikely is it, that when we are endeavouring to bring out Truth, we should commit two or more such Errors that shall be directly contrary, and balance each other; *i.e.* that the Excesses of those which are too much, should be exactly equal to the Deficiencies of those which are too little: Upon the Whole, that we should bring out a false Product, which shall by this Method prove right, seems not only unlikely, but improbable; and the Chance exceeding great.

86. It may here be remarked, by-the-bye, that Addition may also be proved by casting out the Nines. For an Example let it be required to prove the Addition-Sum in Art. 28.

First say, $5+7=12=9+3$; and $3+6=9$; then 8 is 8, which reserve, it being the Excess after the Nines are cast out of 5768; then, the Nines cast out of 123, leaves 6 to be reserved; and, casting the Nines out of 879, we have 6 to be reserved; then, to cast the Nines out of these reserved Numbers, say $8+6=14=9+5$, and $5+6=11=9+2$; so that, after all the Nines are cast out of the Numbers which were to be added together, we have 2 remaining. Now, since all the Parts taken together are equal to the Whole, the Nines being cast out of the total Sum, there must be also 2 remaining; and, casting the Nines out of 6770, we shall find 2 remain, for Proof.

Note, The same Objection may be made to this Proof as in Art. 85; and the Answer, there given, will also serve as an Answer for this.

 87. Co-

87. *Co-efficients*, (*Con* and *Efficiens*, Lat.) in Algebra, are the Numbers which are placed to the Left-hand of the Letters: Thus, in $3bd$ 3 is the Co-efficient: If there be not a Co-efficient placed before a Quantity, that Quantity is ſuppoſed to have an Unit for its Co-efficient; thus, b may be read $1b$ (one b).

88. In *Algebraic* or *Univerſal Multiplication*, we make Uſe of this Character ×, to denote that two Quantities are multiplied together: Thus, $a\times b$ is a multiplied by b; but we more frequently omit this Character, and write the Letters cloſe together, (without any Character between them) as for $a\times b$ we write ab. When a Line is drawn over any Quantities, and the Sign of Multiplication put between the Letters under the Line (called by Algebraiſts the *Vinculum* (*Vinculum*, Lat.) and ſome other Quantity) it denotes, that the Sum of the Quantities under the Vinculum is to be multiplied by the other Quantity: Thus, $\overline{a+b}\times m$ is to be read the Sum of $a+b$ multiplied by m. Alſo $\overline{a+b}\times\overline{a-b}$ is the Sum of a and b, multiplied by the (Sum of a and $-b$, or the) Difference of a and b.

89. Though the Method of expreſſing the Product of Compound Quantities by the Vinculum, as ſhewn in the laſt Article, is many times convenient; yet, it is more frequently uſeful to be able to expreſs the Product without a Vinculum, in more ſimple Terms; and, in order to ſhew the Method of doing this, it will be proper to lay down a few *Theorems* by way of *Lemma's*.

Theorem 1. The Product of the Sum of any two affirmative Quantities, by any Number, is equal to the Sum of the Products of each of the affirmative Quantities of the Multiplicand, by the Multiplier; that is, $\overline{a+b}\times x=ax+bx$.

Demonſtration. By the Nature of Multiplication (Art. 51.) to multiply $\overline{a+b}$ by x, is only to take

$a+b$ as many Times as there are Units in x; and ∵ $\overline{a+b}\times x$ must be $=ax+bx$; for (by Art. 88.) $ax=a\times x=a$ taken x Times; and $bx=b\times x=b$ taken x Times, and consequently (by Art. 24.) $ax+bx=\overline{a+b}$ taken x Times $=\overline{a+b}\times x$ (by Art. 51.) *Q.E.D.*

90. *Theorem* 2. The Product of the Difference of any two Quantities, by any (affirmative) Number, is equal to the Product of the Subducend by the Multiplier, Minus the Product of the Minorand by the same Number, *viz.* $\overline{a-b}\times x=ax-bx$.

Demonstration. The Product of a by x is ax; but it is evident, that ax is greater than the required Product; because ax is the Product of a by x (or is a taken x Times) whereas we want only the Product of the Difference of a and b by x; ∵ from the Product ax we must subtract the Product of b by x, viz. bx, for x Times the Quantity, expressing the Excess of a above b; and ∵ the Product required is $ax-bx$. *Q.E.D.* But, if this be not clear enough, take the Demonstration otherwise, thus: Let $d=$ the Difference of a and b; then (by Art. 50.) $a=b+d$; and, by multiplying both Sides of the * *Equation* by x, we have by the last Theorem $ax=bx+dx$, and, taking bx from both Sides of this Equation, we have (by Art. 36.) $ax-bx=dx$. *Q.E.D.*

* An *Equation*, (*Æquatio*, Lat.) in Algebra, is when a Quantity, or Quantities, on one Side of $=$ is equal to one or more Quantities on the other Side; the Equality $3=2+1$ is an *Equation*.

91. *The-*

91. *Theorem* 3. I say $\overline{a-b}\times\overline{x-y}=ax-bx-ay+by$.

Demonstration. By Art. 51. to multiply $\overline{a-b}$ by $\overline{x-y}$ is only to take $\overline{a-b}$, $\overline{x-y}$ Times; which may be done thus:

x Times $\overline{a-b}$ is (by the last Art.) $=$	$ax-bx$
and y Times $\overline{a-b}$ $=$	$ay-by$

$\therefore$ by Subtraction and Art. 36. x Times $\overline{a-b}$, Minus y Times $\overline{a-b}$ $= ax-bx-ay+by$. *Q. E. D.* But, as Subtraction may be performed by Addition, (see Art. 46.) by changing the Signs of the Minorand, the Sum will, according to this Method, stand thus:

To	$ax-bx$
Add	$-ay+by$

The Sum is $ax-bx-ay+by$ the same as above.

92. *Corollary* 1. Hence, it may be observed, that the Product of Compound Quantities may be found, by collecting together the several Products of each Quantity of the Multiplicand, by each Term of the Multiplier: And that, in multiplying the Multiplicand by the affirmative Terms of the Multiplier, the Product will have the same Signs as in the Multiplier; but that, in multiplying by the negative Terms, the Signs of the Product must be contrary to those of the Multiplicand.

93. *Corollary* 2. Hence, also, it will appear, by viewing the Work (in Art. 91.) that, taking the simple Terms of the Product separately, the first is ax, which may be considered as $+a\times+x=+ax$, so that an affirmative Quantity, multiplied by an affirmative Quantity, will have an affirmative Product; or, in the Language of Algebraists, $+$ into $+$ is $+$. The second Term of the Product is $-bx$, which may

may be taken for the Product of $-b$ by $+x$; here, a negative Quantity, multiplied by an affirmative Quantity, produces a negative Product; or $-$ into $+$ is $-$. Again, the third Term is $-ay$, which may be considered as the Product of $+a$ by $-y$; so that an affirmative Quantity, multiplied by a negative Quantity, will have a negative Product; or $+$ into $-$ is $-$. Lastly, the fourth Term is $+by$, which may be considered as the Product of $-b$ by $-y$, so that, a negative Quantitybeing multiplied by a negative Quantity, the Product must be taken as affirmative; or $-$ into $-$ is $+$.

Hence these Rules for the Memory:

$$\left.\begin{array}{l} + \text{ into } + \text{ is } + \\ - \text{ into } - \text{ is } + \\ - \text{ into } + \text{ is } - \\ + \text{ into } - \text{ is } - \end{array}\right\} \text{that is,} \left\{\begin{array}{l} \text{Plus into Plus is Plus.} \\ \text{Minus into Minus is Plus.} \\ \text{Minus into Plus is Minus.} \\ \text{Plus into Minus is Minus.} \end{array}\right.$$

But though, with Relation to affirmative Quantities, we may, in the Sense here delivered, talk of the Multiplication of two negative Quantities; yet to consider the Multiplication of two negative Quantities independent, or having no Relation to an affirmative Quantity, seems to be but little, if any thing, better than Nonsense.

94. From either of the two last Articles, the Method of multiplying compound Quantities must appear evident, when compared with an Example, or two.

$$\begin{array}{rcl} \text{Multiply} & & a+b-c \\ \text{By} & & x-y \\ \hline \overline{a+b-c}\,x & = & ax+bx-cx \\ \overline{a+b-c}\,x-y & = & -ay-by+cy \\ \hline \text{Product} & = & ax+bx-cx-ay-by+cy. \end{array}$$

$$\begin{array}{rcl}
\text{Multiply} & & 2a-2b \\
\text{By} & & a+3b \\
\hline
\overline{2a-2b}\times a & = & 2aa-2ba \\
\overline{2a-2b}\times 3b & = & +6ba-6bb \\
\hline
\text{Hence Product} & = & 2aa+4ba-6bb.
\end{array}$$

95. It may be remembered by the Reader, that we have hitherto left, in this Chapter, ſome Things not demonſtrated; and therefore their Demonſtrations ought to be given before we put an End to this Chapter; which take as follows:

That $a\times b=b\times a$.

Theorem 1. If two Numbers are to be multiplied together, whichever is made the Multiplier, the Product will be ſame, *viz.* $a\times b=b\times a$.

Demonſtration. Let $1+1+1+1$, &c. be the Number of Units contained in a; then, ſince (by Art. 51.) Multiplication is only taking the Multiplicand ſo often as there are Units in the Multiplier, it follows, that $b+b+b+b$, &c. as many Times as there are Units in a, is $=$ the Product of $a\times b$; becauſe $b+b+b+b$, &c. (till there are ſo many b's written down as there are Units in a) is each Unit in a taken b Times; (for each b is $=1$ taken b Times) and all the Parts taken together are equal to the Whole. Hence we have proved that b taken as many Times as there are Units in a is $=a\times b$; but (by Art. 51.) b taken as many Times as there are Units in a is $=b\times a$; $\therefore$ * $b\times a=a\times b$. Q.E.D. * 23.

96. If two Numbers are to be multiplied together, and one of the Factors is a compoſed Number, we may, inſtead of multiplying by that Factor, multiply by the Numbers of which it is compoſed. I mean, if $b\times c=p$, that then $a\times b\times c=a\times p$.

Demonſtration. The Reaſon of this may be ſhewn by an Example. Suppoſe it was required to multi-

ply 29 by 30; here, 30 is composed of 5 and 6, for 5x6=30. Now 29x5=145; here we have repeated 29 five Times, which is 145; and consequently, if 145 be taken 6 Times, the Product will be = 29 taken 30 Times; because 145 is each Unit in 29 taken 5 Times, and ∴, if each Unit in 145 be taken 6 Times, we shall have 5 Times as many Units as are in 29 taken 6 Times; or, which is the same, as many Units as are in 29 taken 5x6=30 Times. It may be demonstrated generally thus:
* 51. $a \times b$ * $= a$ taken b Times; and $a \times b \times c = a$ taken b
Times, taken c Times; but b taken c Times $= p$ by
* 23. the Supposition; ∴ a taken b Times, c Times * $= a$
taken p Times; or, which is the same, $a \times b \times c = a \times p$.
Q. E. D.

97. If any 3 Numbers are to be multiplied continually together, in whatever Order the Factors are taken, the last Product will be the same, *viz.* $a \times b \times c = a \times c \times b = b \times c \times a = c \times b \times a = c \times a \times b = b \times a \times c$.

* 95. *Demonstration.* Because $b \times c$ * $= c \times b$, if we call the
Product of $b \times c = n$, $c \times b = n$, also; ∴, writing n for
$b \times c$, and $c \times b$ in the above, we have $a \times b \times c = an$, and
* 95. $a \times c \times b = an$; also $b \times c \times a = na$, and $c \times b \times a = na$; but *
* 23. $an = na$; hence, * $a \times b \times c = a \times c \times b = b \times c \times a = c \times b \times a$. A-
* 95. gain, $a \times b$ * $= b \times a$, ∴, if we denote the Product of
$a \times b$ by p, the Product of $b \times a$ may also be represented
by p; and ∴, putting p for $a \times b$, and for $b \times a$, we
shall have $c \times a \times b = cp$, and $b \times a \times c = pc$, also $a \times b \times c = pc$;
* 95. but * $cp = pc$, ∴ * $c \times a \times b = b \times a \times c = a \times b \times c =$ (by the
* 23. above) $a \times c \times b = b \times c \times a = c \times b \times a$. *Q. E. D.*

98. *Corollary.* We have demonstrated, that in two or three Factors, which Way soever they are taken, the continued Product will come out the same; and therefore may conclude, that if we take in one Factor more, *viz.* make them four, it will still hold good; therefore, in one more, *&c. ad infinitum.*

99. In what Place soever any Digit stands, being taken in its simple Value, it is equal to what will remain, after all the Nines that are contained in its real Value are taken away; unless the Figure is 9, and then

then the Remainder will be nothing. The Meaning of this Theorem is, that (for Example) in 500, after all the Nines are taken away, there will remain 5; and, after all the Nines in 60 are taken away, there will remain 6; and, after all the Nines are taken out of 8000, the Remainder will be 8; alſo, after all in 900 are taken away, there will remain 0, &c.

Demonſtration. Becauſe, in whatever Place of a Number, any Figure b ſtands, it is * = (10 Times =) * 9.
9 Times + 1 Time that Figure b, in the next lower Place (as for Inſtance in 500, the 5 is = 10 Times the 5 in 50;) it muſt follow, by taking 9 Times the Value of b in the next lower Place, that the Value of b in its given Place, Minus 9 Times the Value of b
in the next lower Place, is * = 1 Time the Value of * 36.
b in that next lower Place; but 9 Times any Number is a preciſe Number of Nines, and ∴, that being taken away, we ſhall have the Value of b in that next lower Place; and the Value of b in this laſt found Place is equal to (for the ſame Reaſons as before) 9 Times + 1 Time the Value of b in the next inferior Place; and ∴ the Value of b in the laſt-found Place Minus 9 Times the Value of b in the next inferior Place (which is a compleat Number of Nines) = the Value of b in that next lower Place, and ſo on, till you bring it down to the Place of Tens, and then it is $10b = 9b + 1b$, and ∴, taking $9b$ from both Sides
of the Equation, we have * $10b - 9b = b$, its ſimple * 36.
Value; but (the $9b$, which we have now ſubtracted, is a compleat Number of Nines, and ∴) we have now taken out all the Nines, except b is $= 9$, and then we can take exactly one 9 more, and then the Remainder will be 0. Conſequently the Theorem is demonſtrated. Or, perhaps, the following Demonſtration may, to ſome, appear plainer. In this Demonſtration we ſhall prove it to be true, when the Place that the Figure ſtands in doth not exceed Thouſands; and, after the ſame Manner, it may be demonſtrated in Millions, &c.

To

To proceed then, let b be the Figure, which we ſuppoſe to ſtand in the Place of Thouſands, then
* 9. * $1000b =$ its Value in that Place, and $1000b - 900b,$
* 46. $= 100b, - 90b = 10b, - 9b$ * $= b$: Here it may be obſerved, that from $1000b$ we have taken $900b$, $90b$, and $9b$, and the Remainder comes out b; but $900b$, $90b$, and $9b$, are preciſe Numbers of Nines, for $900b$ is 100 b Nines; $90b$ is 10 b Nines; and $9b$ is b Nines: But $900b$, $90b$, and $9b$, are the greateſt Number of Nines that could be taken out of $1000b$, and leave a Remainder; becauſe, theſe being deducted, the Remainder was only b, which cannot exceed 9, becauſe it is a ſingle Figure; and, conſequently, when $b =$ any Figure leſs than 9, then there cannot be any more Nines deducted: But, when $b = 9$, then there can be one Nine more deducted; but then the Remainder would be $= 0$; becauſe $1000b - 900b - 90b - 9b - b = 0$. *Q. E. D.*

100. In Multiplication-Sums, the Nines being caſt out of the Multiplicand and Multiplier, the Remainders, being multiplied together, will, when their Product does not exceed 9, be equal to what remains, after all the Nines are caſt out of the Product of the given Multiplication-Sum: But, if the Product of the Remainders above-mentioned be more than 9, the Nines being caſt out of it, what remains, will be equal to the Remainder of the given Product.

Demonſtration. Let $m =$ all the Nines in the Multiplier, $d =$ what remains after all the Nines are taken away, then $m + d$ (Art. 24.) $=$ the Multiplier; and let $n =$ the Number in which all the Nines that can be taken out of the Multiplicand are exactly contained; and $x =$ what remains after the Nines are taken out
* 24. of the Multiplicand; then $n + x$ * $=$ the Multiplicand. Now the Work of multiplying $n + x$ by $m + d$ (according to Art. 92, 93, and 94) will ſtand thus:

$n+x$

$$\begin{array}{r} n+x \\ m+d \\ \hline nm+xm \\ +nd+xd \\ \hline \end{array}$$

Hence the Product is $=nm+xm+nd+xd$; but nm, xm, and nd, are certain Number of Nines; becauſe n and m are by the Suppoſition a certain Number of Nines, and ∵ any Number, multiplied by n or m, muſt be a certain Number of Nines; and ∵, theſe Terms being rejected, there remains in the Product only xd: But the Product of x (the Number remaining in the Multiplicand) multiplied by d (the remaining Number of the Multiplier) is xd, which being the ſame as what remains in the Product, we have proved what was propoſed; for, when xd is leſs than 9, we have already caſt out the Nines of the Product; but, if xd is more than 9, then, caſting out the Nines in xd of the Product, we ſhall have caſt out all the Nines that were contained in the Product, and the Remainder muſt be equal to what remains after the Nines are caſt out of the Product xd; (*viz.* the Product of the Remainders of the two Factors) becauſe the Quantities out of which they are to be caſt, are both expreſſed by xd. *Q. E. D.*

101. From the two laſt Articles, the Reaſon of the Method ſhewn in Art. 84. for proving Multiplication-Sums will eaſily appear. For, becauſe * in whatever Place any Figure ſtands, taken in its ſimple Value, it is equal to what will remain after all the Nines that are contained in its Value, according to the Place in which it ſtands, are taken away; it follows, that the Sum of all the Figures of which any Number conſiſts, taken ſimply as ſo many Units, is equal to the Remainder after all the Nines are taken out of that Number, that can be found in the real Value of each Figure of which it conſiſts; and ∵, if this Sum be leſs than 9, it muſt be equal to the Remainder, after as many Nines are taken out of the given

* 99.

given Number as possible: But if this Sum is, or exceeds 9, then, taking the Nines out of this Sum, the Remainder will be equal to what remains after all the Nines are cast out of the the given Number; because the Number of Nines in any Number must
* 24. be * equal to the Nines which are contained in the several Parts, and in the Sum of the Excess of Nine in those Parts. To make this as plain as may be, let this be illustrated in casting the Nines out of
* 9 & 24. 25476 * = 20000 + 5000 + 400 + 70 + 6. Now, the
* 99. Nines cast out of 20000, there will remain * 2; out of 5000 there will remain 5; out of 400 the Remainder will be 4; and out of 70 there will remain 7; the Sum of these Remainders is 2+5+4+7+6=24, out of which the Nines being cast, the Remainder is 6: Thus we have shewn the Reason of the Method of casting out the Nines by adding the Figures together; and the Reason of the remaining Part of the Proof is shewn in the last Article.

CHAP. VI.

Of DIVISION.

102. DIVISION (*Divisio*, Lat.) is a Rule by which we find how many Times one Number is contained in another. Or, which is the same, it is a (compendious) Method of subtracting one Number from another, as often as it is contained in that other: For, as often as one Number is contained in another, so often can it be taken out of that other.

103. The Number to be divided (*viz.* that Number which is considered as the containing Number) we call the *Dividend*, (from *divide*, from *divido*, Lat.)

104. The measuring Number, or Number we are to divide by, (*viz.* that which is considered as contained

tained in the Dividend, is called the *Divisor* (*Divisor*, Lat.)

105. The Number sought (*viz.* that shewing how often the Divisor is contained in the Dividend) is called the *Quotient*, (*Quoties*, Lat.)

106. As the Divisor is not always contained exactly a certain Number of Times in the Dividend, there will, sometimes, after the Divisor has been taken out of the Dividend as often as possible, be a Number remaining; which we therefore call the *Remainder* (from *Remain*, Eng. from *Remaneo*, Lat.)

107. A *Sub-multiple*, (from *Sub* and *Multiplex*, Lat.) or *Aliquot* (*Aliquot*, Lat.) Part, is a Number greater than an Unit, and which is contained in another Number a certain Number of Times. Thus, 2 is a Sub-multiple of 6, for 2 is contained in 6, exactly 3 Times. Or, in other Words, a Sub-multiple is a Number greater than an Unit, that will measure another Number, exactly, without a Remainder.

108. An *Axiom*. If equal Things be divided by equal Things, their Quotients will be equal.

109. To divide one Number by another, when both the Divisor and Dividend are single Digits; or the Divisor a single Digit, and the Dividend not consisting of more than two Figures; we may subtract the Divisor as often as possible out of the Dividend: But if the Learner be perfect in his *Pythagorean*, or Multiplication-Table, he will be able readily, by Memory, to tell how often the Divisor is contained in the Dividend; that is, he will be able to take a Digit, by which multiplying the Divisor, the Product will be (either) equal to, or the next less than the Dividend; and the Digit, so taken, will be the integral Part of the Quotient; and the Remainder (which must be always less than the Divisor, because, if at any Time we should bring out a Remainder greater than, or equal to, the Divisor, we can take the Divisor out of the Remainder, and therefore have not taken it out as many Times as possible) being

ing put over the Divisor as a Fraction, (see Art. 6.) compleats the Quotient.

Examples. Divide 10 by 5; here 5 is contained in 10 two Times, for $5\times2=10$; ∵ $10\div5=2$. Also, $13\div5=2\frac{3}{5}$, for $5\times2=10$, and $13-10=3$; ∵ the integral Part of the Quotient is 2, and the fractional Part is 3 out of 5, or 3 Fifths; so that 5 is contained in 13 two Times, and 3 Parts of 5, of a Time more.

110. To divide any Number by another. First, see how many Times the Divisor is contained in as many Places of the Left-hand of the Dividend, as the Divisor consists of; (but, if you cannot go once, then take in one Figure more, and try how often the Divisor is contained in that Number, which cannot be more than nine Times;) and place the Figure expressing how many Times you can go, in the Quotient; (but remember, first, to see that the Dividual Minus the Product of the Figure which you go and Divisor be less than the Divisor;) then multiply the Divisor by that Quotient Figure, and subtract the Product from the before-mentioned Number, and to the Remainder bring down the next Figure of the Dividend (proceeding to the Right) and try how many Times the Divisor is contained in this Number, and multiply, subtract, &c. as before, 'till all the Figures of the Dividend are taken down. We must here observe, that, when any Remainder with one Figure of the Dividend annexed, as just now mentioned, is less than the Divisor, then, as we cannot take the Divisor out of it, we must put 0 in the Quotient, and take down another Figure of the Dividend, and then try how many Times we can go. But many Times the greatest Difficulty is in finding how many Times to go, which is to be found only by Trials; however, the following Observations may be of some Use in nearly determining the Times, *viz.* 1. By Art. 132. we can never go more than 9 Times. 2. If the Number, out of which the Divisor is to be taken, is of the same Number of Places, see how

many Times the first Figure of the Divisor is contained in the first (Left-hand) Figure of that Number; but, if that Number consists of one Place more than the Divisor, then see how often the first Figure of the Divisor is contained in the two first Figures of that Number, (called by some Authors the *Dividual* (*Dividuus*, Lat.) or *Partial* (*Partial*, Fr.) *Dividend*;) and the Number of Times we can take the Divisor out of the aforesaid Number, may be equal to, but cannot exceed the Times thus found. *Note*, It will be convenient, in order to prevent Mistakes, to make a Dot (.) under each Figure of the Dividend as we use it.

111. *Example* 1. Divide 2371 by 5.

```
               Dividend. Quotient.
Divisor 5 ) 2371 (474⅕
            20
            ——
             37   First Dividual.
             35
             ——
              21   Second Dividual.
              20
              ——
               1   Remainder.
```

First, See how many Times 5 in 23, which is 4 Times, (because $5\times5=25$ is too much; and $5\times3=15$, and $23-15=8$, which is greater than the Divisor, and ∵ 3 Times is too little, and consequently we must go 4 Times) ∴ place 4 in the Quotient, and, subtracting $5\times4=20$ from 23, there remains 3; to which take down the next Figure of the Dividend 7, and the first Dividual will be 37: Then the Divisor 5 is contained in 37 seven Times, ∴ place 7 in the Quotient, and subtract $5\times7=35$, from the first Dividual 37, the Remainder will be 2; to which having brought down the next Figure of the Dividend 1, the second Dividual is 21; in which the Divisor is contained 4 Times; ∴ put 4 in the Quotient, and then $5\times4=20$, and $21-20=1$, the Remainder.

mainder. As we have now taken down all the Figures of the Dividend, the integral Part of the Quotient is 474, and the fractional Part 1 out of 5; so that 5 is contained in 2371, 474 Times and $\frac{1}{5}$ (one Fifth) of another Time.

112. *Example* 2. Divide 800 by 25.

First, In order to know how many Times 25 in 80, try how many Times 2 in 8, which is 4 Times, because $25\times4=100$; ∴ try 3 Times, which we can go, for $25\times3=75$, and $80-75=5$, to which bring down the next Place of the Dividend, which is 0; then we have, how many Times 25 in 50? Which is 2 Times. See the Operation itself.

```
25)800(32
   75
   --
    50
    50
    --
     0
```

113. In long Operations the Method of tabulating the Divisor (as shewn in Multiplication, in tabulating the Multiplicand) is very useful, as we may see by Inspection the Times we can go, and the Product of each Time; so that there is very little Difficulty in performing Division by Help of a *Tariffa*; take an Example.

Divide 88350768 by 25476.

```
      The Tariffa.
      1 | 25476    25476)88350768(3468
      2 | 50952          76428
      3 | 76428          ------
      4 |101904          119227
      5 |127380          101904
      6 |152856          ------
      7 |178332           173236
      8 |203808           152856
      9 |229284          -------
        |------           203808
Proof 10|254760           203808
```

It is supposed that, if the Reader compares the Work of this Division with the Tariffa, he will want no other Explanation.

114. If

114. If the *Tariffa* is made on ſeparate Pieces of Paper, we may ſave the Trouble of writing the ſeveral Products under the Dividuals; for we may apply the Slips of Paper to them, and, ſo ſubtracting, only put down the Remainders, and then the Work will appear ſhortened thus:

```
25476) 88350768 (3468
       ________
        119227
       ________
         173236
       ________
          203808
       ________
               0
```

115. The Rule given in *Art.* 110, for finding how many Times one Number is contained in another, may be thus demonſtrated: It is plain, that by *Art.* 110. we part the Dividend into ſeveral Parts; for we firſt take a Part of the Dividend for a Dividual, and, having divided this, to the Remainder we add another Part of the Dividend; which being alſo divided, another Part of the Dividend is added to this Remainder for a new Dividual; and ſo on, 'till all the Parts of the Dividend have been added, and the Number of Times the Diviſor is contained in thoſe Parts been ſeparately found. Therefore, if the Method here laid down will find how many Times the Diviſor can be taken out of thoſe Parts, it will be all that is required: * For, as often as the Diviſor is contained in the Parts which make up the Dividend, ſo often muſt the Diviſor be contained in the whole Dividend. As to the Finding of each Figure of the Quotient ſingly, as the true Quotient of the Diviſor, out of the ſeveral (Parts of the Dividend or) Dividuals, conſidered by themſelves, we need no Demonſtration; becauſe they are found by Trials, and are not written down, 'till it is found that the reſpective Dividual, Minus the Product of the correſponding Quotient Figure into the Diviſor, is leſs than the Diviſor, and that therefore the Figure is taken right; ſince, the Remainder being leſs than the Diviſor, the Diviſor cannot be taken once more out of it; and, conſequently, we have taken the Diviſor out of the Dividual as often as poſſible. Whence

the only Thing which remains now to be ſhewn, is, that though we have taken the ſeveral Dividuals without Regard to their Places in the whole Dividend, and ſo have taken them in leſs than their true Value; yet that we have ſupplied that Defect by placing the ſeveral Quotes, or Parts of the whole Quotient, in ſuch Order, that they have the real Value they ought to have, if the ſeveral Dividuals had been taken in their real Values; and that, by ſo writing them, they will, when taken together as one Number, be equal to the Sum of the complete Values of the ſeveral Quotes; or, which is the ſame Thing, be equal to the whole Quotient, for the Reaſon above given *(at this Mark*)*. Now that this Defect, of taking the Dividuals in leſs than their real Value, may be ſupplied, will be ſhewn by conſidering; that the complete Value of each Dividual is 10, 100, 1000, &c. Times the Value in which it was taken in the Work, according as there is one, two, or three, &c. Figures to the Right-hand of its firſt Right-hand Figure in the Dividend: And therefore, if the reſpective Quotient Figure is placed ſo as to be in its real Value as many Times its ſimple Value, as the real Value of the Dividual is of its ſimple Value, which was taken in the Operation, the Defect will be ſupplied: (For ſuppoſe that in the ſimple Value of any Dividual the Diviſor is contained *m* Times, and that the real Value of the Dividual is *t* Times its ſimple Value in the Operation; now, if the Diviſor was contained *m* Times in the ſimple Value of the Dividual, certainly in *t* Times that Dividual the ſame Diviſor muſt be contained *t* Times *m*.) Now, that the Quotient Figure will ſtand in ſuch a Place, that its true Value may be 10, 100, or 1000, &c. Times its ſimple Value, according as the real Value of that Dividual is 10, 100, or 1000, &c. Times the ſimple Value of that Dividual, will plainly appear, by only conſidering, that there will be as many other Quotient Figures placed on the Right-hand of this Figure of the Quote, as there are remaining Figures of the Divi-

dend

dend to be taken down; becauſe, for each of theſe Figures, there is, by the Rule, a Figure to be put in the Quotient: Therefore, each Quotient Figure will have as many Places on the Right-hand of it, as there are Figures in the Dividend on the Right-hand of the firſt (Right-hand) Figure of the Dividual: Conſequently, its complete Value will be as many Times its ſimple Value, as the real Value of the reſpective Dividual is of the ſimple Dividual; and, therefore, each Figure in the Quotient will be ſo placed, as to have its true Value. Whence all thoſe Quotient Figures, taken together as one Number, muſt be equal to the required Quotient. *Q. E. D.*

116. As there are ſeveral Contractions in Multiplication, ſo are there alſo in Diviſion, the moſt uſeful of which are the following:

Caſe 1. When the Diviſor is 10, 100, or 1000, *&c.* we may cut off ſo many Figures from the Right-hand of the Dividend as there are Cyphers in the Diviſor: Thus, $257 \div 100 = 2\frac{57}{100}$; $23071 \div 1000 = 23\frac{71}{1000}$, *&c.* This is ſelf-evident.

117. *Caſe* 2. If the Diviſor has any Number of Cyphers annexed to the Right-hand of the Digits, we may cut off as many Figures from the (Right-hand of the) Dividend, as there are Cyphers on the Right-hand of the Diviſor, and divide the remaining Figures by each other; which will produce the ſame Quotient. For * the cutting off ſo many Figures from the Right-hand of the Dividend and Diviſor, as there are Cyphers on the Right-hand of the Diviſor, is dividing each of them by 10, 100, or 1000, *&c.* But it is evident, that, as often as the whole Diviſor is contained in the whole Dividend, ſo often muſt any Part of the Diviſor be contained in a like Part of the Dividend; therefore, the Quotient, found by this Article, muſt agree with the Quotient found by dividing the whole Dividend by the whole Diviſor. *Q. E. D.*

* 116.

Example. Divide 2576 by 2100.

Here, having cut off the two Cyphers from the Divisor, and the 76 from the Dividend, divide 25 by 21, the integral Quotient is 1, and 4 the Remainder, to which annexing the 76 (cut off) it is 476. Whence the Quotient is $1\frac{476}{2100}$.

$$21|00)\ 25|76\ (\ 1\tfrac{476}{2100}$$
$$\underline{21}$$
$$476$$

118. *Case* 3. When the Divisor is but one Figure, or can be reduced to one, by *Art.* 117; then the Operation may be easily performed in one Line, as is shewn in the following Example:

Divide 254 by 3.

Here, say the $\frac{1}{3}$ (third) of 25 is 8, and 1 remaining, (or, which is the same, how many Times 3 in 25? Which is 8 Times, and 1 remains) put down the 8, and suppose (in your Mind) the 1 which remains to be placed before the 4, and then we have the $\frac{1}{3}$ of $14 = 4$, and 2 remaining; put down the 4, and then the Quotient will appear to be $84\frac{2}{3}$.

$$\frac{1}{3}\ \frac{254 \text{ Dividend.}}{84\frac{2}{3} \text{ Quotient.}}$$

119. Again, divide 2476 by 200.

The Operation will stand thus: And the Answer is 12 and $\frac{76}{200}$.

$$2|00)\ 24|\ (\ 76$$
$$\frac{1}{2}\ \overline{}$$
$$12\tfrac{76}{200}.$$

120. When the Divisor is a composed Number, we may divide by the Parts of which it is com-
* 137. posed, and produce the * same Quotient as in the common Method. As to the fractional Part of the Quotient (if any) multiply the given Divisor by the integral Part of the Quotient, and deduct the Product from the Dividend, and the * Remainder will be the fractional Part of the Quotient.

* Put $d =$ the Divisor; $m =$ the Dividend; $q =$ the integral Part of the Quotient, $r =$ the Remainder, or fractional Part; then
* 123. * $dq + r = m$; ∴, subtracting dq from both Sides of the Equation, we have † $r = m - dq$. *Q. E. D.*
† 136.

Example.

Example. Divide 6431 by 48.

Here $8 \times 6 = 48$, ∴ we may divide by 8, and by 6, as shewn in the Operation annexed; which is sufficient to explain itself, if the Reader be already Master of the 118th Article.

$$8)\ 6431$$
$$6)\ 803 \mid 7 \text{ Remains.}$$
$$133 \mid 5 \text{ Remains.}$$

The Divisor 48×133 (the integral Part of the Quotient is) $= 6384$, ∴ the Remainder is $= 6431 - 6384 = 47$.

Note, The several Remainders may also be reduced to one by the following Rule: Multiply the last Remainder by the last Divisor but one, and to the Product add the Remainder belonging to that Divisor; multiply this Sum by the next preceding Divisor, to which add its correspondent Remainder; and thus proceed, 'till you have multiplied by the first Divisor, and added in the first Remainder. (The Reason of this may be seen in *Art.* 138.) Thus, in the above Example, the last Remainder is $= 5$, which, multiplied by the preceding Divisor 8, gives 40, for $5 \times 8 = 40$; to which adding the correspondent Remainder 7, we get $40 + 7 = 47 =$ the Remainder belonging to the Divisor 48; so that the Quotient is $6384\frac{47}{48}$.

121. When the Learner is become ready in the common Method of Division, he may shorten his Work, by multiplying the Divisor by each Figure of the Quotient in his Mind, and subtracting each Figure of the Product, one by one, as they come out from the respective Dividual, setting down only the Remainder either above, or below, the Dividend, according to the Pleasure of the Arithmetician.

Take an *Example.*

Divide 3583 by 25, and place the Remainders over, and the Quotient under the Dividend. To make this appear as clear as possible, we will represent the Work as it stands at the getting each Quotient Figure.

$25\,)\,\begin{matrix} 10 \\ 3583 \\ 1 \end{matrix}$ Remainders. Dividend. Quotient.	$25\,)\,\begin{matrix} 108 \\ 3583 \\ 14 \end{matrix}$	$25\,)\,\begin{matrix} 108(^{8} \\ 3583 \\ 143\frac{8}{25} \end{matrix}$ Rem. Quot.

First, We have 25 in 35, ∵ 1 is the first Quotient Figure, which placed, say $5 - 5 = 0$, which put down, and $3 - 2 = 1$, which being also written down, the first Remainder is 10; ∵ the first Dividual is 108, in which 25 is contained 4 Times, and there remains 8; then, the third Dividual being 83, out of which 25 can be taken 3 Times, say $5 \times 3 = 15$, and, as we cannot take 5 from 3, we have $13 - 5 = 8$, which put over; then $2 \times 3 = 6$, + 1 we carry $= 7$, + 1 we borrowed (because, as we could not take 5 from 3, we took 5 from 13, and ∵, as the 3 was called $13 = 10 + 3$, we must also increase the Product by 10, which is done by adding 1 to it) $= 8$, and $8 - 8 = 0$. Hence the Quotient is $143\frac{8}{25}$.

It is common, to prevent Confusion, to dash out the Figures as they are used; and then the Steps would appear thus:

$25\,)\,\begin{matrix} 10 \\ \cancel{3}\cancel{5}83 \\ 1 \end{matrix}$	$25\,)\,\begin{matrix} \cancel{1}\cancel{0}8 \\ \cancel{3}\cancel{5}83 \\ 14 \end{matrix}$	$25\,)\,\begin{matrix} \cancel{1}\cancel{0}\cancel{8}(^{8} \\ \cancel{3}\cancel{5}\cancel{8}\cancel{3} \\ 143\frac{8}{24} \end{matrix}$

Whence this Method is commonly called Scratch Division. It will appear sufficiently clear, without dashing out the Figures, by only placing the Remainders under the Dividend, as is here shewn:

$$25\,)\,\begin{matrix} 143\frac{8}{25} \\ 3583 \\ 10\cdot\cdot \\ 8\cdot \\ 8 \end{matrix}\quad\begin{matrix} \text{Quotient.} \\ \text{Dividend.} \\ \left.\begin{matrix} \\ \\ \\ \end{matrix}\right\} \text{Remainders.} \end{matrix}$$

Note,

Note, It will be proper to obſerve, to prevent forgetting the adding 1 (when we borrow) to the Product, that, whenever looking back we ſee the next Right-hand Figure of any Remainder greater than the reſpective Figure of the Dividual, it is plain we (borrowed, or in other Words) increaſed the Dividual by 1, in the next higher Place; and, therefore, we muſt again take away 1 from that Place, or (which is the ſame in Effect) add 1 to the next Figure of the Product.

122. If we perform the Example in the laſt Article, by what the late ingenious Mr *Lowe* calls his Rule (given below) the Figures would ſtand thus:

Dividend	
3583	) 25 Diviſor.
1088	) $143\frac{8}{2}$ Quotient.

But this is ſo very near the Method given in the laſt Article, that it ſeems ſcarce able to bear the Name of a different Method.

Mr. *Lowe*, ſpeaking of this Method, ſays, he offers it * "as much the ſhorteſt and eaſieſt, and, by the Diſ-
" poſition of the Figures, the moſt commodious for
" Operation, Proof, Valuation of Remainders, &c."

His RULE is,

1 | † Dív. (1) ſay, How-óft-ſor in-dénd;
Or the 1ſt. in the íſt. (2) By the
Anſwer (which is to be plac'd in the Quotient)
2 | Múltiply-ſór; (3) and the Product ſubtract
From the-dend, by Addition
3 | Thén, for next Stép, advance í in the-dénd;
And count-báck, i'th'-Remainder,
4 | Só many ás are i'th'-ſór: There begín, as at
fírſt; and ſay, How-oft, &c.

* P. 40. of his *Arithmetick*.

† This is what Mr. *Lowe* calls Hexameter Verſes, and will ſerve as a Specimen of that Gentleman's Method of Writing on Arithmetick.

To illustrate this Rule, take an Example, with the Explanation in Mr. *Lowe*'s own Words.

Example. " Divide 365365 by 121.

" Say (first) for " the first Dividual, " or partial Divi- " dend, How-oft " 1 in 3, *&c.* Then " (2) for the 2d Step, " advance one Place " in the Dividend, " to wit to 3; and " count-back (in " the Remainder) " so many Figures " as there are Places " in the Divisor, to wit 3: so the Reckoning will end " in 0; which (since you cannot have 1 in 0) enter " in the Quotient. Then (3) for the third Step, " do as in the second, and the Reckoning will end in " 2: and There begin as at first; and say, How-oft " 1 in 2, *&c.* Then (4) for the fourth Step, do " as in the last; and the Reckoning will end at " 11: and There, again, begin as at first; and say, " as afore, How often 1 in 11, *&c.*"

First Step.	365365 02	Divisor. 121 3 Quot.
2d Step.	365365 02	121 D. 30 Q.
3d Step.	365365 0215 1	121 D. 301 Q.
4th Step.	365365 02156 106	121 D. 3019 Q.

Scholium. As a further Specimen of this Method of delivering Rules, may be given the following Rule for performing Multiplication in one Line, shewn in *Art.* 81, *viz.*

1. Unit's Place óf-cator ínto-cand: Múlt, and add
   áll between each Step;
   Múltiplying báckward in-cator; fórward in-cánd;
   'till the-cand's out.
2. Thén, by Steps, rést of the-cator ínto the first
   of the-cand Mult:
   And the Fígures between each Stép, as afore,
   mult, and add.

Note,

Note, These Verses having the Cadence of *Latin* * Hexameters, the Accent, which is added to ascertain the right Reading of them, denotes the first Syllable of a Dactyl, or that the two following are to be pronounced short.

123. Division may be proved, by adding the Remainder to the Product of the Divisor into the integral Part of the Quote; for that Sum must be † equal to the Dividend. Thus, in the last Example, the Divisor 121 x 3019 the integral Part of the Quotient, is $= 365299$, $+$ the Remainder $66 = 365365$, the Dividend, for Proof.

124. Division may be proved by Division, for, the Dividend being divided by the Quotient, the Quotient, found by this Division, must be equal to the former Divisor, if there was no Remainder in the given Division. But, if there was a Remainder, first subtract that Remainder from the Dividend, and then divide as before directed ‡.

125.

* Hexameter Verses consist of six Feet; for Kind, Dactyls and Spondees; a Dactyl contains a long and two short Syllables; a Spondee has but two Syllables, and both long. In all Hexameters the fifth Foot is a Dactyl, the sixth a Spondee.

† Let $d =$ the Divisor, $m =$ the Dividend, $q =$ the integral Part of the Quotient; $r =$ the Remainder; now, r being the Remainder, it is what m is more than a Multiple of d, and ∴, taking r from it, we shall have $m - r$ a Multiple of d, which, by Supposition, is contained in that Multiple q times; that is, * $\overline{m - r} \div d = q$; ∴ multiplying by d we have $\overline{m - r}$ † $= dq$, and by adding r to both Sides of this Equation we have m ‡ $= dq + r$. *Q. E. D.* * 102. † 56. ‡ 22.

Corollary. When $r = o$, it is $m = dq$.

‡ Things being as in the Note to *Art.* 123, we have (there) $m - r = dq$, ∴, dividing both Sides of the Equation by q, we have $\overline{m - r} \div q$ ‖ $= d$. *Q. E. D.* ‖ 108.

Corollary. When $r = o$, then $m \div q = d$.

Scholium. Mr. *Malcolm* says, "the Dividend being divided by "the integral Quote, the Quote of this Division will be equal to "the former Divisor with the same Remainder. Thus, 3 is con- "tained 4 Times in 14, and 2 remains: But 4 Times 3 = 3 "Times 4; ∴ 4 must be contained 3 Times in 14, with the same "Remainder 2, as it actually is. The same Reason is good in all "Cases." But this Gentleman (who is justly esteemed for his Learning) by making Use of particular Numbers, instead of a general

130. As to the Reason of the above Rule, it may be shewn thus: In the first Example it was
* 102. required to divide 347631 by 997. Now, * Division being the taking the Divisor from the Dividend as often as possible, the Reason will easily appear; for first, dividing by the New Divisor 1000, the in-
† 116. tegral Quotient will be † 347, and the Remainder 631: Now, if this be the true Quotient, then
‡ 123. $347\times997+631$ will be ‡ = the Dividend; but, as it is too little, there must still remain of the Dividend $347631-\overline{347\times997+631}=347000-347\times997=347\times3$ (because, if the Minorand was 347×1000, it would be $=347000$, and so nothing remain; and ∴, as 997 wants but 3 of 1000, there can remain only 3 Times 347) $=1041$: Now, as this is Part of the Dividend, we must take the Divisor out of it as often as possible; that is, we must divide it by 997; but, if we divide it by 1000, the integral Quotient will be 1, and 1 remaining; which it is evident is too little, (because we took a greater Divisor than we ought) and ∴ there is still remaining of the Dividend $1041-\overline{1\times997+41}$ (by the above Method of reasoning) $=1\times3=3$; but we cannot take the Divisor out of this, (3) and ∴ it must be a Remainder. Now, since all the Parts taken together are equal to the Whole, the Sum of these several integral Quotients and Remainders must be equal to that required. That is, in this Example, since the Divisor is contained in the Dividend 347 Times, and 631 remaining, + 1 Time, and 41 remaining, + 0 Time, and 3 remaining, it must, when taken out as often as possible, be = 348 Times, and 675 remaining. And it is evident this Method of reasoning holds good in all other Cases, and ∴, this Method being the same as given in the above Rule, its Truth is manifest.

131. In Algebraick or Universal Division, we make Use of this Character ÷, to express Division by. Thus $a \div b$ is to be read, a divided by b. The Vinculum is used in Division as well as in Multiplication:

cation; thus, $\overline{a-b} \div b$ is to be read, from a subtract b, and divide the Remainder by b; this, according to another Way of writing it, would stand thus, $\frac{a-b}{b}$; it being common amongst Algebraists to express a Division by placing the Divisor under the Dividend.

132. When the Dividend is a Multiple of the Divisor, the Quotient may be represented in more simple Terms than it can be by the last Article; and for the Method of finding such Quotients observe this Rule: Set down the Divisor and Dividend, and work in the same Manner as in Division of Numbers, shewn in *Art.* 110; and then for the Signs of the Quotient remember (1.) that + divided by + gives + in the Quotient. (2.) — divided by — gives + in the Quotient. (3.) + divided by — the Quotient will be —. (4.) — divided by + is —. The Reason of the Signs will easily appear by considering, that the Dividend is equal to * the Divisor in the * 123.
Quotient; and the Nature of the Signs + and — in Multiplication.

133. For the first Example, divide ax by a.

Here say, how often a in ax, which it is plain is x Times, for $a \times x = ax$; and, subtracting this ax from the Dividend, the Remainder is nothing, and ∵ therefore the Quotient is x.

$$\begin{array}{rl} & \text{Dividend.} \\ \text{Divisor } a\,) & ax\ (x \text{ Quot.} \\ & \underline{ax} \end{array}$$

134. Let the second Example be the Converse of *Art.* 90. *viz.* divide $ax - bx$ by x.

Here we have, first, how often the Divisor x in ax the first Term of the Dividend, which is a Times, for $x \times a = ax$; and ∵, subtracting ax from the Dividend, there remains $-bx$, in which x is contained $-b$ Times; ∵ the Quotient is $a-b$.

$$\begin{array}{rl} x\,) & ax - bx\ (a-b \\ & \underline{ax} \\ & -bx \\ & \underline{-bx} \\ & \cdot \end{array}$$

‖ 19. is less than ‖ the lesser Number with the Cypher
annexed; but the adding this Cypher was multiply-
‡ 67. ing by 10 ‡; consequently 10 Times the lesser Number is greater than the greater Number, and therefore cannot be taken from it.

137. *Theorem* 2. If any Divisor be produced by the continued Multiplication of any Quantities a, b, c, &c. that is, if a, b, c, &c. = the Divisor; then, dividing the Dividend by a, and the first Quote by b, the next Quote by c, &c. the last Quote will be the same as that found by the Dividend by the whole Divisor.

Demonstration. Put $x =$ the Dividend, and let $\frac{x}{a}$
* 123. $= m$, $\frac{m}{b} = n$, $\frac{n}{c} = q$, &c. then n * $= cq$, $m = bn$,
$x = am$: but, by multiplying both Sides of the Equa-
† 56. tion $n = cq$ by b, we have bn † $= bcq$, ∵ ‖ $m = bcq$;
‖ 23. again, multiplying both Sides of this Equation by a,
we have $am = abcq$, and ∵ $abcq$ ‡ $= x$, ∵ dividing
‡ 23.
* 108. by abc we have *$q = \frac{x}{abc}$. Q. E. D.

138. *Theorem* 3. If, when the Divisor is a composed Number, the integral Part of the Quote be found by dividing by the Parts of which the Divisor is composed as shewn in *Art.* 120; then we say the Remainder, which would have been found by dividing the whole Dividend by the whole Divisor, may be found by the Rule given in the latter Part of that Article.

Demonstration. We shall demonstrate this Theorem to be true when the Divisor is composed of three Numbers; and after the same Manner it may be demonstrated when the Divisor is composed of four or more Numbers. Let $abc =$ the Divisor, $x =$ the Dividend, $m =$ the first integral Quote, $r =$ the corresponding Remainder, $n =$ the second Quote, $s =$ the Remainder, $q =$ the third Quote, $t =$ the Remainder; R = the Remainder found by dividing
† 123. the whole Dividend by the whole Divisor. From the Nature of Division $abcq + R$ † $= x$; ∵, taking $abcq$ from

from both Sides of the Equation, we have R * $= x - abcq$; and † $cq + t = n$, $bn + s = m$, $am + r = x$; whence, putting $am + r$ for x in the Expression for R, we get R $= am + r - abcq$; and, taking t from both Sides of the Equation $cq + t = n$, we get ‡ $cq = n - t$, which divided by c is q ‖ $= \frac{n-t}{c}$; which put for q, in the last Expression for R; and then $R = am + r - abc \times \frac{n-t}{c} = am + r - ab \times \overline{n-t}$ (because, the last Term being both multiplied and divided by c, it is plain that c may be omitted) $= am + r - abn + abt$. The Equation $bn + s = m$, being multiplied by a, will give $abn + as$ * $= am$, and ∴, writing this Value of am for am in the last Equation for R, we shall have $R = abn + as + r - abn + abt$, out of which striking away the contradictory Terms, it will become $R = as + r + abt$ or, which is the same, $= tba + sa + r$, which is the same as the Rule in the latter Part of the Article 120. *Q. E. D.*

* 36. † 123. ‡ 36. ‖ 108. * 56.

This Theorem may be demonstrated otherwise: Thus, let $q =$ the integral Part of the last Quotient, $c =$ the last Divisor, $t =$ the last Remainder; then $qc + t$ † $=$ the last Dividend, which in this Case is the last Quotient but one; ∵ if we put b and s for the last Divisor but one, and last Remainder but one, respectively, we shall have ($\overline{qc + t} \times b + s =$) $qcb + bt + s$ ‡ $=$ the next preceding Dividend; hence, if there are but two Divisors c and b, the Remainder is $bt + s$; because, dividing $qcb + bt + s$ by cb, the Quotient is $q +$ ‖ $\frac{bt+s}{cb}$, and ∵ the Rule is true in

† 123 ‡ 123.

F

‖ If the young Student should not see readily the Reason, that $bt + s$ is the Remainder, that is, why $\frac{bt+s}{cb}$ is the fractional Part of the Quote, or, which is the same, why cb cannot be contained once in $bt + s$, he is desired to observe, that, t being a Remainder belonging to the Divisor c, it must at least be one less than c, and ∴, in

in two Divisors. Q. E. D. But, if there are three Divisors, then $qcb + bt + s =$ the integral Part of the first Quotient, and ∵ if we put $a =$ the first Divisor, and $r =$ its Remainder, we shall have
* 123. $(\overline{qcb + bt + s} \times a + r =)$ $qabc + bta + sa + r$* = the first or given Dividend; and ∵ in three Divisors the Remainder is $bta + sa + r$, because, $qabc + \overline{bta + sa + r}$ being divided by abc, the integral Part of the Quotient is q, and the fractional Part $\frac{bta + sa + r}{abc}$. Q. E. D.

CHAP. VII.

Of Applicate Numbers,

With Tables of the Money, Weights, and Measures of Great Britain.

139. WE hitherto have, in Addition, Subtraction, Multiplication, and Division, confined ourselves to abstract whole Numbers; that is, we have considered Numbers barely as to the Number of Things, without any Relation to (for we abstracted from, *i. e.* did not attend to) the particular Kind of Things numbered. Whence abstract Numbers are those which are considered as pure Numbers, without being applied to any particular Subject; but we must now proceed to apply Numbers to particular Things, and the Numbers so applied are called applicate Numbers, or we are then said to consider Numbers in the Concrete. Thus, 3, when taken abstractedly,

*∵, in the Product cb, the b must at least be taken once more than in the Product bt; and ∵ cb must at least be more than bt by bt; and ∵, since s is the Remainder belonging to the Divisor b, it must be less than b, and ∵, though it be added to bt, it cannot make up the Defect b; whence, $bt + s$ is always less than cb, and ∵ cb cannot be taken once out of that Sum; and consequently $bt + s$ must be a Remainder.

ſtractedly, ſignifies in general only 3 Things; but, in the Concrete, we ſay, 3 Men, 3 Pounds, 3 Yards, &c.

140. When we conſider Numbers in the Abſtract, whatever is true of thoſe Numbers of Things, is true of the ſame Numbers in whatever Things they are found; for Inſtance, 2 Things is more than 1 by 1 Thing; ſo 2 Yards is 1 more than 1 Yard: But when we conſider them in the Concrete, as we muſt have Regard to the Subjects in which the Numbers are found, as well as to the Numbers themſelves, what is true of them taken in the Abſtract, may not be ſo in the Concrete; for Inſtance, 2 Feet is leſs than 1 Yard, becauſe a Yard is 3 Feet; whereas, in the Abſtract, 2 is more than 1; whence it appears, that, in applicate Numbers, we do not barely conſider the Number of Things, but have alſo Reſpect to the Lengths or Quantities of thoſe Things, in which the Numbers are found. Farther, it muſt be obſerved, that, in comparing applicate Numbers, they muſt be all of one Kind, *viz.* all Lengths, or Weights, &c. For what Compariſon can there be between Yards and Pounds? Or what Relation between Ounces and Buſhels? This it was proper to hint, becauſe on it depends the true Senſe, and Poſſibility or Impoſſibility, of ſome Queſtions; for Example, if it was required to add 2 Ounces and 2 Buſhels together, it is plain the Queſtion would be not only improper, but alſo Nonſenſe and impoſſible.

141. As, for the Conveniency of Commerce, it was neceſſary to make Uſe of ſome Standards, for Weights, Meaſures, &c. adapted to the different Kinds of Things to be weighed or meaſured, it will be neceſſary, before we proceed to the Rules relating to applicate Numbers, to give ſome Account of thoſe Weights and Meaſures, and of the Money made Uſe of in eſtimating the Value of the Things weighed or meaſured. At preſent we ſhall confine ourſelves to thoſe commonly uſed in *Great Britain.*

Of Money.

4 Farthings	=	a Penny,
12 Pence	=	a Shilling
20 Shillings	=	a Pound.

Note also,

4 Pence	=	a Groat
6 Pence	=	a Tester
5 Shillings	=	a Crown
6 Shillings and 8 Pence	=	a Noble
10 Shillings	=	an Angel
13 Shillings and 4 Pence	=	a Mark
21 Shillings	=	a Guinea.

The real Coins now current, and commonly known, are these. 1. Of Copper Money, a Farthing, a Half-penny. 2. Of Silver, Sixpence, a Shilling, Half-crown, a Crown. 3. Of Gold-money, Half-guinea, a Guinea.

We have also some foreign Gold Coins current amongst us, *viz.* a Moidore which passes for 27 Shillings; another *Portugal* Piece for 3 Pounds and 12 Shillings, the Half, Quarter, &c. of that Piece.

Note, In *Scotland*, Accounts are kept in Pounds, Shillings, and Pence, *Scotch*; 12 Pounds *Scotch* being = 1 Pound *English*. But (as Mr. *Malcolm* informs us) they now begin to use *English* Money in their Accounts.

Note also, that when *£. s. d. q.* are written over any Figures (or to the Right-hand of the Figures) they denote Pounds, Shillings, Pence, and Farthings respectively. Thus, 3 Pounds, 7 Shillings, and 2 Pence 3 Farthings, may be written thus, *£. s. d. qs.* 3 : 7 : 2 : 3, or 3*£.* 7*s.* 2*d.* 3*q.* Some also write the Farthings like Fractions, thus, 3*£.* 7*s.* 2*d.* $\frac{3}{4}$.

The Goldsmiths, &c. express the Fineness of Gold by Caracts, which are not any particular Weight; a Caract only denoting the 24th Part of the

the Weight of the whole Quantity; and if, in any Quantity of Gold, there are 22 ſuch Parts of pure Gold, and the remaining two Parts Silver (as our Guineas are, or any other Allay, as Copper, which makes the Gold Coins in which it is uſed, appear ſomething different in Colour) then it is ſaid to be according to Standard. But if, upon aſſaying a Piece of Gold, it is found to contain more or leſs than 22 Caracts of pure Gold, it is ſaid to be ſo many Caracts, or Parts of a Caract, better or worſe than Standard. In Aſſaying of Silver, the Fineneſs is computed in Penny-weights. See *Troy-weight*.

142. *Of* TROY WEIGHT.

24 Grains	=	1 Penny-weight	mark'd thus	Dwts.
20 Penny-weights	=	1 Ounce	mark'd thus	Oz.
12 Ounces	=	1 Pound	mark'd thus	℔.

Mr. *Ward* found by Experiment, that 14 oz. 11 dwts. 15½ Grains Troy, = 1 ℔ Avoirdupois.

According to Mr. *Chamberlayne* in his *Preſent State of Great Britain*, the Moniers (as he calls them) divide the Grain into the following Parts, *viz.*

24 Blanks	=	1 Perit
20 Perits	=	1 Droite
24 Droites	=	1 Mite
20 Mites	=	1 Grain.

But, certainly, the Diviſions lower than Mites muſt be imaginary only; for to conſtruct a Scale for weighing the lower Diviſions ſeems to me impoſſible; for, if Blanks have a real Exiſtence, the Grain will be divided into 230400 Parts, a Thing ſurpaſſing the Belief even of the moſt Credulous.

By Troy Weight, are weighed Gold, Silver, Jewels, and Liquors.

The Fineneſs of Silver is computed in Ounces and Penny-weights, thus: If, in a Pound of Silver, there are 11 Ounces 2 Penny-weights of fine Silver, and the other 18 Penny-weights of Copper, or any other

other Allay, it is then the Standard for our own Silver Coin; and it is then said to be 11 Ounces 2 Penny-weights fine.

A TABLE *shewing into how many Shillings a Pound Weight of Silver hath at several Times been coined, from Mr.* Lowndes *and Bishop* Fleetwood, *being very useful to Readers of the History of* England, *we have transcribed the following from the* 23*d Vol. of the* Gentleman's Magazine.

Years.	Fineness.		Shillings	Pence.
	oz.	dwts.		
28 E. I.	11	2	20	3
20 E. III.	11	2	22	6
27 E. III.	11	2	25	
9 H. V.	11	2	30	
1 H. VI.	11	2	37	6
4 H. VI.	11	2	30	
24 H. VI.	11	2	30	
49 (39) H. VI.	11	2	37	6
5, 8, 11, 16. 24 E. IV. 1 R. III. 9 H. VII.	11	2	37	6
1 H. VIII.	11	2	45	
34 H. VIII.	10	0	48	
36 H. VIII.	6	0	48	
37 H. VIII.	4	0	48	
1 E. VI.	4	0	48	
3 E. VI.	6	0	72	
5 E. VI.	3	0	72	
6 E. VI.	11	1	60	
2 Mary	11	0	60	
2 Eliz.	11	2	60	
19 Eliz.	11	2	60	
43 Eliz.	11	2	62	

Which Standard has continued ever since.

A Pound Troy of Gold is cut into $44\frac{1}{2}$ Parts, each Part is a Guinea, or 21 Shillings. At first, each Part was to pass for 20 Shillings; and some Time after, by the Scarcity of Gold imported, each Part passed current for 21 Shillings and 6 Pence.

143. *Of* Apothecaries Weight.

20 Grains	=	1 Scruple	marked thus	℈
3 Scruples	=	1 Dram		ʒ
8 Drams	=	1 Ounce		℥
12 Ounces	=	1 Pound		lb.

Note the lb is = the Pound Troy.

By this Weight Apothecaries compound their Medicines, but buy and sell their Drugs by the Avoirdupois Weight.

The *Scotch* Ounce is 4 Grains less than the *English* Troy Ounce.

144. Avoirdupois Weight.

16 Drams	=	1 Ounce	marked thus	oz.
16 Ounces	=	1 Pound		lb.
28 Pounds	=	1 Quarter		qr.
4 Quarters	=	1 Hundred		C.
20 Hundred	=	1 Tun		Tun.

By this Weight are weighed all Kinds of Grocery Wares, and most other Things in common Business.

Note, A Pound Troy is = $13\frac{1}{6}$ oz. Avoirdupois, very exact; therefore 72 Ounces Troy are = 79 Ounces Avoirdupois, very near.

In weighing Wool, the Avoirdupois Pound is used; but the greater Weights are thus:

7 Pounds, according to most Authors, but Mr. *Lowe* says 8 Pounds,	=	1 Clove
2 Cloves	=	1 Stone
2 Stones	=	1 Tod
$6\frac{1}{2}$ Tods	=	1 Wey
2 Weys	=	1 Sack
12 Sacks	=	1 Last.

Note, Wool is commonly bought by the Tod; but, when it is stapled or sorted, it is sold by the Pack. See *Pack* in the next Table.

145. LONG MEASURE.

12 Inches	=	1 Foot
3 Feet	=	1 Yard
$5\frac{1}{2}$ Yards	=	1 Pole, Perch, or Land Yard, or Rod.
40 Poles	=	1 Furlong
8 Furlongs	=	1 Mile, (the greatest Measure of Length commonly used.)
3 Miles	=	1 League, (used chiefly by Sailors.)

Note, Four Statute Poles = 1 *Gunter*'s Chain, which Chain is divided into 100 equal Parts called Links. This Chain is very much esteemed in measuring of Land.

Sailors divide a Degree of the Earth into 60 equal Parts, which they call a Geographical Mile; but a Degree is $69\frac{1}{2}$ Statute Miles nearly; of which more hereafter. The *Scotch* Foot is = $12\frac{1}{11}$ *English* Inches.

Cloth is measured by the Yard of 3 Feet, which is thus divided:

4 Nails	=	1 Quarter
4 Quarters	=	1 Yard.

We have also a Measure for Cloth, called an Ell; which is = 5 Quarters of a Yard. Also, an Ell *Flemish* (by which Tapestry is measured) is = 3 Quarters of a Yard. As to the *Scotch* Ell, according to the Standard at *Edinburgh*, it is = $37\frac{2}{10}$ *English* Inches, *i. e.* an Inch and $\frac{2}{10}$ longer than the *English* Yard.

146.

146. LIQUID MEASURE.

1. *Of* Wine.

2 Pints	=	1 Quart
4 Quarts	=	1 Gallon
42 Gallons	=	1 Tierce
31 $\frac{1}{2}$ Gallons	=	1 Barrel
63 Gallons, or 1 $\frac{1}{2}$ Tierce,	=	1 Hogshead
84 Gallons, or 1 $\frac{1}{3}$ Tierce,	=	1 Punchion
2 Hogsheads, or 1 $\frac{1}{2}$ Punchion,	=	1 Butt
2 Butts, or Pipes,	=	1 Tun.

By the Wine Gallon, are measured and sold all Wines, Spirits, Brandies, Mead, Perry, Cyder, Vinegar, Honey, Oil, *&c.*

The Beer or Ale Gallon is greater than the Wine Gallon, in Proportion of 282 to 231; (these Numbers being the Number of Cubic Inches in each) or in smaller Numbers as 94 to 77.

The Measures for Ale or Beer in *London* are,

2 Pints	=	1 Quart
4 Quarts	=	1 Gallon
8 Gallons of Ale / 9 Gallons of Beer	=	1 Firkin
2 Firkins	=	1 Kilderkin
2 Kilderkins	=	1 Barrel
1 $\frac{1}{2}$ Barrels.	=	1 Hogshead.

But this Distinction of Ale and Beer Measures is only used in *London*; for in all other Places of *England*, Mr. *Ward* says, 8 $\frac{1}{2}$ Gallons of Ale or Beer = a Firkin.

In *Scotland* (Mr. *Malcolm* says) the common Denominations of Liquid Measure are these, Hogshead, Gallon, Pint, Mutchkin, Gill; and 4 Gills = 1 Mutchkin, 4 Mutchkins = 1 Pint, 8 Pints = 1 Gallon, and 16 Gallons = 1 Hogshead. They also call 2 Mutchkins, 1 Chopin, and 2 Pints, 1 Quart. The Excise in *Scotland*, since the Union of the two Nations, is calculated upon *English* Measure.

The

The *Scotch* Pint contains $103\frac{4}{10}$, and the Beer Firlot $2150\frac{40}{100}$ Cubic Inches.

147. DRY MEASURE, *called also* CORN MEASURE.

2 Pints	=	1 Quart
2 Quarts	=	1 Pottle
2 Pottles	=	1 Gallon
2 Gallons	=	1 Peck
4 Pecks	=	1 Bushel of Corn
5 Pecks	=	1 Bushel of Water
4 Bushels	=	1 Coomb or Sack
2 Coombs	=	1 Quarter
4 Quarters	=	1 Chalder
5 Quarters	=	1 Tun or Wey
2 Weys	=	1 Last.

This Table is agreeable to most Authors, but Mr. *Ward* says, 10 Quarters = 1 Wey, 12 Weys = 1 Last of Corn.

Note, The *Winchester* (or Corn) Bushel, having a plain round Bottom $18\frac{1}{2}$ Inches wide, and being 8 Inches deep, is, according to the Standard in his Majesty's *Exchequer*, a lawful Bushel. Hence it may be found, by any one acquainted with the first Principles of Gauging, that the Corn Gallon contains $268\frac{4}{5}$ Cubic Inches; therefore, the Ale Gallon is to the Corn Gallon as 282 to $268\frac{4}{5}$, or, which is the same, as 235 to 224.

In *Scotland*, Mr. *Malcolm* says, the common Denominations of Corn Measure are, 4 Quarters = 1 Peck, 4 Pecks = 1 Bushel, 4 Bushels = 1 Boll, 16 Bolls = 1 Chaldron. But they are different Measures from the *English* of the same Name.

According to the Experiments of Mr. *James Gray*, (in the Physical and Literary Essays of a Society in *Edinburgh*) the Wheat Firlot of *Scotland* contains $2197\frac{34}{100}$ Cubic Inches. To this Gentleman it is we are obliged for the Comparison of the other *Scotch* Measures abovementioned.

148. Of Time.

By Time, we only mean the Quantity or Measure of any Duration of Things, which Duration is in a continual and equable Flux; and therefore, that we may be able to communicate our Thoughts of so long or so short a Duration, we must make Use of some Contrivance to measure Durations with: And the most natural that occurred to Mankind, and which hath been agreed on by the Consent of all Nations, is, that the Interval of Time elapsed between the Instant when the Center of the Sun is on the Meridian, or due South, and its Return after one Revolution (real or apparent) to the same Meridian, should be called a Day; which Day they subdivided into 24 equal Parts called Hours, and each of these again into 60 equal Parts called Minutes, and each Minute into 60 equal Parts called Seconds, &c.

Further, the Time which is elapsed between the Instant when the Sun is in the Equinox, or first Point of *Aries*, as the Astronomers express themselves, to the Time of its Return to the same Place after one Revolution, is found by them to contain 365 Days, 5 Hours, 48 Minutes, 55 Seconds; which they call a tropical Year: But, as these odd Hours, Minutes, and Seconds, are not convenient in common Affairs, they made a common Year to be 365 Days; but as by this Reckoning they omitted $5^h. 48'. 55''$, which in 4 Years would amount to near a Day, they made every fourth Year, called Bissextile, or Leap-Year, consist of 366 Days. Whence this Table:

$$\left.\begin{array}{r l} 60 & \text{Seconds} \\ 60 & \text{Minutes} \\ 24 & \text{Hours} \\ 365 & \text{Days} \\ 366 & \text{Days} \end{array}\right\} = \left\{\begin{array}{l l} 1 & \text{Minute} \\ 1 & \text{Hour} \\ 1 & \text{Day} \\ 1 & \text{Common Year} \\ 1 & \text{Leap-Year} \end{array}\right.$$

Here it is evident, that the calling a common Year 365 Days, and every fourth Year 366 Days, is in

Effect

Effect the same as calling every * Year 365 Days 6 Hours.

In

* In the Year 325, when the *Nicene* Council was held, History informs us, that, amongst other Things, that Assembly agreed, that *Easter* should be celebrated on the *Sunday* next after the 14th Day of the Moon, that should follow next after the vernal Equinox, which they then fixed on the 21st of *March*. And we may remember, that in the Year 1752 the vernal Equinox was on or about the 10th of *March*, so that, through some Cause, the Equinox happened about 11 Days sooner in the Year 1752, than it did in the Year 325; which may be thus accounted for: In making each Year 365 Days 6 Hours, called *Julian* Years, instead of 365 Days 5 Hours 48 Minutes 55 Seconds, we make each Year about 11′. 5″ too much, which in 1427 Years, the Time from the Year 325 to 1752, will amount to near 11 Days. This being the Case, most Nations in *Europe* before, and *Great Britain* in the Year 1752, adjusted the Calendar, so as to bring the Equinox on the same Day it was on at the Time of the *Nicene* Council. And (that it might agree with foreign Accounts) it was done after the following Manner, *viz.* In the Year 1751 it was ordered by an Act of Parliament, "that the "natural Day, next immediately following the Second of *September* "1752, should be reckoned and accounted the 14th Day of *September*, the next to that the 15th. And so on in the natural Order." By which Omission of 11 nominal Days in *September* 1752, it is evident that that natural Day in 1753, which would, if the Old Stile had continued, have been called the 10th of *March*, was by this Means called the (10 + 11 =) 21st of *March* New Stile; and therefore the Equinoxes and Seasons were reduced to the same nominal Days, as at the Time of the *Nicene* Council abovementioned. But since the 11 Minutes 5 Seconds, which the tropical Year is less than the *Julian*, will in 400 Years amount to 3 Days, in Order to prevent the Falling back of the Equinoxes in future Ages, it was further enacted, "that in the several Years of our Lord 1800, "1900, 2100, 2200, 2300, or any other hundredth Years of our "Lord in Time to come (except only every fourth hundredth "Year, whereof the Year of our Lord 2000 shall be the first) shall "not be taken to be Leap-Years, but shall be common Years, con- "sisting of 365 Days; and that the Year of our Lord 2000, 2400, "2800, and every fourth hundredth Year of our Lord, from the "Year 2000 inclusive, and also all other Years of our Lord, which "by the present Supputation are Leap-Years, consisting of 366 "Days, shall be Leap-Years as is now used." And by this Method it is evident, that the Falling back of the Equinox 3 Days, in four hundred Years, will be prevented.—But perhaps some Astronomers may object, that in the Year 325 the vernal Equinox was on the 20th of *March*, and in the Year 1752 on *March* 9th. Be it so, yet the Difference is 11 Days as above. They may further object, that, in correcting the Calendar, the Time of the Equinox ought to have

been

In Payment of Men's Wages belonging to the Royal Navy, each Year is reckoned to consist of 364 Days, and is thus subdivided:

$$\left.\begin{array}{r}7 \text{ Days}\\ 4 \text{ Weeks}\\ 13 \text{ Months}\end{array}\right\}=\left\{\begin{array}{l}1 \text{ Week}\\ 1 \text{ Month}\\ 1 \text{ Year.}\end{array}\right.$$

This is sufficient for many Uses, where Exactness is not required.

In Almanacks the Year is divided into 12 Months, called *January*, *February*, *March*, *April*, *May*, *June*, *July*, *August*, *September*, *October*, *November*, *December*. The Days in each Calendar Month may be known by the following memorial Verse:

——Thirty Days hath *September*,
April, *June*, likewise *November*;
February hath Twenty-eight,
But, in Leap-Year, more one's it's right;
The other Months, it does appear,
Have Thirty-one, in ev'ry Year.

Or by this,

April ter denos, June, Septemberque November;
Uno plus reliqui, Viginti Februus Octo;
At, si Bissextus fuerit, superadditur unus.

Thus much concerning Time is sufficient at present, till we come to a more proper Place to treat of the different Kinds of Years, Solar, Lunar, &c.

149. A

been found for the Year of the *Julian* Reformation, or at least for the first Year of the *Christian* Æra. To this it may be sufficient to answer, that the principal Design of the Parliament was to make our Calendar agree with most Countries in *Europe*. In a Word, our Calendar is now so accurately adjusted, that it will not anticipate a Day in less than 5760 Years, supposing the World to continue in the same State so long. We have only to remark further, that, whereas our civil Year used to begin on the 25th of *March*, it is now always to begin on the First of *January*.

149. *A* TABLE *of such Quantities of Goods, &c. whether Number, Weight, or Measure, as could not properly be inserted in the foregoing Part of this Chapter.*

ACHISON, a Coin = $\frac{1}{7}$ of a Farthing.

AEM, Awme, or Awame; Gallons (of Wine) 35 from *Antwerp*; 40 from the *Rhine*; 50 from *Dordrecht*.

ANGELET, a Coin = 4 Shillings.

ANKER, $\frac{1}{4}$ of a Aem.

BAG, — Bushel; 1 of Lime; — hundred Weight; 3 of Almonds; 4 of Currants.

BALE, — Bolts; 100 of *Lyons* and *Paris* Thread; —hundred Weight; $1\frac{1}{2}$ of Cochineal, Indigo; 2 of Cardamoms, thrown Silk; $2\frac{1}{4}$ of *Spanish* Wool; 3 of Carraway-Seeds; 6 of Safflower.— Pieces; 20 of Boultels; $22\frac{1}{2}$ of *Bevernix* and *Holmes* Fustians.—Reams; 10 of Paper; 100 of unbound Books.

BAND, — Strikes; 10 of Eels.

BANDLE, — Feet; 2 in Length; an *Irish* Measure.

BARREL, — Bushels; 3 of Apples, Pears. — Dozen; 10 of Candles.—Gallons; $31\frac{1}{2}$ of Oil; 32 of Herrings, Ling; 42 of Eels, Mum, Salmon. — Hundred Weight; 1 of Gunpowder, Lipora, Raisins. Number, 300 of black or white Plates; 1200 of Herrings, Stock-fish. V. Hundred Pounds, 16 (the little) 30 (the great) of Anchovies; 120 of Candles; 200 of Barilla, Oat-meal; 224 of Butter; 240 of Soap. — Gallons, 30 of Herrings, Eels; 84 of Salmon.

BASKET, — Bushels, 2 of Medlars.

BAMBEE, a Coin = $\frac{3}{10}$ of a Farthing.

BEZANT, a Coin = 3*l.* 15*s.*

BILLET, — Feet, (of Wood) 3 in Length; whereof there should be 3 Sorts. (1.) a single Billet, 7 Inches

Inches about. (2.) A Cask, 10 Inches. (3.) A Cask of Two, 14 Inches. V. WOOD.

BIND, — Strikes, 10 of Eels.

BODLE, a Coin = $\frac{1}{16}$ of a Farthing.

BOLT, — Ells, 28 of Poldavies, or Canvas.

BOX, — Gross, 2 of Rings for Keys; — Pounds, 14 of Prunelloes.

BUNDLE, — Feet, 3 (about at the Band) of Basket-rods; — Load, $\frac{1}{16}$, of Bulrushes. — Number, 10 of Necklaces, Glovers-knives, Harness-plates, Bass-ropes; 16 of Sets of Instruments for Barber-Surgeons; 100 of Laths 5 Feet long; 120 of Laths 4 Feet long; — Skains, 20 of *Hamburgh* Yarn.

BURDEN, — Pounds, 180 of Gad-Steel.

BUSHEL, of Salt † and Sea Coal, is 5 Stricken, or 4 heaped Pecks *. — † Salt (formerly) used to be bought and sold by Measure, as Corn now is. But it is (now) sold from the Pits, only by Weight, reckoning 7 ℔ Avoirdupois to a Gallon; 56 ℔ to the Bushel; and 42 Bushels to the Tun, for Freight; and 5 Bushels is one Sack; and 4 C is 1 Quarter. * Coal, Bushels are different in many Places.

BUTT, — Gallons, 84 of Salmon.

CADE, — Number, 500 of Red Herrings; 1000 of Sprats.

CARACT, by Jewellers the Ounce is divided into 152 Parts called Caracts, which are divided into Grains, or $\frac{1}{4}$ and $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, &c. Parts, — *N. B.* (1.) The Caract Price is the 24th Part of the Value of a Piece of Gold; thus, if the Value of the Piece be 24 *l.* the Caract Price is 1 *l.* (2.) The Caract fine is $\frac{1}{24}$ of the Goodness of a Piece of Gold. (3.) The Caract Weight is $\frac{1}{24}$ of the Weight of any Piece.

CARAT, — Grains, 24 of Gold; 4 of Diamonds, Pearls, &c.

CARDECUE, a Coin, = 1 s. 7 *d* $\frac{1}{2}$.

CAROLUS, a Piece of Gold coined by King *Charles* I. = 1 *l.* 3 *s.*

CAROTEEL, — Hundred Weight, 3 of Mace.

CARRIAGE, — Bushels, 64 of Lime.
CARUCATA, V. HIDE.
CASATA, V. HIDE.
CASE, — Feet, 120 of *Normandy* Glass, — Number, 5 of Recorders; 120 of Window-Glasses; — Pieces, 21 of *Holland* Linnen; — Tables, 24 of *Ratcliff* Crown-Glass; 25 of *French* Glass; 35 of *Newcastle* Glass.
CASK, — Hundred Weight, 2 of Wheat Flour; 3 of Almonds.
CHAIN, one in Breadth, and 10 in Length, is an Acre.
CHALDRON, or Chaudron, — Bolls; 16 of Corn; — Bushels, 32 of Corn; 36 of Coals.
CHEST, — Hundred Weight, 1½ of Cochineal; 3½ of Benjoin, Ising-glass.
CHIEF, — Ells, 10 of fine Linnen, Silk; 14 of Fustian.
CLOVE, or half Stone, of Cheese or Butter, is 8 lb.
CROSS-DAGGER, a Coin = 11 Shillings.
CORD, — Feet, (of Wood) 8 long, 4 broad, 4 deep. (*N. B.*) That, called the 14 Foot Cord, is to be 14 Feet in Length, 3 in Breadth, and 3 in Depth.
CUBIT, — Feet, 1½.
DAKIR, a Dicker, which see.
DENARIATA, — Acre, 1 of Land.
DICKER, — Number, 10 of Hides; Pair, 10 of Gloves.
DISH, — Cubic Inches, 1073 $\frac{1}{2}\frac{1}{7}$ (near 4 Corn Gallons) of Lead Ore; which, if pretty good, will yield about 3 hundred Weight of Lead.
DOZEN, — (generally means 12 of most Things) but 13 of tanned Calf-Skins; — 14 of Rolls.
DUPPER, — Hundred Weight, 1 of *Roman* Vitriol.
FAGOT, — Feet, 3 in Length (of Wood) and, at the Band, 24 Inches about, beside the Knot. — Pounds, 120 of Steel.
FAMILIA, V. HIDE.

FAN-

FANGOT, — Hundred Weight, 1 ½ of thrown Silk of *Naples*.

FARDEL, — Land-Yard, ⅛ of Land.

FARDING-DEAL, or Farundel, — Acre, ¼ of Land. (*N. B.*) In a Survey-Book of *West Slapton*, in the County of *Devon*, is entered thus: *A. B.* holds 6 Farthings of Land, at 126*l. per Annum*.

FATHOM, is a Measure of 6 Feet long.

FATT, or Vat, — Bushels, 8 of Corn; — Hundred Weight, 5 of Bristles; — Maunds, 1 ½ of unbound Books; — Pieces, 200 of narrow *German* Linnen.

FIRKIN, — Pounds, 56 of Butter; 60 of Soap.

FIRLOT, 31 Pints.

FLOOR, — Feet (of Wood) 18 long, 18 broad, 1 deep.

FLORENCE, — a Coin = 6 Shillings.

FORTNIGHT, 2 Weeks.

FOTHER, or Fodder, — Hundred Weight, 19 ½ of Lead among the Plumbers; 21 at *Newcastle*; 22 at *Stockholm*; 22 ½ at the Mines.

FURR, — Pains, 4 of Budge-poults.

GALLON, — Pounds, 7 ½ of Train-Oil.

GILL, ¼ of a Pint.

GOAD, — Ell *English*, 1 * of *Welch* Frizes and Frizados, — * 55 Inches, *Hayes*'s Negotiator's Magazine, p. 206.

GRAINS, used in weighing Diamonds, are somewhat lighter than those used in Gold, *&c.*

GUNNY, — Hundred Weight, ¼ of Cinnamon; 1 of Aloes Hepatica, Benjoin; 1 ¼ of Saltpetre.

HAND, — Inches, 4 in measuring a Horse.

HARPER, a Coin = 9 Pence.

HIDE, (Synonyms are) Hyde, Hyda, Carucata, Cassata, Familia, Manens, Mansum, Plough-Land, Sullinga; — Acres, 100, or 120 of Land.

HOGSHEAD. The Distillers weigh their Vessels when full; and for a Hogshead allow 4 C. 2 qrs. 22 lb. Cask and Liquor.

HUNDRED, — of moſt Things 100, — Bags, 25 (each one Buſhel) of Lime; — Bundles, 70 of Pipe-Hoops; 90 of Hogſhead-Hoops; 120 of Barrel and Kilderkin Hoops; — Ells, 120 of Canvaſs (except quilted, ſtriped, and tufted) and Linnen Cloth, — Number, 80 of Pales 6 Feet long; 120 of Anchor-Stocks, Balks, Barlings, Barrel-Boards, Battens, Cabbage-Plants, Capravens, Clapholt, Deals, Eggs, Cod-Fiſh, Cole-Fiſh, Stock-Fiſh, Hand-Spikes, Headings (for Barrels, Pipes, *&c.*) Red Herrings, Laths of 3 Feet Length, Morkins, Oars, Pack-Duck, Pales of 4 Feet Length, Sackcloths, Coney-Skins, Lamb-Skins, Sheep-Skins, Boom-Spars, Bow-Staves, Wainſcots, Walnuts; 124 of Haberdine or Ling; — Tuns, 14 of Salt at *Amſterdam*. V. QUINTAL. Great Hundred = 24 Small Hundred of Clap-Board.

HYDE. V. HIDE.

JACOBUS, an imaginary Piece of Money = 1 *l.* 5 *s.*

JARR, — Pounds, 52 of Wheat, 100 of Green Ginger.

KINTAL. V. QUINTAL.

KNIGHT's-FEE, — Hides, 12 of Land, or ſo much Inheritance as is ſufficient to maintain a Knight, with a ſuitable Retinue; which, in *Henry* IIId's Time, was reckoned at 15 *l.* But Sir *Thomas Smith* rates it at 40 *l.*

LAST, — Barrels, 12 of Pot-Aſhes, Cod-Fiſh, White Herrings, Oatmeal, Pitch, Tar; 24 of Gunpowder; — Cades, 20 of Red Herrings; — Dickers, 20 of Leather; — Dozen, 12 of Hides; — Hundred Weight, 17 of Flax; — Number, 1000 of Stock-Fiſh; Pair, 3 of Dog-Stones; — Pounds, 384 of any Commodity in *Scotland*; 1700 of Feathers, Flax; — Quarters, 9 of Meal; 10 of Rape-Seed; Tuns, 12 in eſtimating the Contents of Ships.

LIBRATA, — Acres, 240 of Land, or 20 Solidata's.

LOAD,

LOAD, — Bundles, 60 of Bulrushes; — Bushels, 40 of Corn, Lime (says Mr. *Lowe*, but, according to Mr. *Langley*, 30 of Lime in many Countries is accounted a Load) 18 heaped Bushels of Sand; — Dishes, 9 of Lead-Ore; — Feet solid, 50 of Timber and Planks; — Number, 50 of Fagots; 100 of Bavins; 500 of Bricks; 1000 of Tiles; — Pounds, 175 of Lead; — Trusses, 36 of Hay; — Yard solid, 1 of Earth. V. POKE, SEAM.

MAILE, a Coin = $\frac{1}{2}$ a Farthing.

MANENS, or Mansum. V. HIDE.

MARK, — Ounces, (Avoirdupois) 8 of *French* Copper-Thread, Gold Thread, Silver Thread.

MAST, — Pounds (Troy) 2 $\frac{1}{2}$ of Amber, *Colegn* Gold and Silver Thread.

MAUND, — Bales, 8 of unbound Books.

MONY, a Coin = 4*s*. 6*d*.

NEST, — Chest, 3 of Cypress-Wood, Coffers.

NOBLE, — Half Nobles were called Half-pennies of Gold, Quarter-Nobles, Farthings of Gold. V. ART. 141.

NOOK, or Nocata, — Acres, 12 $\frac{1}{2}$ of Land.

OBOLATA, — Acre, $\frac{1}{2}$ of Land.

OX-GANG, or Oskin, — Acres, 15 of Land, or as much as can be ploughed in a Season by one Ox.

PACE, (geometrical) — Feet, 5 in Length.

PACK, — Number, 52 of Cards; 20000 of Teasels; — Pounds, 240 of Wool; 480 of *Irish* Yarn; also a Weight of 120 ℔.

PACKET, — Number, 250 of Needles.

PALM, — Inches, 3 in Length.

PECK, — the legal *Winchester* Peck is 2 Gallons; but, besides this, there are local Pecks, containing, some more, some less: The *Lancaster* Peck is 6 Gallons.

PEISA, V. WEIGHT.

PERCH, Pole or Rod, — Feet, 16 $\frac{1}{2}$ is a Statute Perch: But there are other customary Perches, *viz.* the Wood-Land Pole = 18 Feet; the Forest, *Lancashire*, and *Irish* Perch = 21 Feet; and the *Scotch*

Pole = 18¼ Feet. The Statute Pole is generally used in meaſuring of Meadow, Arable, and Paſture Land, and Brick-Works, *&c.* the Wood-Land Pole in the Menſuration of copious Woods, *&c.* and the Foreſt Pole in meaſuring of large Chaces, Foreſts, *&c.* V. ROD.

PIECE, — Ells, 13 of Lawns, 106 of Lockrams, 120 of moſt Linnens. — Yards, 2½ of Carpets of *Tunis*; 7½ of Scamoty; 10 of Checks; 15 of Baffins, Bombaſins, Buſtians, Carrels, Dornix, Fuſtians, Raſhes, Sackcloths, Sayes; 24 of Broad-Cloth (the ſhort Piece) Friſados; *Hounſcot* ſays, *Newberry* Whites, and other Kerſeys of like Make; 25 of *Spaniſh* Cloth; 28 of ſorting *Hampſhire* Kerſeys; 30 (the double Piece) of Fuſtians, Cloth-Serges; 32 (the long Piece) of Broad-Cloth; 36 of Caddas. — Pounds, 13 of *Devon* Dozens; 28 of ordinary Peniſtones; 35 of Northern Dozens ſingle, ſorting Peniſtones (unfrized;) 22 of narrow *Yorkſhire* Kerſeys; 43 of *Spaniſh* Cloth; 32 of ſorting *Hampſhire* Kerſeys.

PIG, — Stones, 21½.

PLACK, a Coin = 2*d*½.

PLOUGH-LAND, ſo much as may be tilled with a ſingle Plough. V. HIDE.

POCKET, Sarplar, Serpliathe, — Pack, ½ of Wool.

POKE, — Hundred Weight, 20 of Wool, called (in ſome Places) a Load, being a Waggon-Load.

POLE. V. PERCH.

POT, — Gallon, ½ in *Guernſey* and *Jerſey*.

POUND, — Raw, Long, Short, *China*, *Morea* Silk, *&c.* are weighed by a ℔ called the great ℔, becauſe it contains 24 Ounces Avoirdupois; but Ferret, Filoſella, Sleeve-Silk, *&c.* are weighed by the Avoirdupois ℔ of 16 oz.

PRIME, $\frac{1}{24}$ of a Grain-Weight.

QUADRANTATA, — Acre, ¼ of Land.

QUINTAL, or Kintal, or Hundred Weight, — Buſhels, 25 of Lime; — Pounds, 75 at *Leghorn*; 100 of Cloves, Cochineal, Fiſh (at *Newfoundland*, and

and in the *Streights*) Ginger, Indigo, Mace, Nutmegs, Pepper, Sugars, (in the *English* Settlements in *America*) *Brasil*, *St. Christopher's*, *Spanish*, and *Verinus* Tobacco, Mohair raw, and Linnen Yarn; 120 (called Long Weight) of Cheese (in *Cheshire*, *Derbyshire*, *Lancashire*, *Leicestershire*, *Shropshire*, *Sturbridge Fair*) coarser Metals *, and *Irish* Yarn. * It is also called the Stannary Hundred; Tin being hereby weighed to the King's Farmers.

QUIRE, contains 24, or 25 Sheets, of Paper.

RATION, — Pecks, 9 $\frac{3}{4}$ or a Day's Allowance of Bread or Forage, for Man or Horse.

REAM, — Number, 20 Quires of Paper.

RIAL, in the Reign of *Henry* VI, was 10*s*.

RING, — Number, 240 of Clap-Boards.

ROD, — Number, (of Candles) 12 of six in the ℔; 16 of eight in the ℔; 24 of twelve in the ℔. By the Custom of several Counties, the Measure called by this Name is of different Length. V. PERCH. — In *Herefordshire*, a Perch of denshired Ground is 12 Feet; — of Ditching 21; — in the Forest of *Sherwood* 25; — in *Staffordshire* 24.

ROLL, — Dozens, 5 of Skins of Parchment; — Ells, 1100 of Minsters and Ozenbrigs; — Quintals, $\frac{1}{4}$ of *Barbadoes* Tobacco.

ROPE, Feet, 20.

SACK, — Bushels, 3 of Coals; 4 of Corn; 5 of Salt; — Stone, 26 of Sheep's Wool (14 ℔ to the Stone; but in *Scotland* 24 of 16 ℔ to the Stone.)

SALUTE, a Coin = 6 $\frac{24}{25}$ Shillings.

SARPLAR, V. POCKET.

SAUME, — Pounds, 315 of Quicksilver.

SCORE, (generally means 20, in Numbering of most Things, but) — Chaldrons, 21 of Coals.

SCRUPLE, = $\frac{1}{1680}$ of an Hour.

SEAM, — Bushels, 8 of Malt; — Horse-Load, 1 of Wood; — Pounds, 120 of Glass.

SEMIBOLE, — Pipe, 1 of Wine.

SERON, — Hundred Weight, 2 of Almonds; 3 of Barilla.

SET, — Number, 5 of Recorders; 24 of Alphabets.
SERPLIATHE, V. POCKET.
SEXLING, — Shillings, 15.
SHID of Wood, — Feet, 4 in Length, and in Girth, according as they are marked. If they have but 1 Notch, they are to be 16 Inches about; if they have two Notches, they are to be 23; if of 3, 28; if of 4, 33; if of 5, 38.
SHOCK, — Ells, 13 of Lawn; — Number, 60 of Soap-Boxes, Canes, Trays.
SKIN, — Hundred Weight, $\frac{3}{4}$ of Cinnamon.
SOLIDATA, — Acres, 12 of Land; or 12 Denariatas.
SORT, — Dozen, 4 of Balances; — Ells, 106 of Lockrams; 120 of ſeveral Linnens.
SOVEREIGN, — Shillings, 22$\frac{1}{2}$.
SPAN, — Inches, 9 in Length.
STACK, — Feet, (of Wood) 3 long, 3 broad, 12 high.
STICK, — Rods, 30 of Candles.
STONE, — of Beef, is 8 ℔, (but in *Hertfordſhire* and Parts adjacent 12; in *Pembrokeſhire*, &c. 18; in the northern Counties 16) — of Glaſs, 5℔, *&c.* — In Racing, Hay, Iron, Shot, *&c.* 14 ℔.
STOOK, — Sheaves, 12 of Corn.
STRIKE, — Number, 25 of Eels.
SULLINGA, V. HIDE.
SUM, — Number, 10000 of Copper, Harneſs, Roſe, Sadlers, or Sprig-Nails.
TESTOON, — Pence, 18$\frac{21}{25}$.
THOUSAND, — Herrings, 1200.
THRAVE, — Sheaves, 24 of Corn.
THRYLING, = $\frac{7}{100}$ of a Farthing.
TIMBER, — Skins, 40 of Furs, Filches, Grays, Jennets, Martins, Minks, Sables.
TRUSS, — Pounds, 56 of Hay (except in the Months of *July* and *Auguſt*, when it is 60 ℔;) of Forage, as much as a Trooper can carry on his Horſe's Crupper.
TUB, — Pounds-Weight, 60 of Tea.

TUN,

TUN, — Bales, 5 of Feathers; 8 of Paper; 10 of Cork; — Barrels, 2½ of Brandy; 3 of Syrup; 4 of Prunes; — Bushels, 20 of Chesnuts, Wheat, and other Grain; 42 of Salt. V. Bushel.

Dozen, 1 of Planks; 2 of Walnut-tree Tables; — Feet of Timber. V. Load.

—— Gallons, 236 of Oil, by the Custom of *London*, called by Merchants the Civil Gauge, is ordinarily sold for a Tun; except Whale-Oil, or Oil from *Greenland*, which has 252 Gallons to the Tun.

Pounds, 1709 of Barley; 2000 the Sea-Tun, by which the Contents of a Ship are estimated; — Quarters, 5 of Corn is usually reckoned a Tun in Freight.

VAGA, V. Weight.

VAT, V. Fatt.

UNICORN, = 6 Shillings.

UNIT, — Shillings, 22.

URCHIN, — Pence, 3.

WEIGH, Wey, Waga, Vaga, Peisa; — Bunches, 60 of *Rhenish* Glass; — Cases, 60 of Window-Glass; — Pounds, 224 of Cheese, by 9 *H.* 6. 8; 248 in *Essex*; 336 in *Suffolk* (of Bay Salt) — Quarters, 6 of Barley and Malt; 5 of other Grain.

WEY, — Cloves, 32 (or 25 ℔) of Cheese or Butter in *Suffolk*; but in *Essex* 42 Cloves, or 336 ℔.

WOOD, is assized into Shids, Billets, Fagots, Falwood, and Cordwood. V. Shid, *&c.*

YARD-LAND, — Acres, 15 at *Wimbledon* in *Surry*; 20 in most other Places; 24, 30, 40, in some.

As the Nature of this Table is such, that we have been obliged, for the most Part, barely to transcribe the Articles from several Authors, (*viz.* Mess. *Chamberlayne*, *Dilworth*, *Langley*, &c. but chiefly from Mr. *Lowe*) we will not presume to say this Table is correct. However, if such Persons as may discover any Errors therein, or who have any new Articles proper to be inserted in it, would send the Corrections, *&c.* to the Au-

Author, (Post paid) in Order to their being corrected; we may by this Means (and not otherwise) hope to have a correct as well as well as useful Table.

CHAP. VIII.

ADDITION of APPLICATE NUMBERS.

150. THIS may be properly divided into two *Cases*. 1. To add simple Numbers of one Denomination together.

This is done in all Respects as Addition of abstract whole Numbers, already treated of. For, as 2 and 3 is 5, so 2 Men and 3 Men is 5 Men; 2 ℔ and 3 ℔ is 5 ℔; 2 Yards and 3 Yards is 5 Yards. However, it may not be improper to add a few Questions, to put the young Beginner on Reflection.

151. *Example* 1. The Author was born in the Year 1729. It is demanded, when he will be 30 Years of Age?

Solution. Here the Learner will reason thus with himself:

If the Author was born 1729 Years after *Christ*, certainly, in 30 Years after that, he will be 30 Years of Age; and ∴ his Age must be 30 Years, in $1729 + 30 = 1759$ Years after *Christ*; or, which is the same, in the Year of the *Christian* Æra 1759.

152. *Example* 2. A Man has two Sons, the Youngest 25 Years of Age; and the Eldest 3 Years older than his Brother; and the Father 10 Years older than both his Sons Ages, when taken together: What is the Age of the Father?

Solution. If the eldest Son be 3 Years older than the Youngest, who is 25 Years of Age; it is plain, his Age is $25 + 3 = 28$ Years; and ∴ the Ages of both will amount to $28 + 25 = 53$ Years; but, by the Question, the Father's Age was 10 more than both his Sons Ages, and, consequently, must be =

53

53 + 10 = 63 Years. Or, it might have been found thus, 25 + 25 + 3 + 10 = 63 Years, the Age of the Father.

153. *Example* 3. Let us suppose, that, from the Creation of the World to the Beginning of the Deluge was 1656 Years; from thence to the Destruction of *Troy* 1162 Years; from thence to the Æra of *Nabonassar* (by which the *Chaldeans* and *Egyptians* reckoned their Years) 436 Years; from the Æra of *Nabonassar* to that of the Death of *Alexander* the Great 423 Years; from thence to the Æra of the City of *Antioch* 275 Years; from which to the Æra of the *Julian* Reformation of the Calendar 4 Years; from thence to the Æra *Actiaca* (so named from the Victory obtained by *Augustus* over *Anthony* at *Actium*) 15 Years; and, from thence to the Birth of *Christ*, 30 Years: According to this Chronology, it is required to find in what Year of the World *Christ* was born?

Solution. Here, it is evident, that 1656 + 1162 + 436 + 423 + 275 + 4 + 15 + 30 = 4001 Years must be the Answer, which was required.

154. *Case* 2. To add mixed Numbers together.

The Method of doing this will be made sufficiently clear by a few Examples. 1. Suppose a Man spends, at one Time, 2£. 7*s*. 6*d*; at another Time, 15£. 11*s*. 8*d*. 2*qrs*; and, at another Time, 3£. 10*s*. 7*d*. 3*qrs*. What did he spend in all?

£.	s.	d.	qrs.
2· :	7 :	6 :	0
15 :	11· :	8· :	2·
3 :	10 :	7 :	3
21 :	9 :	10 :	1

Solution. Having placed the Numbers in a proper Order for Addition, add up the Rows thus: Beginning with the Row of Farthings, say, 3 + 2 = 5; but as 4 of this Row is one in the next (because 4 Farthings = a Penny) make a Dot (·) for the Penny; and, as 5 Farthings is 1 more than 4, put down the odd Farthing. Then, looking on your Sum, you find 1 Dot, *viz.* 1 Penny to be carried

ried to the Row of Pence; therefore say, $1+7=8$, $+8=16$, which is 4*d.* above 12 Pence, or a Shilling; make a Dot for the Shilling, and proceed, saying, mentally, $4+6=10$, which put down. Then, looking on your Sum, you will find 1 Dot, or 1 Shilling, which carry to the Column of Shillings, and then you will have $1+10=11+11=22=$ 1£. 2*s.* (because 20 Shillings = 1 *l.*) make a Dot for the 1 *l.* and proceed with the 2 Shillings, saying, $2+7=9$, which write down. Now looking on your Sum you will find 1 Dot, or 1 *l.*; which carry to the Row of £*s*, and, adding them up as in Case 1, you will find it come out to 21 *l.* Whence, the Answer is 21 *l.* 9*s.* 10*d.* 1*qr.*

155. Addition of mixed Numbers may also be performed, by Help of Division, much easier than otherwise, when the Number of Things in one Row, that makes one of the next Row, is something great. *Example.* A Merchant has sold to different Men the following Quantities of Wine, *viz.* to one Man 8 Hhds. 39 Gall. to another 7 Hhds. 32 Gall. to another 3 Hhds. 18 Gall. and to another 1 Hhd. 12 Gall. What Quantity he sold in all is required?

	Hds.	Gall.
sold to one Man	8 :	39
to another	7 :	32
to another	3 :	18
and to another	1 :	12
In all	20 :	38

Solution. Having placed the Numbers in the annexed Order, collect the Gallons together by common Addition; *viz.* $12+18+32+39$ will be found to be $=101$ Gallons; now, 63 Gallons being = 1 Hogshead, it is evident, that, as often as 63 can be taken out of 101, so many Hogsheads are contained therein; to do which is the Property of Division; therefore, dividing 101 by 63, we shall have 1 Hogshead and 38 Gallons remaining; therefore, under the Row of Gallons put 38, and carry the 1 Hogshead to the Row of Hogsheads; saying, 1 (you carry) $+1+3+7+8=20$ Hogsheads, which

which being wrote down will complete the Sum, *viz.* 20 Hogsheads and 38 Gallons. But, if you chuse to know how many Tuns it is, you may by Division see how often 4 is contained in 20 (because 4 Hogsheads = 1 Tun) *viz.* 5 Times, and nothing remaining; whence the Sum may be read thus, 5 Tuns and 38 Gallons.

156. When the Number of Things in any Row, which make one in the next superior Column, is a certain Number of Tens; we may add them up, as is shewn in this Example. Let it be required to add up the Sum annexed.

H.	′
10 :	22
11 :	18
36 :	54
18 :	18
17 :	2
10 :	12
104 :	6

Suppose the Column of Minutes parted into two Rows, then, since 60 Minutes = 1 Hour, we may add up the Row of Units by 10, and the other by 6; which will give the same as if added up by 60, because 6 Tens = 60. Say then, 2 + 2 + 8 + 4 + 8 + 2 = 26; put down 6, and carry the 2 Tens to the next Row; saying, 2 + 1 + 1 + 5 + 1 + 2 = 12; which, being to be added up by 6, gives 2 Hours exactly; for 6 is contained in 12 two Times; ∴ carry the 2 to the Row of Hours; and, adding them up as whole Numbers, the Sum will be 104 Hours; whence, the required Sum is 104 Hours and 6 Minutes.

N. B. The first Column is always to be taken as Integers, and, therefore, to be added up as whole Numbers.

157. *Corol.* From what has been said, we may deduce this general Rule, for performing Addition of mixed Numbers, *viz.* First, begin with the Column, whose Denomination is the lowest, and find its Sum as in Addition of whole Numbers; then find, how many Units of the next superior Denomination are contained in it, *viz.* divide this Sum by the Number of Units of this Column, which makes one of the next;

next; the Remainder (if any) is to be put down under this Column, as Part of the required Sum; and the Quotient is to be carried as Units to the next superior Column, and added up with it. After this Manner, proceed through every Denomination, till you come to the last or highest; which, being considered as simple Numbers, must be added up purely as whole Numbers.

CHAP. IX.

SUBTRACTION *of* APPLICATE NUMBERS.

158. AS the Method of subtracting applicate whole Numbers is exactly the same as that of subtracting abstract whole Numbers, which has been already explained, we shall here only give one Question, and then proceed to explain the Method of subtracting applicate mixed Numbers.

The *Question*. According to Dr. *Wells*, the Æra of *Yezdegrid*, or the *Persian* Æra, began 632 Years after the *Christian* Æra; and the Æra of the *Hegira*, or the Flight of *Mahomet* from *Mecca* to *Medina*, used by the *Turks* and *Arabs*, began 10 Years before that; and the *Dioclesian* Æra, or the Æra of the Martyrs (so called from the Number of *Christians* that were slain, in the *Dioclesian* Persecution) otherwise called the Æra of the *Abyssinians*, began 338 Years before that of the *Hegira*: It is required to find in what Years of the *Christian* Æra the *Dioclesian* and *Hegira* Æra's commenced?

Solution. Since the Æra of *Yezdegrid* was in the Year of our Lord 632, and the Æra of the *Hegira* was 10 Years before that, it is plain that 632 — 10 = 622 must be the Year of the *Christian* Æra, when the *Hegira* commenced; and, for the like Reason, 622 — 338 = 284 Years after *Christ*, when the *Dioclesian* Æra began.

Ex-

Examples in mixed Numbers.

159. *Example* 1. *A* borrowed of *B* 200*l.* 10*s.* 6*d.* of which *A* has ſince paid 102*l.* 18*s.* 5*d*: What has *A* more to pay?

Solution. Having placed the Numbers in a convenient Order, ſay, 6 — 5 = 1, which put under Pence; then, ſince you cannot take 18 from 10,

	£.	s.	d.
Borrowed	200 :	10 :	6
Paid	102 :	18 :	5
Unpaid	97 :	12 :	1

increaſe the 10*s.* by one Pound, *viz.* call it 30 Shillings; then 30 — 18 = 12 Shillings, which put under Shillings: Now, as we have increaſed the Money borrowed by 20 Shillings, it is evident, that the Remainder cannot be the ſame as if it was not increaſed, unleſs we augment the Money paid by 20 Shillings, or 1*l.*; therefore, add 1*l.* to the 102, *viz.* call it 103*l*; then ſay, 200 — 103 = 97*l*; hence, *A* ſtill owes *B* 97*l.* 12*s.* 1*d.* *Q. E. I.*

160. *Example* 2. What is the Difference between 2 Yards, 3 Quarters, 2 Nails, and 1 Yd. 2 qrs. 3 Nails of Cloth?

Solution. Place the Numbers in proper Order; then, becauſe we cannot take 3 from 2, we muſt increaſe the two Nails by 1 Quarter; ſaying, 6 Nails — 3 Nails = 3 Nails; then, becauſe we have increaſed the

	Yds.	Qrs.	Nails.
From	2 :	3 :	2
Sub.	1 :	2 :	3
Anſ.	1 :	0 :	3

greater Quantity by 1 Quarter, we muſt (that we may have the ſame Remainder, as if it had not been ſo augmented) make the leſſer Quantity, or Minorand, more by one Quarter; ∵ ſay, 3 Quarters — 3 Quarters = 0, which put under Quarters; laſtly, 2 — 1 = 1; hence, the required Difference is 1 Yard 3 Nails. And after this Manner may the Difference of any Quantities of Money, Weights, or Meaſures be found; and therefore we ſhall content ourſelves with

with laying down the following Rule, by Way of Corollary.

161. The *Rule*. Begin with the lowest Denomination, subtracting the Number in that Place of the Minorand from that in the corresponding Place of the Subducend (if Subtraction can be made) and put down the Remainder underneath: But, if the Number of any inferior Species of the Minorand is greater than its Correspondent in the Subducend, you must increase this Number by a Number which is equal to an Unit in the next superior Denomination; (*viz.* for Example, in the Column of Pence, add 12; and, in Shillings, add 20, *&c.*) and from this Sum subtract the Number in the Minorand, and put down the Remainder; but then you must remember to carry or add 1 to the Number in the next superior Place of the Subducend. Proceed in this Manner, till you come to the highest Place; which subtract as whole Numbers.

CHAP. X.

MULTIPLICATION of APPLICATE NUMBERS.

162. AS to simple applicate Numbers, nothing need be said; and even Multiplication of mixed Numbers will be sufficiently explained by two Examples. 1. Multiply 3 Feet 6 Inches by 8.

Feet	Inch.
3 :	6
	8
28 :	0

Solution. Say, $6 \times 8 = 48$ Inches $=$ (by dividing it by 12) 4 Feet, therefore, under Inches put 0, and carry the 4 to the Feet; then $3 \times 8 = 24$, $+ 4$ you carry $= 28$ Feet, for Answer.

163. *Example* 2. Multiply, 2*l.* 10*s.* 11*d.* 3*qrs.* by 57.

Here,

Here, since it would be troublesome to multiply at once by 57, observe, that $8 \times 7 = 56$, which wants but 1 of 57; ∴ the Work may stand thus:

First, multiply by 8, *viz.* $3 \times 8 = 24$ Farthings = (by dividing by 4) 6 Pence; ∴ put 0 under Farthings, and carry 6 Pence; saying, $11 \times 8 + 6$ (you carry) = 94 Pence = (by dividing by 12) 7 Shillings and 10 Pence; ∴ under Pence put 10, and carry the 7 Shillings,

	£.	s.	d.	qrs.
	2 :	10 :	11 :	3
				8
	20 :	7 :	10 :	0
				7
	142 :	14 :	10 :	0
Add	2 :	10 :	11 :	3
Ans.	145 :	5 :	9 :	3

saying, $10 \times 8 = 80 + 7$ (you carry) = 87 Shillings = (by dividing by 20) 4*l.* 7*s.*; ∴ write down 7 under Shillings, and carry 4 to the Pounds, saying, $2 \times 8 = 16$, $+ 4 = 20$*l.* which write down. Then multiply this Product by 7, after the same Manner, which will give 142*l.* 14*s.* 10*d.*; thus, we have multiplied by 56, for $8 \times 7 = 56$; but, since this wants 1 of 57, we must add once 2*l.* 10*s.* 11*d.* 3*qrs.*; which will give 145*l.* 5*s.* 9*d.* 3*qrs.*, for the Product which was required. *Vide* Art. 182.

164. *Scholium.* Since Multiplication is * the repeating a Number, a certain Number of Times, it follows, that, when two Numbers are to be multiplied together, one of them must be an abstract Number; for, suppose it was required to multiply 3*l.* by 4*l.*, I would ask the Proposer, how many Times he would have 3*l.* taken or repeated? If he should answer (as he must according to his Question) 4*l.*, it plainly appears to be Nonsense; but, if it had been required to multiply 3*l.* by 4, it would be only to repeat 3*l.* 4 Times, which is very proper, and the Answer $3 \times 4 = 12$*l.* Hence it appears, that, when Authors propose to multiply Money by Money, Weight by Weights, &c. they talk very im-

* 51.

improperly and absurdly. We shall here only further observe, that as, in Multiplication of abstract Numbers, we may make the Multiplicand the Multiplier, and the Multiplier the Multiplicand, so we may here also, by only considering the applicate Number as the abstract one, and the abstract as the applicate
* 95. Number; for Instance, as $3 \times 4 = {}^{*} 4 \times 3$, $3l. \times 4$
† 95. must be $= 4l. \times 3$; or, generally, as $a \times b = \dagger\, b \times a$, $al. \times b = bl. \times a$.

CHAP. XI.

DIVISION of APPLICATE NUMBERS.

165. THIS admits of two *Cases*. 1. To divide applicate Numbers by applicate Numbers. *Example*. Divide 24*l*. by 8*l*. Dividing this, as in abstract Numbers, the Quotient will be 3; shewing, that 8*l*. is contained in or can be taken out of 24*l*. three Times. As it would be something difficult to divide mixed applicate Numbers by mixed applicate Numbers, without some Knowledge of Reduction, we shall omit it here, and refer to *Art*. 182. And shall only further observe in this Place, that, when an applicate Number is to be divided by an applicate Number, the Quotient must be an abstract
‡ 120. Number; for Division shews to find ‡ how many Times one Number is contained in another, and, therefore, such Authors, as divide an applicate Number by an applicate Number, and make the Quotient an applicate Number, speak not only improperly, but absurdly; for if 4*l*. was to be divided by 2*l*. and we should ask how many Times 2*l*. is contained in 4*l*., and should be answered 4*l*.; what would be more ridiculous?

166. *Case* 2. To divide an applicate by an abstract Number. Here it is proper to hint, that, besides

sides the Definition of Division already given, it may, consistently with that Definition, be defined to be ‡ a Rule, which shews to find a Number, which is contained in a given Number, a given Number of Times; for let x denote a Number in which y is contained z Times, then y, taken z Times, $=$ * $y \times z$ $= x$; ∴, dividing both Sides of the Equation by z, we have $y =$ † $\frac{x}{z}$. That is, any Number, being divided by the Number expressing the Times another Number is contained in it, will give that other Number. * 51. † 108.

Example. Divide 142*l.* 14*s.* 10*d.* by 7; or, which the same, divide 142*l.* 14*s.* 10*d.* amongst 7 Men, and give each Man an equal Sum.

Solution. Here it is evident we are to find a Number (which, being multiplied by 7, shall be equal to 142*l.* 14*s.* 10*d.*; or in other Words) which is contained in 142*l.* 14*s.* 10*d.* seven Times; and therefore, by the above, 142*l.* 14*s.* 10*d*, divided by 7, will give the Number required; which, it is plain, must be applicate, and may be found thus: Say, the $\frac{1}{7}$ of 14 is 2, which put down; and the $\frac{1}{7}$ of 2 is 0, and 2 remaining; put down the 0, and, the Remainder being 2*l.* $=$ 40 Shillings, carry it to the Shillings; and then we shall have 40 + 14 $=$ 54 Shillings, the Seventh of which is 7, and 5 remaining; therefore put down the 7 Shillings, and carry

	£.	s.	d.
	142 :	14 :	10
$\frac{1}{7}$	20 :	7 :	10

H

‡ Besides this and the former Definition, Division will admit of others, *viz.* 1. To find what Part the Divisor is of the Dividend; for, if the Divisor is contained 3 Times in the Dividend, it must be a third Part of the Dividend; because, being repeated 3 Times, it will be $=$ the Dividend; and for the same Reason, if it be contained 4 Times in the Dividend, it will be the 4th Part of the Dividend, &c. ∴ the Quotient shews the Part. *Q. E. I.*

2. To

carry the remaining 5 Shillings, or $5 \times 12 = 60$ Pence, to the Pence; and then we have $60 + 10 = 70$ Pence; the Seventh of which is 10*d.* which, being written down compleats the Anſwer, *viz.* 20*l.* 7*s.* 10*d.* This may be proved by Multiplication of applicate Numbers.

167. *Example* 2. What is the fourth Part of 2 *Yds.* 2 *Feet.* 10 *Inches?*

	Yds.	Feet	Inch.
	2 :	2 :	10
$\frac{1}{4}$	0 :	2 :	2

Solution. Since 4 cannot be contained in 2 Yards, under the Yards put 0, and carry the 2 Yards to the Feet, ſaying, (2 Yards =) 6 Feet + 2 Feet = 8 Feet; the Fourth of which is 2 Feet, which put down; then the Fourth of 10 Inches is 2, and 2 remaining; put down the 2 Inches, and the 2 remaining, being 2 Parts of 4, is $\frac{1}{2}$ of an Inch; Whence the Anſwer is 2 Feet 2 Inches.

168. If the Number we are to divide by be ſo great, as to be troubleſome to divide in one Line, the Operation may be put down as in common Diviſion; for *Example*, divide 145*l.* 5*s.* 9*d.* 3*qrs.* by 57. See *Art.* 167.

Solu-

2. To find a Number, which is ſuch Part of a given Number, as a given Number expreſſes. Let $x =$ the required Number, $a =$ the Number of which it is a Part, $b =$ the Number expreſſing the Part; then, by the Nature of the Thing, $x \times b = a$, ∴ dividing by b, we have $x = {}^{*}\frac{a}{b}$. *Q. E. I.*

* 108.

Solution. Here say, how many Times 57 in 145, which is 2 Times; put the 2 in the Quotient, which must be 2 *l.* as we are dividing *£s.* by an abstract Number; then, since 2 *l.* × 57 = 114, we have 31 *l.* remaining; and ∵, since 20 Shillings = 1 *l*, in 31 *l.* there must be 20 Times 31 Shillings, for which Reason we multiply by 20, and add in the 5 Shillings which brings out 625 Shillings; this divided by 57 gives 10 *s*, and 55 remaining; and, since 12 Pence = 1 Shilling, in 55 Shillings + 9 *d*, there must be 55 × 12 + 9 = 669 *d*, which divided by 57 gives 11 *d*, and 42 remaining; which, multiplied by 4, because 4 Farthings = 1 *d*, and the 3 *qrs.* added, gives 171 Farthings; and 171 Farthings, divided by 57, give 3 Farthings. Whence the Answer is, 2 *l.* 10 *s.* 11 *d.* 3 *qrs.* *Q. E. I.* Vide *Art.* 182.

```
       £.   s.  d. qrs.
  57) 145 : 5 : 9 : 3 (2 l.
      114
      ---
       31
       20
      ---
  57) 625 (10 s.
       57
      ---
       55
       12
      ---
  57) 669 (11 d.
       57
      ---
       99
       57
      ---
       42
        4
      ---
  57) 171 (3 qrs.
      171
      ---
        0
```

CHAP. XII.

REDUCTION.

169. **R**EDUCTION (Reduction *French*) is the Rule for changing Numbers of any Denomination into Numbers of another Denomination,

ſo as to retain the ſame Value, though changed to a different Name: As, for Inſtance, Pounds into Shillings, or Shillings into Pounds equal thereto. This admits of three *Caſes*.

170. *Caſe* 1. To bring Numbers of one Denomination into Numbers of a Leſs Denomination. The *Rule*. Multiply by as many of the leſs, as makes one of the greater Denomination. A few Examples will explain this *Rule*, and alſo ſhew the Reaſon thereof.

171. *Example* 1. In 213*l.* how many Shillings?

Solution. Since 20 Shillings = 1 *l.* in 213 *l.* there muſt be 213 Times 20 Shillings; and ∵ 20 Shillings multiplied by 213 taken as an abſtract Number, or, which is the ſame, 213 x 20 taken as abſtract Numbers, will be = the Shillings in 213 *l*, = 4260 Shillings: Whence we ſuppoſe the Reaſon of the *Rule* is plain; and therefore, in the two following *Examples*, we ſhall be more compendious in explaining the Operations.

£.
213
20
4260

172. *Example* 2. In 14*l.* 10*s.* 6*d.* how many Pence?

	£. s. d.
	14 : 10 : 6
Mult. by	20, becauſe 20*s.* = 1 *l.* and add in the 10*s.*
	290 = 14 x 20 + 10 = Shillings in 14*l.* 10*s.*
Mult. by	12, becauſe 12 *d.* = 1 Shilling, and add in 6*d.*
	3486 = 290 x 12 + 6 = Pence in 14*l.* 10*s.* 6*d.*

173.

173. *Example* 3. In 5C. 2Qrs. 14℔. how many ℔.

C. Qrs. ℔.
5 : 2 : 14
× by 4, becauſe 4 *qrs.* = 1 C. and add in the 2 *qrs.*

22 = 5 × 4 + 2 = *qrs.* in 5 C. 2 Qrs.
× by 28, becauſe 28 ℔. = 1 *qr*; and add in the 14℔.

180
45

C. Qrs. ℔.
630 = 22 × 28 + 14 = ℔s. in 5 : 2 : 14.

It is common amongſt Merchants to bring it into ℔s. thus: Put down the 5C. twice, *viz.* one directly under the other; then put down the Hundreds, removed one Place to the left Hand; again, put down the 5C. removed one Place more towards the left Hand; then the Work would appear as here annexed, and, if added up, would give the Number of ℔s. contained in 5C. But, ſince there yet remain 2 *qrs.* 14℔. to be taken Notice of, in the Row of Units they would put 6, and in the Column of Tens 5 (2 *qrs.* being = 2 × 28 = 56 ℔.) then, for the 14℔, in the Row of Units they write 4, and in that of Tens 1; then, adding up the Rows, the Work will be compleated, and appear thus:

```
   5
   5
  5
 5
```

```
   5
   5
  56
 554
  1
 ---
 630
```

The Reaſon of this Method of operating will eaſily appear, by conſidering, that 1 C. = 112 ℔. (for 1 C = 4 *qrs*, and 1 *qr.* = 28 ℔, and ∵ 1 C. = 4 × 28 = 112 ℔.) and ∴, if we multiply the Number expreſſing the Number of Cs, which is to be brought into ℔s, by 112, the Product will be the Number of ℔s. contained in the Cs, which was required: Now, in multiplying by 112, we firſt multiply by 2, which is only doubling, and in the above Method

 is

is done by writing it down twice; then multiplying by 10 (*viz.* the 1 ſtanding in the Place of Tens) is only putting the ſame Number down, removed one Place to the left Hand; laſtly, multiplying by 100 (*viz.* the 1 in the Place of Hundreds) is only removing the Multiplicand two Places to the Left; conſequently, the Sum of all theſe muſt be equal to the Multiplicand multiplied by 112, and, therefore, the Pounds in the Quarters and ℔s, if any, being added thereto, the Sum will be the ℔s, contained in the given Hundreds, Quarters, and Pounds.

174. *Caſe* 2. To bring Numbers of a Leſs into Numbers of a greater Denomination. The *Rule.* Divide by as many of the Leſs, as makes one of the greater Denomination.

175. For an *Example*, take the Reverſe of *Art.* 172, *viz.* In 3486 Pence, how many Pounds?

Solution. Divide the Pence by 12, the Quotient is 290 Shillings, and 6 Pence over; then 290 ÷ 20 gives 14*l.* and 10*s.* over: Whence, the Anſwer is 14*l.* 10*s.* 6*d.* The Reaſon will eaſily appear, by comparing it with *Art.* 172; and, as this Caſe is only the Reverſe of the laſt, we ſhall paſs on, without any further Delay, to the next *Caſe.*

$$\begin{array}{l} \quad 3486 \\ \hline \tfrac{1}{12})29|0(6 \\ \hline \tfrac{1}{2}\ \ 14 : 10 \\ \hline \end{array}$$

176. *Caſe* 3. is, when we are to find how many of one Species, which is not an aliquot Part of another, is contained in any Number of that other Species. The *Rule.* By *Caſe* 1. bring the Denominations both into one Name, and then divide one by the other; and the Quotient may be brought into a higher Denomination, by *Caſe* 2.

177. *Example* 1. In 89 Piſtoles, each 16*s.* 6*d*, how many Pounds?

Solution. In 16*s.* 6*d.* (found by *Caſe* 1.) is 198 Pence; ∴, ſince in 1 Piſtole there is 198 *d*, in 89 Piſtoles there muſt be 89 Times 198 *d*, which is=17622 Pence; then, to find how many Shillings are contained therein, we divide by 12, becauſe 12 *d.* = 1 Shilling; which gives 1468 Shillings, and 6 Pence remaining; laſtly, 1468 Shillings by *Caſe* 2. are = 73 *l.* 8*s.* Whence the Anſwer is 73*l.* 8*s.* 6*d.*

```
 s.  d.
16 : 6      198
12           89
----      -----
198        1782
----      1584
          -----
          17622
          -----
  1/12)146|8(6d. remaining
          -----
     1/2 73l.(8s. remaining
          -----
```

It might have been ſolved ſeemingly more agreeable to the Rule above given, by finding by *Caſe* 1. how many Pence are contained in 1 *l.* and, dividing by it, the Quotient would give the Pounds, and Parts of a £; but, as the odd Shillings and Pence would not have been ſo naturally diſcovered, the Method juſt given ſeems the beſt; and is in Effect exactly the ſame, for, ſince the Pence in 1 *l.*=12 x 20=240 *d*, it is * the ſame Thing whether we divide by 12, and then by 20, or by 240 at once. * 137.

Corollary. Whence it appears that by Reduction one Kind of Money may be changed into another; but we ſhall not here give any more Examples, becauſe we ſhall make the Exchange of Money a particular Chapter.

178. *Example* 2. Allowing the Diſtance between *York* and *London* to be 204 Miles, how many Times will a Coach-Wheel turn round, whoſe Circumference is 6 Yards, in going from one of theſe Places to the other?

Solution. A Mile being 8 Furlongs, 204 × 8 = 1632 Furlongs; and ∴ 16320 × 40 (because 40 Perches = 1 Furlong) = 65280 Perches; and, since 5 ½ Yards = 1 Perch, in 65280 Perches there must be 5 Times and ½ so many Yards; ∴, to multiply by 5 ½, first multiply by 5, to which add ½ of the Multiplicand; the Sum, or Product, gives 359040 Yards, which divided by 6 gives 59840 Times.

	204
	8
	1632
	40
	65280
	5 ½
	326400
½	32640
	359040
⅙	59840

179. *Example* 3. How many 10 Feet Rods will reach 8 Miles?

Solution. Here we shall observe, that a Mile is 1760 Cloth-Yards, (for 1 × 8 × 40 × 5 ½ = 1760) and, therefore, the best Way to bring Miles into (Cloth) Yards, is to multiply by 1760; whence 8 × 1760, or rather 1760 × 8, = 14080 Yards; and ∴ 14080 × 3 = 42240 Feet. *Q. E. I.*

Again, 42240, the Feet in 8 Miles, being divided by 10, gives 4224, = the Number of 10 Feet Rods contained in 8 Miles. *Q. E. I.*

180. By this *Case*, we can divide the Value of any Species into different Denominations, the Number of which shall be equal, by reducing the Denominations to the lowest Name, and by the Sum of which dividing the Species reduced to the same Name. *Example* 4. A Gentleman having two old-fashioned Tankards of Silver, one weighing 1 ℔, the other 1 ℔, 4 oz. 3 dwts, he has ordered his Goldsmith to melt them, and with the Metal to make him Spoons of 2 Ounces, Cups of 4 ½ Ounces, Salts of 10 oz. 10 dwts, and Snuff-Boxes of 10 oz. 6 dwts; and of each

each an equal Number: It is required to find the Number of each, allowing 5 dwts. for Waste in melting, &c.

Solution. By *Case* 1.

		dwts.
1	Spoon of 2 oz. (= 2 × 20)	= 40
1	Cup of $4\frac{1}{2}$ oz. (= $4\frac{1}{2}$ × 20)	= 90
1	Salt of 1 oz. 10 dwts. (= × 20)	= 30
1	Snuff-Box of 1 oz. 6 dwts.	= 26
1	of each Sort amounts to	186

		lb.	oz.	dwts.
1 Tankard	=	1	0	0
The other	=	1	4	3
Both	=	2	4	3
Deduct Waste	=			5
Remains	=	2	3	18

× by 12

27 Ounces

× by 20

558 Dwts.

Hence, the Metal out of which the several Utensils are to be made is 558 dwts, and 186 dwts. will make one of each Sort; ∴, as often as 186 can be taken from 558, so many it will make of each Sort; ∵ 558 ÷ 186 = 3 is the Answer; *viz.* it will make 3 Spoons, 3 Cups, 3 Salts, and 3 Snuff-Boxes; which may be proved thus:

			dwts.
By *Case* 1. the	3 Spoons	=	120
	3 Cups	=	270
	3 Salts	=	90
	3 Snuff-boxes	=	78
	All	=	558 dwts.

the same as the Quantity of Silver out of which they were to be made.

Note.

Note. Reduction may be proved by reversing the Questions.

What we call *Case* the first some Authors call Reduction descending; and *Case* second Reduction ascending; and this *Case* Reduction ascending and descending, or descending and ascending.

181. *Corollary.* Hence it appears, that any Person who is acquainted with this Chapter, may make such Tables as the following at his Pleasure.

1. *Of* MONEY.

Farth.			
4 =	1 Penny.		
48 =	12 =	1 Shill.	
960 =	240 =	20 =	1 *l.*

2. *Of* TROY WEIGHT.

Grains.			
24 =	1 Penny-weight.		
480 =	20 =	1 Ounce.	
5760 =	240 =	12 =	1 Pound.

3. APOTHECARIES WEIGHT.

Grains.				
20 =	1 Scruple.			
60 =	3 =	1 Dram.		
480 =	24 =	8 =	1 Ounce.	
5760 =	288 =	96 =	12 =	1 Pound.

4. AVOIRDUPOIS WEIGHT.

Drams.				
16 =	1 oz.			
256 =	16 =	1 lb.		
28672 =	1792 =	112 =	1 C.	
573440 =	35840 =	2240 =	20 =	1 Tun.

5. LONG

5. LONG MEASURE.

Inches.					
12 =	1 Foot.				
36 =	3 =	1 Yard.			
198 =	$16\frac{1}{2}$ =	$5\frac{1}{2}$ =	1 Pole.		
7920 =	660 =	220 =	40 =	1 Furlong.	
63360 =	5280 =	1760 =	320 =	1 Mile.	

6. WINE MEASURE.

Gallons.					
42 =	1 Tierce.				
63 =	$1\frac{1}{2}$ =	1 Hogshead.			
84 =	2 =	$1\frac{1}{3}$ =	1 Puncheon.		
126 =	3 =	2 =	$1\frac{1}{2}$ =	1 Pipe or Butt.	
252 =	6 =	4 =	3 =	2 =	1 Tun.

7. ALE *London* MEASURE.

Gallons.				
8 =	1 Firkin.			
16 =	2 =	1 Kilderkin.		
32 =	4 =	2 =	1 Barrel.	
48 =	6 =	3 =	$1\frac{1}{2}$ =	1 Hogshead.

8. BEER *London* MEASURE.

Gallons.				
9 =	1 Firkin.			
18 =	2 =	1 Kilderkin.		
36 =	4 =	2 =	1 Barrel.	
54 =	6 =	3 =	$1\frac{1}{2}$ =	1 Hogshead.

9. ALE

9. ALE *and* BEER *Country* MEASURE.

Gallons.

$8\frac{1}{2}$ = 1 Firkin.				
17 =	2 = 1 Kilderkin.			
34 =	4 =	2 = 1 Barrel.		
51 =	6 =	3 =	$1\frac{1}{2}$ = 1 Hogshead.	

10. CORN MEASURE.

Gallons.

2 = 1 Peck.			
8 =	4 = 1 Bushel.		
64 =	32 =	8 = 1 Quarter.	

11. TIME.

Seconds.

60 = 1 Minute.				
3600 =	60 = 1 Hour.			
86400 =	1440 =	24 = 1 Day.		
31556935 =	525940 =	8765 =	$365 + 5^h + 48' + 55''$ = 1 Sol. Yr.	

182. *Scholium.* It may not be improper to add here the Method of multiplying and dividing applicate mixt Numbers, by Help of Reduction; and this will be sufficiently explained by three *Examples*.

Ex-

Example 1. Multiply 2 *l.* 10 *s.* 11 *d.* 3 *qrs.* by 57. (See *Art.* 163.)

Solution. By *Case* the first it will be found, that 2*l.* 10*s.* 11*d.* 3*qrs.* =2447 Farthings; ∴, in 57 Times the given Sum, there must be 57 Times 2447 Farthings = 2447 × 57 = 139479 Farthings, which, being brought into Pounds by *Case* the second, give 145 *l.* 5 *s.* 9 *d.* 3 *qrs.* The Operation at large is here annexed:

$$
\begin{array}{r}
\pounds. \quad s. \quad d. \quad qrs. \\
2 : 10 : 11 : 3 \\
\times\ 20 \\
\hline
50 \\
\times\ 12 \\
\hline
611 \\
\times\ 4 \\
\hline
2447 \\
\times\ 57 \\
\hline
139479 \\
\hline
\tfrac{1}{4}\ 34869(3 \text{ remaining} \\
\hline
\tfrac{1}{12}\ 290|5(9 \text{ remaining} \\
\hline
\tfrac{1}{2}\ 145(5 \text{ remaining} \\
\hline
\end{array}
$$

Example 2. Divide 145 *l.* 5 *s.* 9 *d.* 3 *qrs.* by 57. (See *Art.* 168.)

Solution. This being only the Reverse of the last, we shall only observe, that 145 *l.* 5 *s.* 9 *d.* 3 *qrs*, reduced into Farthings by *Case* 1, is = 139479, which, divided by 57, gives 2447 Farthings for the required Sum; which, brought into Pounds by *Case* 2, gives 2 *l.* 10 *s.* 11 *d.* 3 *qrs.*

Example 3. Suppose 145 *l.* 5 *s.* 9 *d.* 3 *qrs.* was divided equally amongst a certain Number of Men; and, it being remembered that each Man had 2 *l.* 10 *s.* 11 *d.* 3 *qrs*, it is required to find how many Men it was divided amongst?

Solution. In 145 *l.* 5 *s.* 9 *d.* 3 *qrs.* there are 139479 Farthings, and in 2 *l.* 10 *s.* 11 *d.* 3 *qrs.* there are 2447 Far-

Farthings; therefore there were as many Men, as 2447 is contained Times in 139479 = by Division 57. Q. E. I.

CHAP. XIII.

The RULE of DIRECT PROPORTION, GOLDEN RULE, or RULE of THREE DIRECT.

183. THIS is the Rule by which having 3 Numbers given, we find a fourth proportional Number, *viz.* one which shall have the same Ratio to the Third, as the Second has to the First. Or such, that the same Ratio that the First has to the Second, shall the Third have to the Fourth. This Rule is called the Rule of Three, from its having three Numbers given to find a Fourth; and from its extensive Usefulness, in the common Affairs of Life, and all the Mathematical Sciences, it is by many called the *Golden Rule.*

184. Four Quantities are said to be in direct Proportion, when the Quotient of the First and Second is equal to that of the Third and Fourth. Or, in other Words, Analogy, or Proportionality, is an Equality of Ratio's.

185. *Ratio,* (Ratio *Latin*) is the Proportion betwixt two homogenious Quantities, with Respect to their Greatness or Smallness; and is expressed by the Quotient of the two Quantities; thus, the Ratio of a to b is $\frac{a}{b}$: The Quantities compared are called the Terms of the Ratio; that which is referred to the other being called the Antecedent, *viz.* a; and that to which it is referred (b) is the Consequent; and the Quotient $\frac{a}{b}$ is named the Exponent of the Ratio.

185. *Lemma.* When four Quantities are in direct Proportion, the Product of the First and Fourth is

is * equal to that of the Second and Third; or, as some chuse to express themselves, the Product of the Extremes is equal to the Product of the Means; the First and Fourth being called Extremes, and the Second and Third the Means.

186. *Theorem:* Whence, three Numbers in direct Proportion being given, the Fourth may be found, by dividing the Product of Second and Third, by the First, and the Quotient will be † the Fourth, or required Number.

187. But, as the Numbers may not be placed in proper Order, in the Question to be solved, it may be proper to give the Learner the following *Rule*, *viz.* That, of the three given Terms, that which moves the Question must be put in the third Place; and may generally be known by these, or the like Words, What comes? What cost? How many? How much? How little? How long? How short? How far? &c. Of the other two given Terms, (which are Terms of Supposition, on Condition of which the Demand is made) that which is (or may be made) of the same Name as the Third, must be placed in the first Place; and consequently the remaining given Number in the second, or middle Place; and here it is proper to observe, that, when the third Term is found by *Art.* 186, it is in the same Name as the middle Number, and therefore, if it be in a low Denomination, it may be brought to a higher by Reduction. Note also, that it may be convenient to reduce the second Term, if of several Denominations, into the lowest mentioned, (if not lower.)

188.

* Let ra, a, br, b, be the four Quantities, which are in direct Proportion *; for $ra \div a = r$, and $br \div b = r$. Now $ra \times b = rab$; and $a \times br =$ † rab; ∵ $ra \times b =$ ‡ $a \times br$. *Q. E. D.* * 184. † 97.

† Let $a : b :: c : d$ be the Analogy, then by the *Lemma* $ad = bc$; ∵, dividing both Sides of the Equation by a, we have $d =$ || $\frac{bc}{a}$. *Q. E. D.* ‡ 23. || 108.

Note. $a : b :: c : d$ is to be read, as a is to b, so is c to d, and the like in other Cases.

188. *Question* 1. As 2 is to 3, so is 6 to a certain Number; what is that Number?

Solution. Here the Numbers stand already in proper Order, ∴, by *Art.* 186, $3 \times 6 = 18$, $\div 2 = 9$, the Number required.

The whole Operation at large would stand thus:

$$\begin{array}{r} \text{As } 2 : 3 :: 6 \\ 3 \\ \hline \tfrac{1}{2}\ 18 \\ \hline 9 \text{ the Answer.} \end{array}$$

189. *Question* 2. * Suppose Sound moves 1142 Feet in one Second of Time, how long then, after the Firing of a Cannon, may the Report be heard, at the Distance of 8 Miles from the Gun?

Solution. First, 5 Miles, being brought into Feet by Reduction, give $1760 \times 3 \times 5 = 26400$ Feet; then, by stating the Question, we shall have, if 1142 Feet : 1″ :: 26400 Feet : the Answer; found thus, $26400 \times 1'' = 26400''$; and $26400 \div 1142 = 23\frac{134}{1142}$ Seconds.

Here it may be proper to observe, that, though many Times the Numbers in the Question may be all applicate, as here, yet, when we have stated it, we consider the First and Third as abstract Numbers, and so do not commit the Absurdity of multiplying applicate by applicate Numbers.

Further, it may be proper to observe, that, though the above Stating is agreeable to the Rule given in *Art.* 187, it will admit of another Method of stating; for it is evident, that, the Time being as the Space over which the Sound passes, the Times must have the same Ratio to each other as the Spaces, and therefore we might say, as 1142 Feet : 26400 Feet :: 1′ : the Answer as above; but here the Quotient would be the Answer in the same Name as the

* *Note.* Dr. *Derham* found by many curious Experiments, that Sound, from whatever Body produced, moves equal Spaces, in equal Times, at the Rate of 1142 Feet *per* Second; and he found nothing to alter Velocity; but the Wind blowing either with, or against it: But of this, perhaps, more in a proper Place.

the third Number. And generally of the two middle Terms it matters not which is placed first in Order; for the Second, multiplied by the Third, is equal to * the Third multiplied by the Second; and therefore their Product will come out the same, and consequently the Quotient or required Answer. * 95.

190. *Question* 3. What comes 14 lb. of Butter to, at 6*d*. ½ *per* lb?

Solution. Here the two Terms of Supposition are 1 lb, and 6*d*. ½, and that which moves the Question is 14 lb; ∴, 6*d*. ½ being = (by Reduction) 13 Halfpence, the Stating, according to *Article* 187, will stand thus: If 1 lb. : 13 ½*d*. :: 14 lb. : the Number required, ∴ 13 x 14 = 182 Half-pence, the Answer (because, the first Number, which is always the Divisor, being an Unit, the Quotient will be the same as the Dividend) which by Reduction is = 7*s*. 7*d*.

191. *Question* 4. What comes 6 C. 1 Qr. 14 lb. of Tobacco to, at 2*l*. 16*s*. *per* C.?

	C. Qr. lb.	£. s.
The Work at large being duly observed will be sufficient Explanation.	6 : 1 : 14	2 : 16
	4	20
	25	56
	28	12
	204	672
	51	
	714	

d.
If 112 : 672 :: 714
672
1428
4998
4284
112)479808(4284
318 1/12 357
940 1/20 17*l*.|17*s*.
448
0

Answer 17*l*. 17*s*.

192. *Question* 5. What come 7 Yards of Linnen, to, at 2*s.* 1*d.* 2*qrs. per* Ell?

	Yds.		*s. d. qrs.*
Solution.	7		2 : 1 : 2
	× 4		× 12
	28	Quarters	25 = 2 × 12 + 1.
			4
			102 = 25 × 4 Farthings.

An Ell *English* is 5 Quarters, ∴ if 5 : 102 :: 28

× 28
816
204
2856
$\frac{1}{5}$ 571(1 remains
$\frac{1}{4}$ 142(3 remains
$\frac{1}{12}$ 11 : 10

Anſ. 11*s.* 10*d.* 3*qrs.* and $\frac{1}{5}$

193. *Question* 6. If 1 Yard coſt 7*s.* 6*d.* what come 6 Pieces of Cloth to, each containing 20 $\frac{1}{2}$ Yards, at that Rate?

Yds. 20 $\frac{1}{2}$ × 2 = 41 = $\frac{1}{2}$ Yds. in 1 Piece; *s. d.* 7 : 6 × 12 = 90
× 6
246 = $\frac{1}{2}$ Yds. in 6 Pieces.

$\frac{1}{2}$ Yds. *d.* $\frac{1}{2}$ Yds.
If 2 : 90 :: 246
90
22140
$\frac{1}{2}$ 11070
$\frac{1}{12}$ 922(6*d.* remaining
$\frac{1}{20}$ 46*l.* 2*s.* 6*d.*

Anſ. 46*l.* 2*s.* 6*d.*

194.

194. *Question* 7. Suppose a Seaman sailed in a Ship from the 10th of *June* 1753 to the 8th of *August* 1754; what comes his Wages to, at 25*s.* *per* Month, allowing 30 Days to a Month?

Solution.

	Days	Days	Days
From the 10th of *June* 1753 to the 10th of *June* 1754 is	365	If 30 : 25 ::	424
Remains in *June* —	29		25
July — — —	31		2120
In *August* — —	8		848
Sum	424		10600
			2 | 32|0
Ans. 16*l.*			16*l.*

195. Sometimes there cannot be found the Proportion, till some Operations in Addition, Subtraction, Multiplication, or Division, are performed, (besides Reduction before hinted at) or perhaps to be done after the Proportion is worked, in Order to find some Number sought: The last *Example* is one Instance, and it may not be improper to give two or three more, *viz.*

Question 8. A certain Messenger goes 6 Miles a Day, for 4 Days, from a Town *A* towards another *B*; at the End of his four Days Travelling he was 20 Miles from *B*; it is required to find how far the Towns *A* and *B* are distant from each other?

	Day	Miles	Days
Solution.	If 1 :	6 ::	4
		4	
		24	

Travelled in 4 Days 24 Miles from *A*.
Distant from *B* — 20

Ans. *A* distant from *B* 44 Miles

Here it may be observed, that there may be superfluous Terms in a Question; or one Thing repeated twice, as four Days in this *Question*; and therefore the superfluous Terms must be omitted in the Operation.

196. *Question* 9. * A certain Messanger goes 6 Miles every Day; 8 Days after another follows him, and he goes 10 Miles a Day. In how many Days will he come up to the First?

Solution. The first Messenger goes $6 \times 8 = 48$ Miles before the Second sets out; therefore the Second must gain 48 Miles upon the First, and then he will get up with him; but, by the *Question*, the Second gains $10 - 6 = 4$ Miles, each Day, upon the First, ∴ the Proportion is

$$\begin{array}{llllr} & \text{Miles} & & \text{Day} & \text{Miles} \\ \text{If} & 4 & : & 1 & :: 48 \\ & & & & 1 \\ \hline & & & & 48 \\ \hline & & & \tfrac{1}{4} & 12 \\ \hline \end{array}$$

Ans. 12 Days.

197. *Question* 10. If the ⅓ of 6 lb comes to 3 Shillings, what will the ⅛ of 40 lb come to, at that Rate?

Solution. The ⅓ of 6 is 2, for $6 \div 3 = 2$; and the ⅛ of 40 is 5; ∴ the ⅛ of 40 lb is 5 lb × 5 = 25 lb; hence, the Analogy is

$$\begin{array}{llllr} & \text{lb.} & & \text{s.} & \text{lb.} \\ \text{If} & 2 & : & 3 & :: 25 \\ & & & & 3 \\ \hline & & & & 75 \\ \hline & & & \tfrac{1}{2} & 37(1 \\ \hline & & & \tfrac{1}{2} & 3|7 \\ \hline & & & & 1\,l.\ 17\,s. \\ \hline \end{array}$$

Ans. 1*l*. 17½*s*. or 1*l*. 17*s*. 6*d*.

* This Question is from *Hill's* Arithmetic. See *Question* 12.

198.

198. *Question* 11. Admit a Merchant buys 3 Hogsheads of Tobacco, at 13 Shillings *per* C; the Weight and Tare of each as under, what comes the whole to?

	C.	Qrs.	lb.	Tare lb.
N° 1 —	6 :	2 :	11 —	34
2 —	7 :	1 :	14 —	52
3 —	8 :	3 :	6 —	28

Solution. The Weight of the 3 Hogsheads, being added up, gives

```
 C.  Qrs. lb.                     s.
 22 : 3 : 3                       13
   4                     x by     12
 ----                             ---
  91                             156
  28                               4
 ----                             ---
  731                            624 Farthings.
 182
 ----
 2551
Deduct  114 = the lb s of Tare added up.
Neat   2437 lb

    lb.  Farthings   lb.
If  112 : 624 :: 2437
                 624
               -----
                9748
               4874
             14622
         112)1520688(13577
             400     1/4  3394(1
              646    1/12 28|12(10
               868   1/2  14(2
                848
                 64
```

Ans. 14*l*. 2*s*. 10*d*. 3*qrs*.

199. Many Times the Question is so compounded, as to require two or more Statings to find the Thing sought, as in the *Examples* following.

Question 12. *From *Noremberg* to *Rome* is 140 Miles: A Traveller sets out at the same Time from each of the two Cities; one goes 8 Miles a Day, the other 6; in how many Days will they meet one another, and how many Miles will each of them go?

Solution. One travels in one Day 8 Miles.
The other 6

They both together travel in 1 Day 14

Miles Day Miles
∴ If 14 : 1 :: 140
1

140

$\frac{1}{14}$ 10 Days

Then as 1 Day : 8 Miles :: 10 Days : 80 Miles; and, as 1 Day : 6 Miles, :: 10 Days : 60 Miles.

Hence one travelled 80 Miles } in 10 Days
The other 60

Proof 140 Miles the Distance of the Places.

200. *Question* 13. †

Suppose a Man, whose Name is *A*,
A certain Work can end
In 30 Days; another, *B*,
Full 40 Days doth spend
Upon the same; the Question is,
What Time would it require,
If both together work upon't?
Pray answer my Desire.

Solu-

* This, and *Question* 9th are taken from *Hill's* Arithmetic, where they are proposed under the Title of Algebraic Questions, but without *Solutions*. Mr. *Hill* got them from *Alexander's* Algebra, a *Latin* Author.

† This is from the *Monthly Entertainments*, for 1711.

Solution. Since *B* could perform the Work in in 40 Days, find what *A* could perform in that Time, *viz.* If 30 Days : 1 Work :: 40 Days : 1 $\frac{1}{3}$ Work.

Then, in 40 Days *A* could perform 1 Work.
In 40 Days *B* could perform 1 $\frac{1}{3}$ Work.

In 40 Days both can perform 2 $\frac{1}{3}$ Work.
× 3

Gives 2 × 3 + 1 = 7 Thirds Work.

$\frac{1}{3}$ Work Days $\frac{1}{3}$ Work
Hence last Stating is, If 7 : 40 :: 3
3

120

Ans. 17 $\frac{1}{7}$ Days. $\frac{1}{7}$ 17(1 remaining

201. *Question* 14. Admit there are two Merchants *A* and *B*, and that *A* bought of *B* a Hogshead of Sugar, weighing 18 C. 1 Qr. 7 lb, at 3 *d.* $\frac{1}{2}$ *per* lb, deducting 12 lb *per* C for Tare; and that *B* bought of *A* 6 Pieces of Cloth, each Piece containing 20 Yards, at 2 *s.* 6 *d.* *per* Yard: It is required to find whether *A* is indebted to *B*, or *B* to *A*, and how much?

Solution. First find the Value of the Sugar.

lb.
From 112
Take 12

Remains 100

Hence every 112 lb, after the Tare is allowed, will be but 100 lb.

C.
18
18
188
1827

2051 = lb in 18 C. 1 Qr. 7 lb.

lb. lb. lb.
Therefore, as 112 : 100 :: 2051

100

112)205100(1831 lb, the Neat, or lbs to be paid for.

931

350

140

28

Note, 28 is $\frac{1}{4}$ of 112, ∴ 1831 $\frac{28}{112}$ lb = 1831 $\frac{1}{4}$ lb, ∴, the last being easiest, we will work by it, thus

lbs.	d.	$\frac{1}{4}$lbs. $\frac{1}{2}$d. $\frac{1}{4}$lbs.
1831 $\frac{1}{4}$	3 $\frac{1}{2}$	If 4 : 7 :: 7325
4	2	7
7325	7	51275
		$\frac{1}{4}$ 12818(3 remaining
		$\frac{1}{2}$ 6409
		$\frac{1}{12}$ 534(1 remains
		$\frac{1}{20}$ 26*l.* 14*s.*

Hence the Sugar comes to 26*l.* 14*s.* 1*d.* and $\frac{3}{4}$ of a Halfpenny; which might have been found with 1831 $\frac{28}{112}$ lb, by bringing it into 112ths of a lb, by multiplying by 112, and adding in the 28 Parts, which might be taken without any Work from the above Dividend, *viz.* 205100, and saying, if 112 112ths of a lb : 7 $\frac{1}{2}$*d.* :: 205100 112ths of lbs : same as above; but it is sufficient to have hinted it.

Now to find the Value of the Cloth.

Yds.	*s.* *s.* *d.*	Yd. $\frac{1}{2}$*s.* Yds.
20	2 $\frac{1}{2}$=2 : 6	If 1 : 5 :: 120
6	2	5
120=Yds. in 6 Pieces.	5 =$\frac{1}{2}$*s*, in 2*s*.6*d*,	60|0
		$\frac{1}{2}$ 30|0
		$\frac{1}{2}$ 15:0

Whence,

	£.	s.	d.	½d.
Whence, *A* bought of *B* Sugar valued at	26	14	1	0¼
B bought of *A* Cloth valued in —	15	00	0	0
Therefore *A* owes *B* — —	11	14	1	0¼

202. *Question* 15. A Factor has 15 Hogsheads 18 Gallons of Wine sent him for Sale; for which he is allowed 5 *l. per Cent*; he sold the Wine at 4 *s.* 8 *d. per* Gallon, to return the Neat Produce in Tobacco, at 8 *d. per.* lb; now the Charges on the same amounted to 19 *l.* 18 *s.* 8 *d.* how much Tobacco must the Factor return?

Hhds. Gall. 15 : 18; ×63 = 945 (53; 91); 963

s. d. 4 : 8; ×12 = 56

Gall. d. Gall. If 1 : 56 :: 963
56
5778
4815
Sold Wine for 53928 Pence.

Now we may say, if 100 *l.* : 5 *l.* :: the Money the Wine was sold for : the Commission required; but then we should be obliged to bring all three Numbers into Pence; ∴ * it is better to state thus, * 189.

£. d. £.
If 100 : 53928 :: 5
5
2696|40

£. s. d.
19 : 18 : 10
20
398
12
4786

	d.
Commission	2696
Charges	4786
Commission and Charges	7482

	d.
Sold Wine for	53928
Commission and Charges	7482
Remains due to the Merchant, which is to be sent	46446 *d.*

him in Tobacco; ∴ we have now to find what Quantity of Tobacco can be bought for 46446 Pence, at 8 *d. per* lb, whence this Stating;

If

	d.	lb.	d.
If	8 :	1 ::	46446
			1
			46446
			$\frac{1}{8}$ 5805|6 remains
			$\frac{1}{28}$ 207|4 lbs. remain
			$\frac{1}{4}$ 51|3 Qrs. remain

Ans. 51 C. 3 Qrs. 4 lbs. $\frac{6}{8}$ must be sent to the Merchant.

N. B. We have entirely omitted taking Notice of the $\frac{2}{100}$ of a Penny (in the Commission above) in the Operation, because it was too inconsiderable to be taken Notice of amongst Merchants; and, for the same Reason, we might have omitted the $\frac{28}{112}$ Part of a lb in the last *Question*; but we retained it here to shew the Learner how to manage such fractional Parts.

203. *Scholium.* The *Rule* of Proportion being very extensive, and there being innumerable Ways of proposing a *Question*, it may be so complicated, as many Times to require a considerable Judgment to know what Things are proportional, in Order to state the *Question*; and for these Reasons it is impossible to give any general Direction, that shall reach all *Cases*; for, after all that is, or can be done, the bringing *Questions* out of the complicated Language of the *Question* into numeral Expressions must chiefly depend on the Judgment of the Arithmetician; all that can be done to help the young Arithmetician is to propose such a Variety of *Questions*, as, when he becomes Master of them, it may be supposed he will be able to solve any other that may fall in his Way; and to this Purpose serve most of the following *Rules* of Arithmetic.

204.

204. Before we put an End to this Chapter, it may be proper to hint to the young Arithmetician, that it is absolutely necessary, before he states a *Question*, to consider whether the Terms are in direct Proportion to each other; for, otherwise, he may commit gross Errors by taking such Things to be in simple Proportion which are not so; thus, though, in Buying and Selling, the Price of the Goods increases or decreases, in the same Proportion with the Quantity of the Goods, yet, in geometric, philosophic, *&c. Cases*, those Things which at first Sight may to many Persons appear to be in simple Proportion to each other, may not be so, upon mature Consideration; wherefore, such Persons, as would solve such *Questions*, must first acquaint themselves with the Laws thereof; the Necessity of which Knowledge may be shewn by an *Example*. Let us suppose then, that there are two Towers, one of 16 Feet in Height, from the Top of which a Stone, being let fall, fell to the Ground in one Second of Time; it is required to find how high the other Tower is, from which a Stone falls in 3 Seconds?—Here, a Tyro may conclude, that, since the higher the Tower is, the longer Time the Stone must be in falling, that the Space the Stone falls through, will be in simple Proportion to the Time; and, therefore, would state the *Question* thus, as 1″ : 3″ :: 16 Feet : 48 Feet, for the Height of the Tower, which was required; but, if he asks a Person acquainted with the Laws of falling Bodies, he will be informed, that falling Bodies do not fall equal Spaces in equal Times; but that, the greater Space a Body has fallen through, the greater is its Velocity; and that the *Question* ought to be thus stated, as 1″ x 1 : 3″ x 3 :: 16 Feet : the Answer, or as, 1 : 9 :: 16 Feet : 144, the true Height of the Tower which was required. See the *Inverse Rule of Proportion*, in the next Chapter.

205. Further, it may be proper to observe, that some may object, that in several Statings, in the foregoing *Questions*, we have made the first and se-

cond Terms of different Names; (as, for Instance, in *Question* the second we have this Stating: If 1142 Feet : 1″ :: 26400 Feet) and so may demand, what Ratio can there be betwixt the first and second Terms (here Feet and Seconds) and thence conclude, that we talk improperly and absurdly; to which we shall only answer, that they may imagine the second Term to be placed in the third Place, and the third in the second Place, as we have hinted in *Art.* 189, and all Things will be clear; otherwise consider them all as abstract Numbers. Our Reason for placing them otherwise is only to conform to the common and general *Rule*, in *Art.* 187. And it may be observed, that we have put the Word If, (not As) before such Statings as have the first and second Terms of different Names, to hint that such Stating may be read properly thus, If the first Number be (give or cost) the second Number, what will the third Number be (give or cost) and so the above-mentioned Stating might be read very properly thus: If 1142 Feet give 1 Second, what will 26400 Feet give?

206. For the Sake of such of our Readers as may have the Curiosity to look into ancient Writers, we shall put an End to this Chapter with the Explanation of such Terms as were used by the Ancients, in expressing particular Ratio's; *viz.* when the Ratio, or the Antecedent divided by the Consequent, is Unity, the Ratio is said to be that of Equality. Multiple Ratio is, when the Antecedent divided by the Consequent is equal to any whole Number; and to express the particular Multiple Ratio's, if the Quotient was 2, 3, 4, 5, &c. it was respectively called double, triple, quadruple, quintuple, &c. and such are 2 to 1, 3 to 1, 4 to 1, 5 to 1, &c. But the Ratio of a lesser Number to a greater they distinguished by the Word *sub*; thus, the contrary to these, or such, whose Antecedent divided by the Consequent is equal to any Fraction, whose * Numerator is an Unit.

Whence

* In any Fraction $\frac{a}{b}$, a is called the Numerator, and b the Denominator.

Whence the Ratio of 1 to 2, 1 to 3, 1 to 4, 1 to 5, &c. or such whose Antecedent divided by the Consequent is = $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c. is called, respectively, sub-duple, sub-triple, sub-quadruple, sub-quintuple, &c.

Super-particular Ratio is, when the Quotient of the Antecedent by the Consequent is an Unit, and a Fraction whose Numerator is one; and such are 3 to 2, 4 to 3, 5 to 4, &c. And, to express the several Kinds of these Ratio's, they write the Word *Sesqui* before the Name of the lesser Term; thus, the Ratio of 3 to 2 was Sesqui-alteral; 4 to 3, Sesqui-tertian, 5 to 4, Sesqui-quartan, &c. And the contrary to these, *viz.* such whose Quotient of the Antecedent by the Consequent (by some called the Exponent of the Ratio) is a fractional Number whose Numerator is greater than Unity; as are these Ratio's, 2 to 3, 3 to 4, 4 to 5, &c. are called sub-superparticular Ratio's; and these particular Ratio's, respectively, sub-sesquialteral, sub-sesquitertian, sub-sesquiquartan, &c.

Super-partient Ratio is, when the Quotient, or Exponent of the Ratio, is an Unit, and a Fraction whose Numerator is greater than 1; as 5 to 3, 7 to 4, &c. And, to express the particular Kinds of super-partient Ratio, they put the Name of the Number by which the Antecedent exceeded the Consequent, betwixt the Words *super* and *partient*, and the lesser Term of the Ratio's after all; thus, the above-mentioned Ratio's were called super-bis-partiens tertias, and super-tri-partiens-quartas, respectively, &c. and the contrary to these are called sub-super-partient; thus, the Ratio of 3 to 5 was named sub-super-bipartiens tertias, &c.

Multiple-superparticular Ratio is, when the Exponent of the Ratio is any Integer greater than an Unit, and a Fraction whose Numerator is an Unit, as 5 to 2, 10 to 3, &c. and, to express these particular Ratio's, they put the Word *Sesqui* before the Name of the lesser Term; and before the Word

Sesqui

211. *Question* 1. If 6 Men could do a Piece of Work in 10 Days, in how many Days could 12 Men do it?

Men Days Men

Solution. If 6 : 10 :: 12 reciprocally to the Answer.
× 10 (See *Art.* 209.)

12)60(5 Anſ. 5 Days.

0

It is evident, that 5 is the true Anſwer; for, if 6 Men could do it in 10 Days, conſequently twice 6, or 12 Men, could do it in Half that Time, *viz.* in 5 Days.

212. *Queſtion* 2. If, when Wheat is 4 Shillings a Buſhel, the 20 Penny Loaf weighs 18 ℔, what ought it to weigh, when Wheat is 6 Shillings *per* Buſhel?

s. ℔. s.

Solution. If 4 : 18 :: 6 reciprocally,
× by 18

72

$\frac{1}{6}$ 12 Anſ. 12 ℔.

Note, The Number 20 in this *Queſtion* is ſuperfluous; for it does not affect the required Price; becauſe we were to find the Price of the ſame Quantity, for which Reaſon it was omitted in the *Solution*.

Let it be noted once for all, that theſe *Queſtions* may be ſolved by the *Rule* of direct Proportion; for *Example*, this *Queſtion* by *Art.* 208. may be ſtated thus,

s. s. ℔.

As 6 : 4 :: 18 directly : the Anſwer.
× by 4

72

$\frac{1}{6}$ 12 ℔ the Anſwer as before.

213. *Question* 3. How many Yards of Damask of 3 Quarters of a Yard wide must there be to line a Carpet, that is 12 Yards in Length, and 7 in Breadth?

		$\frac{1}{4}$ Yds.	$\frac{1}{4}$ Yds.	$\frac{1}{4}$ Yds.
Solution.		As 28 in Breadth :	48 in Length ::	3 in
Yds.	Yds.		x by 28	Breadth.
12	7			
4	4		1344	
48	28		$\frac{1}{3}$ 448	
			$\frac{1}{4}$ 112	Anſ. 112 Yds.

214. *Question* 4. Suppoſe that in a Garriſon there are 90 Men, with Meat ſufficient for 40 Days; how many Men muſt be turned out, that the Meat may laſt 60 Days?

	Days Men Days		
Solution.	If 40 : 90 :: 60 reciprocally.		Men
	40	The Garriſon	90 Men
	60)3600	Meat will ſerve	60
	$\frac{1}{6}$ 60	Anſ. turn out	30 Men.

215. As the Learner may be apt to take Things to be in ſimple direct Proportion, which are not ſo, as we have already hinted in *Art.* 204; ſo in this *Rule*, if he does not reaſon with himſelf, before he ſtates the *Question*, he may take ſome Things to be in ſimple reciprocal Proportion, which are not ſo; for *Example*, ſuppoſe that in a Room, where two Men *A* and *B* are ſitting, there is a Fire; from which *A* is 3 Feet, and *B* 6 Feet diſtant; and it is required to find, how much hotter it is at *A*'s Seat, than at *B*'s? In ſolving this *Question*, at firſt Sight, the Learner thinking, that as it is evident that, the nearer a Perſon is to the Fire, the greater Heat he muſt feel, may conclude that this is a *Question* in the *Rule* of Three reverſe, and therefore to be ſtated thus, if

6 Feet : 1 Degree of Heat :: 3 Feet reciprocally : 2 Degrees of Heat; or that the Heat is twice so great at *A*'s, as it is at *B*'s Seat: But let the Tyro go to a Philosopher, a Person who is acquainted with these Things, and he will be told, that, according to the Principles of Philosophy, it should be thus stated, if $6 \times 6 : 1 :: 3 \times 3$ reciprocally, or as $3 \times 3 : 1$ Degree :: 6×6 directly : 4 Degrees of Heat, or that it is 4 Times so hot at *A*'s Seat, as at *B*'s. Whence it appears, that, in solving some *Questions* which may seem to belong to common *Rules* of Arithmetic, there is not only required the Knowledge of Arithmetic, but also of some other Science.

216. We shall put an End to this Chapter, with observing, once for all, that in the following Part of this Treatise, when we would be understood to mean a reciprocal Proportion, it is always mentioned, and therefore, when a Proportion is not said to be reciprocal, it is direct.

CHAP. XV.

Of PRACTICE.

217. BY Practice (Πρακτικὴ) is understood some short Methods of solving such *Questions* of of the *Rule* of Three as are frequent in Business; so that this *Rule* might properly go by the Name of Compendiums in the *Rule* of Three; and, therefore, all the Compendiums in Multiplication and Division may be supposed to belong to this *Rule*; but, these being known already, we shall proceed to lay down a few other *Rules* adapted to particular *Cases*; but, first, the following Table must be committed to Memory.

Even

Even or aliquot Parts of a Shilling.			Even or aliquot Parts of a Pound.		
Parts of 1*d.*			*s.* *d.*		
$\frac{1}{4}$	is =	$\frac{1}{48}$ of 1*s.*	1 0	is =	$\frac{1}{20}$ of a £.
$\frac{1}{2}$		$\frac{1}{24}$	1 8		$\frac{1}{12}$
$\frac{3}{4}$		$\frac{1}{16}$	2 0		$\frac{1}{10}$
1*d.*		$\frac{1}{12}$	2 6		$\frac{1}{8}$
$1\frac{1}{2}$		$\frac{1}{8}$	3 4		$\frac{1}{6}$
2		$\frac{1}{6}$	4 0		$\frac{1}{5}$
3		$\frac{1}{4}$	5 0		$\frac{1}{4}$
4		$\frac{1}{3}$	6 8		$\frac{1}{3}$
6		$\frac{1}{2}$	10 0		$\frac{1}{2}$

218. *Case* 1. When the Price of an Unit, *viz.* one Yard, one Pound, *&c.* is an even Part of a Shilling, take the Part expressed in the Table, and the Quotient will give the Answer in Shillings; which (if more than 20) bring into £*s*, by cutting off the last Figure, and taking the Half of the others.

219. *Example* 1. What come 114 lb to, at a Halfpenny *per* lb?

Solution. A $\frac{1}{2}d.$ is $\frac{1}{24}$ of a Shilling.

$$\begin{array}{ll} & 114 \\ \hline \tfrac{1}{2}d. \text{ is } \tfrac{1}{24} & 4 \text{ Shillings and 8 remains} \end{array}$$

Which is 18 Halfpence, or 9 Pence.

$$\begin{array}{ll} & 114 \\ \hline \text{Or thus, } 1\,d. \text{ is } \tfrac{1}{12} & 9 \text{ Shillings and } 6\,d \text{ remaining.} \\ \hline \text{a Halfpenny is } \tfrac{1}{2} & 4:9 \\ \hline \end{array}$$

Otherwise thus: Because 2 Halfpence are 1 *d*, taking the $\frac{1}{2}$ will give the Price in Pence; and the $\frac{1}{12}$ of the Pence will be the Price in Shillings.

$$\begin{array}{ll} & 114 \\ \hline \tfrac{1}{2} & 57 \text{ Pence.} \\ \hline \tfrac{1}{12} & 4:9 \\ \hline \end{array}$$

220. *Example* 2. 120 Yards at 6*d. per* Yard.

	120
6*d.* is $\frac{1}{2}$	6|0 Shillings.
$\frac{1}{2}$	3 : 0 . Anf. 3*l.*

221. *Cafe* 2. When the Price of an Unit is not an aliquot Part of a Shilling, as 5*d.* 7*d.* 8*d.* 9*d.* 10*d.* or 11*d*, part them into 2 or 3 even Parts of a Shilling, and take the Parts refpectively belonging to them.

222. *Example* 1. 215 oz. at 9*d. per* oz.

Solution. Here 9*d.* may be parted into two Parts 6*d.* and 3*d.* ∴ work for 6*d.* and 3*d.* and add the Sums together.

	215
6*d.* is $\frac{1}{2}$	107 : 6
3*d.* is $\frac{1}{4}$	53 : 9
$\frac{1}{2}$	16|1 : 3
	Anf. 8*l.* 1*s.* 3*d.*

223. *Example* 2. 115 Yards at 8*d. per* Yard.

	115
4*d.* is $\frac{1}{3}$	38 : 4
4*d.* is $\frac{1}{3}$	38 : 4
	7|6 : 8
$\frac{1}{2}$	3*l.* 16*s.* 8*d.*

224. *Cafe* 3. When the Price is Pence and Farthings, work for the Pence, as in the former *Cafes*; and, if the Farthings are an even Part of the Pence, you may work by taking fuch Part of the Pence; otherwife you may take that Part the Farthings are of a Shilling.

225. *Example.* 84 ℔. at 4*d.* ½ *per* ℔.

	84	Or thus,		84
4*d.* is ⅓	28		4*d.* is ⅓	28
½*d.* is $\frac{1}{24}$ of a 1*s.*	3 : 6		½*d.* is ⅛ of 4*d.*	3 : 6
	3|1 : 6			3|1 : 6
½	1 : 11 : 6		½	1 : 11 : 6

226. *Scholium.* The Butchers have a Method of computing, which is in many *Cases* very compendious, and is illustrated in the following *Example*, 9 ℔ of Beef at 2*d.* ¾ *per* ℔. Here, the Price of a ℔ wanting but 1 Farthing of 3*d*, they first compute at 3*d.* *per* ℔. thus, 9 Threepences is 27*d.* (for $3 \times 9 = 27$) and 9 Farthings is 2*d.* ¼, and 27*d.* less than 2*d.* ¼ is 24*d.* ¾, or 2*s.* 0*d.* ¾.

227. *Case* 4. When the Price is Shillings, or Shillings and Pence, and the even Part of a Pound; take the Part you find in the Table.

228. *Example* 1. 200 C. at 10*s.* *per* C.

	200
10*s.* is ½	100*l.*

229. *Example* 2. 150 Yards, at 6*s.* 8*d.* *per* Yard.

	150
6*s.* 8*d.* is ⅓	50*l.*

230. *Case* 5. When the Price of an Unit is Shillings, or Shillings and Pence, or Shillings, Pence, and Farthings, and not an aliquot Part of a £. multiply by the Shillings, and work for the Pence and and Farthings, as you did in the three first *Cases*.

231. *Example* 1. 210 Yards at 3*s. per* Yard.

210
3
63|0*s.*
$\frac{1}{2}$ 31*l.* 10*s.*

232. *Example* 2. 114 lb at 6*s.* 4*d. per* lb,

	114
	6
	684
4*d.* is $\frac{1}{3}$	38
	72|2
	$\frac{1}{2}$ 36*l.* 2*s.*

233. But, when the Price of an Unit is an even Number of Shillings, the Value of any Quantity may be more compendiously found by multiplying by $\frac{1}{2}$ the Number of Shillings; remembering to double the Units Place of the Product for Shillings, the other Places will be £*s.*

234. *Example.* 214 lb at 8*s. per* lb,

	214
$\frac{1}{2}$ of 8 is =	4
Anſ.	85*l.* 12*s*,

214
8
171|2
$\frac{1}{2}$ 85*l.* 12*s.*

Here 4 Times 4 is 16; the first Figure 6, being doubled, is 12 Shillings; then 1 × 4 + 1 carried = 5, and 2 × 4 = 8, whence the Anſwer is 85*l.* 12*s.* The Reaſon for this Method will appear by comparing it with the Operation done according to *Art.* 230, which is here annexed. For it is evident, that, if any Quantity be multiplied by $\frac{1}{2}$ of any Number, the Product muſt be the ſame as if multiplied by the whole Number and $\frac{1}{2}$ of that Product be taken.

235. Here, by Way of *Corollary*, may be shewn the Method of working for Pence by the Aliquots of 2 *s*, which some are very fond of, and take to be of late Invention; though it is to be found, as Mr. *Lowe* acquaints us in Dr. *Record*'s Arithmetic, as ancient as the Reign of *Edward* the Sixth.

One *Example* will be sufficient to shew the Method, which is this: What come 115 Yards to, at 8 *d*. *per* Yard?

Here it is evident, * that at 2 *s*. *per* Yard, by a bare Inspection, 115 Yards would come to 11 *l*. 10 *s*; and consequently, as 8 *d*. is $\frac{1}{3}$ of 2 *s*, at 8 *d*. *per* Yard, 115 Yards must come to $\frac{1}{3}$ of that Money, *viz*. 3 *l*. 16 *s*. 8 *d*. Whence it appears, by comparing this Operation with that in *Art*. 223, that in many *Cases* this Method is shorter than the common Method. * 232.

	£. s.
	11 : 10
8 *d*. is $\frac{1}{3}$ of 2 *s*.	3 : 16 : 8

236. *Scholium*. Perhaps it may not be improper, before we proceed any further, to shew the Reasons of the *Cases* already delivered.

1. As to those which are done by aliquot Parts, the Reason is, that the given Quantity, suppose n Things, at one Shilling, or 1 *l*. each, must come to n Shillings, or £*s*; and therefore, if the given Price of ones be $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$, &c. of a Shilling, or Pound, the required Price must be $\frac{1}{2}$, $\frac{1}{3}$, or $\frac{1}{4}$, &c. of n Shillings, or £*s*; and generally, if $\frac{1}{m}$ Part, the required Price of the Whole will be $\frac{1}{m}$ Part of n.

Secondly and lastly. In such *Cases*, wherein we multiply the Number of Things by the Price of one, the Reason is evident; for certainly, if 1 Thing cost s Shillings, or £*s*, &c. 9 Things must cost 9 Times s Shillings, or £*s*, &c. that is, $s \times 9$, or † $9 \times s =$ Price of the whole Quantity, which was required. † 95.

237. *Case* 6. When the Number of Things are few, it is a ready Way to multiply the Price of one by

by the Number of Things, by the Method already explained in *Articles* 162 and 163.

238. *Example* 1. 57 C. at 2*l.* 10*s.* 11*d.* 3*qrs.* *per* C. It is evident that 57 Times 2*l.* 10*s.* 11*d.* 3*qrs.* is the Price of 57 C, which * is 145*l.* 5*s.* 9*d.* $\frac{3}{4}$.

* 163.

239. *Example* 2. What come 14 Yards of Cloth to, at 1*l.* 2*s.* 6*d.* *per* Yard?

£.	s.	d.
1	2	6
		7
7	17	6
		2
15	15	0

Or, as 2*s.* 6*d.* is an aliquot Part of a £, it might have been worked thus:

1*l.* × 14 = 14*l*, and by *Case* 4. 14 Yards at 2*s.* 6*d.* *per* Yard will be found to be 1*l.* 15*s.* and therefore the whole Price = 14*l.* + 1*l.* 15*s.* = 15*l.* 15*s.* as before.

240. *Case* 7. When the Quantity is a Fraction, multiply the Price of an Unit by the Numerator, and divide by the Denominator, according to the Methods explained in the 10th and 11th Chapters. Or divide by the Denominator, and multiply the Quotient by the Numerator, either of which *Rules* will give the true Answer.

241. *Example* 1. If a Parcel of Land cost 142*l.* 14*s.* 10*d.* what is the Value of a seventh Part?

It is evident that a seventh Part of 142*l.* 14*s.* 10*d.* must be the Answer; which, by *Art.* 166, is 20*l.* 7*s.* 10*d.*

242. *Example* 2. What is the Value of $\frac{3}{5}$ of a Yard, a Yard being valued at 7*d.* 3*qrs?*

Here we multiply by 3, and take the $\frac{1}{5}$, which must give the Answer; for, the Quantity being $\frac{3}{5}$ of a Yard, it is evident, that its Price must be $\frac{3}{5}$ of the Price of 1 Yard;

	d.	qrs.	
	7	3	
		3	
	23	1	
$\frac{1}{5}$	4	2 $\frac{3}{5}$	the Answer.

1 Yard: But, if we multiply the Price of 1 Yard by 3, it makes it 3 Times as much, and consequently $\frac{1}{5}$ of this Product is just as much as $\frac{3}{5}$ of the Price of 1 Yard. Or you may, if you please, take $\frac{1}{5}$ of the Price of 1 Yard, and multiply that $\frac{1}{5}$ by 3; (because the Price of $\frac{3}{5}$ is required) and the Product, it is evident, will be the same as above.

The same Method of Reasoning will hold good in all possible *Examples*, and might be easily made general; but what is already said is sufficient to shew the Reason of this *Case*.

243. *Case* 8. When the Price of an Unit is given to find the Value 12, the *Rule* is, for every Penny reckon a Shilling. The Reason is plain, for 12 Things, at 1 *d*. each, come to 12 *d*. or 1 Shilling.

244. *Example*. 12 Yards, at 2 *d*. $\frac{1}{2}$ *per* Yard, is 2 *s*. $\frac{1}{2}$, or 2 *s*. 6 *d*.

245. When the Price of 12 Things is given, to find the Price of one, the Answer is found mentally, by only considering the Shillings as Pence.

246. *Example*. If 12 Yards cost 2 *s*. 6 *d*, that is, 2 *s*. $\frac{1}{2}$, one Yard comes to 2 *d*. $\frac{1}{2}$.

247. *Scholium*. This *Case* and *Corollary* are useful to such Persons as sell many Things by the Dozen (12).

248. *Case* 9. When the Price of one (Thing) is given, to find the Price of 112, or, which is the same, the Price of 1 lb being given, to find the Price of 1 C Weight; Multiplying 9 *s*. 4 *d*. by the Pence that one cost will give the Answer.

249. *Example*. 112 lb. at 3 *d*. $\frac{1}{2}$ *per* lb.

	s.	d.
	9 :	4
		3 $\frac{1}{2}$
	28 :	0
$\frac{1}{2}$	4 :	8
	3|2 :	8
$\frac{1}{1}$	1 *l*. : 12 *s*. :	8 *d*.

The Reason of this *Rule* is, that 112 at 1 *d*. comes to 9 *s*. 4 *d*.

250.

250. When the Price is small, it is a ready Way to multiply 2 *s.* 4 *d.* by the Farthings that one cost; thus, the last *Example* may be worked by multiplying 2 *s.* 4 *d.* by 14 (there being 14 Farthings in 3 *d.* $\frac{1}{2}$.) because 112 at 1 Farthing comes to 2 *s.* 4 *d.*

251. *Case* 10. When the Price of 112 is given, to find the Price of one; or the Price of 1 C, to find the Price of 1 ℔; multiply the Price in Shillings by 3, and divide by 7, the Quotient will be the Equal to * the Price of one in Farthings.

252. *Example* 1. At 14 *s.* *per* C. what is that *per* ℔?

$$\begin{array}{r} 14 \\ 3 \\ \hline \div 7 \;\; 42 \\ \hline \end{array}$$

Ans. 6 Farthings = 1 *d.* 2 *qrs.*

253. *Example* 2. At 23 *s.* 8 *d.* *per* C. what is that *per* ℔?

$$\begin{array}{rl} s. & d. \\ 23 : & 8 \\ & 3 \\ \hline 71 & \\ \div 7 \;\; 10 & (1 \text{ remaining} \\ \hline \div 4 \;\; 2\,d. & 2\,qrs. \\ \hline \end{array}$$

Ans. 2 *d.* 2 *qrs.* $\frac{1}{7}$.

This, and the last *Case*, are useful to Grocers, *&c.* who buy and sell by the C Weight.

254. *Case* 11. When the Price of an Unit is given, to find the Price of 100, multiply 2 *s.* 1 *d.* by the Farthings that one comes to, the Product will give the Answer; (because 100, at a Farthing each, comes to 2 *s.* 1 *d.*) or, which is the same, as many Farthings as one

* Let s = the Price of 112 in Shillings, p = the Price of an Unit in Farthings, then $112\,p$ = the Price of 112 in Farthings; and
* 56. ∵, a Shilling being = 48 Farthings, $\frac{112p}{48} = s$; ∴, multiplying by
† 108. 48, we have * $112\,p = 48\,s$, and, dividing by 16, we get $7\,p =$ †
‡ 108. $3\,s$, whence dividing, by 7, we find $p =$ ‡ $\frac{3s}{7}$. Q. E. D.

one costs, count twice as many Shillings, and once as many Pence.

255. *Example.* 100 Yards at 2 *d.* $\frac{1}{4}$ *per* Yard.

s. *d.*
2 : 1
2 *d.* $\frac{1}{4}$ = 9 Farthings

Anf. 18 : 9

256. Or this *Case* may be solved by multiplying 8 *s.* 4 *d.* by the Pence that Unity comes to.

257. *Example.* 100 Knives at 4 *d.* each,

s. *d.*
8 : 4
4

3|13 : 4

Anf. 1 *l.* 13 *s.* 4 *d.*

258. *Case* 12. The Price of 100 being given, to find the Price of one; multiply the Shillings by 12, and divide by 100, the Quotient will be the * Answer in Pence.

259. *Example.* If 100 Yards cost 18 *s.* 9 *d.* what is that *per* Yard?

s. *d.*
18 : 9
Mult. by 12 and add in 9 *s.* for the 9 *d.*

2|25

Anf. 2 *d.* $\frac{25}{100}$ or 2 *d.* $\frac{1}{4}$.

Here we might have multiplied the 9 *d.* by 12, but, as we must then have divided by 12, to bring the Pence into Shillings, to carry to the Shillings, it is better, since the Pence would by such Operation become Shillings, to multiply only the Shillings by 12,

* Let p = the Price of an Unit in Pence, s = the Price of 100 in Shillings, then $\frac{100p}{12} = s$; ∴, multiplying by 12, we have $100p$ = * $12s$, and dividing by 100 we find p = † $\frac{12s}{100}$. *Q. E. D.*

* 56.
† 108.

12, and add in the Pence as Shillings.—This, and the 11th *Case*, will be of Use to such Persons as have frequent Occasion to buy or sell by 5 Score to the Hundred, called the neat or small Hundred.

260. *Case* 13. When the Price of one is given, to find the Price of 1000; multiplying 1 *l.* 0 *s.* 10 *d.* by the Farthings that one comes to, will give the Answer; (because 1000 at 1 Farthing comes to 1 *l.* 0 *s.* 10 *d.*)

261. *Example.* 1000 Tiles, at 1 *d.* 2 *qrs.* each.

	£.	s.	d.
	1 :	0 :	10
1 *d.* $\frac{1}{2}$ =			6 Farthings
Anf.	6 :	5 :	0

262. *Case* 14. When the Price of 1000 Things is given in Pounds, to find the Value of one; multiplying the Pounds that 1000 comes to by 12, and dividing by 50, * give the Value of one, in Pence.

263. *Example.* If 1000 comes to 6 *l.* 5 *s.* what is the Value of one?

£. s.
6 : 5
12

7|5 : 0

$\frac{1}{5}$ 1|2 remains.

Anf. 1 *d.* $\frac{25}{50}$ or 1 *d.* $\frac{1}{2}$.

See *Art.* 117. and 120.

This, and the 13th *Case*, may be useful to such as buy or sell Things by the Thousand.

264. *Case* 15. The Price of one being given, to find the Value of 144, or, which is the same, to find the

* Let p = the Price of one in Pence, l = the Price of 1000 in Pounds, then $1000\,p$ = Price of 1000 in Pence, and dividing by
* 56. 240 (because 240 Pence = 1 *l.*) gives $\frac{1000p}{240} = l$; which multi-
† 108. plied by 240 gives $1000\,p =$ * $240\,l$; and dividing by 20 gives
‡ 108. $50\,p =$ † $12\,l$; and dividing this by 50 gives $p =$ ‡ $\frac{12}{50}l$. *Q. E. D.*

the Value of a great Grofs, the Price of a Dozen being given: Multiply 12 Shillings by the Price that one comes to in Pence, the Product will be the Anfwer, becaufe 144, at 1 *d*. each, comes to 12 Shillings.

265. *Example* 1. At 6 *d*. each, what comes 144 to?

£.	s.
0 :	12
	6
3 :	12

266. *Example* 2. At 4 *d*. $\frac{1}{2}$ each, what comes 144 to?

	£.	s.
	0 :	12
		4 $\frac{1}{2}$
	2 :	8
$\frac{1}{2}$ of 12 *s*. =		6
Anf.	2 *l*.	14 *s*.

Or thus, 4 $\frac{1}{2}$
Mult. by 12

5|4

Anf. $\frac{1}{2}$ 2 *l*. 14 *s*.

267. *Cafe* 16. When the Price of 144 is given, to find the Value of one: Multiply the Pounds that 144 come to by 5, and divide the Product by 3, the Quotient will give the * Anfwer in Pence.

268. *Example*. If 144 coft 2 *l*. 14 *s*. what is the Value of one?

	£.	s.
	2 :	14
		5
	13 :	10
$\frac{1}{3}$	4 :	10

Anf. 4 *d*. $\frac{10}{20}$ or 4 *d*. $\frac{1}{2}$.

268.

* Let p = the Price of one in Pence, l = the Price of 144 in Pounds, then $\frac{144p}{240} = l$, (becaufe 240 Pence = 1 Pound) ∴, multiplying by 240, we have $144p =$ * $240\,l$, which divided by 48 gives $3p =$ † $5\,l$; confequently, dividing by 3 brings out $\frac{5l}{3} =$ ‡ p. *Q. E. D.* * 56. † 108. ‡ 108.

275. Tare (from *Teeren Dutch*) is an Allowance for the Weight of the Chest, Bag, Hogshead, *&c.* in which the Goods are; and it is sometimes marked on the Chest, Bag, *&c.* and then it is called *Invoice Tare:* It is sometimes at so much *per* Bag, Chest, Hogshead, *&c.* and sometimes at so much *per* C.

276. Tret (perhaps from *Tritus Latin*) is an Allowance of 4 lb on every 104 lb, made to the Freemen of *London*, in Consideration of Moats or Dust, *&c.* in the Goods.

277. Cloff, or Clough (Clough *Saxon*) is another Allowance made to the Citizens of *London*, for the Turn of the Scale; and is 2 lb *per* every 3 C.

278. Gross (*Gross French*) is the whole Weight, before any Allowances are made.

279. Suttle is what remains after some Allowances are made, but not all.

280. Neat (*Net French*) is what remains after all the Allowances are made.

281. 14 and 16 lbs are called Standards for Tare, because by them the Tare for any Number of Pounds *per Cent.* may be found.

282. It is the Custom of Merchants first to allow the Tare, and out of the Remainder (or first Suttle) to allow the Tret; and out of this Remainder (or second Suttle) to allow the Cloff.—As we suppose our Readers to have acquired sufficient Knowledge to find these Things by the *Rule* of Three, we shall only here shew the Methods commonly used by Merchants for this Purpose.

283. When the Tare *per* lb, is an aliquot Part of 112 lb, take the Part you find in the annexed Table; but, if it be not an aliquot Part of 112 lb, you may find it by Help of 14 or 16 lb.

Aliquot Parts of 112 lb are,

lb			
7	is	$\frac{1}{16}$	of 112 lb.
14	is	$\frac{1}{8}$	of 112 lb.
16	is	$\frac{1}{7}$	of 112 lb.
28	is	$\frac{1}{4}$	of 112 lb.
56	is	$\frac{1}{2}$	of 112 lb.
84	is	$\frac{3}{4}$	of 112 lb.

284. *Example* 1. Suppose 1 C. 1 Qr. 11 lb. Tare is allowed on 110 C. 2 Qrs. 8 lb. what is the Neat?

	C.	Qrs.	lb.
From	110	2	8
Sub. Tare	1	1	11
Anf. Neat	109	0	25

285. *Example* 2. What is the Neat of 3 Bags, each 1 C. 2 Qrs. 10 lb, Tare 14 lb. *per* Bag?

14
3
—
42
—
$\frac{1}{112}$ 1 : 14

	C.	Qrs.	lb.
	1	2	10
			3
Whole	4	3	2
Tare		1	14
Anf. Neat	4	1	16

286. *Example* 3. Grofs 21 C, Tare 56 lb *per* C; what is the Neat?

C.
21
—
56 lb. is $\frac{1}{2}$ 10 : 2 Anf. 10 C. 2 Qrs.

287. *Example* 4. Grofs 410 C. 2 Qrs. 12 lb, Tare 20 lb. *per* C; what is the Neat?

	C.	Qrs.	lb
	410	2	12
16 lb is $\frac{1}{7}$	58	2	17 $\frac{1}{4}$
4 lb is $\frac{1}{4}$ of 16 lb	14	2	18 $\frac{1}{4}$
Tare	73	1	8

Note, Finding it to the neareft Quarter of a lb is fufficiently near the Truth.

	C.	Qrs.	lb.	
From the Grofs	410	2	12	
Sub. Tare	73	1	8	
Anf. Neat	337	1	4	

288. *Example* 5. Grofs 410 C. 2 Qrs. 12 lb, Tare 10 lb *per* C; required the Neat?

	C.	Qrs.	lb.	
	410	2	12	
14 lb is $\frac{1}{8}$ From	51	1	8 $\frac{1}{2}$	= Tare at 14 lb *per* C.
2 lb is $\frac{1}{7}$ of 14 lb	7	1	9	
2 lb is $\frac{1}{7}$ of 14 lb	7	1	9 $\frac{1}{4}$	
Deduct	14	2	18 $\frac{1}{4}$	= Tare at 4 lb *per* C.
Remains	36	2	18 $\frac{1}{4}$	= Tare at 10 lb *per* C.

	C.	Qrs.	lb.
Grofs	410	2	12
Tare	36	2	18 $\frac{1}{4}$
Anf. Neat	373	3	21 $\frac{3}{4}$

289. Tret being always 4 lb *per* 104 lb, the Tret may be found by taking $\frac{1}{26}$; becaufe 4 lb is $\frac{1}{26}$ of 104 lb.

290. *Example.* Grofs 410 C. 2 Qrs. 12 lb, Tare 10 lb *per* C, Tret 4 lb *per* 104 lb; what is the Neat?

Solution. By *Article* 288,

	C.	Qrs.	lb.	
the Suttle is $\frac{1}{26}$	373	3	21 $\frac{3}{4}$	
Subtract	14	1	14 $\frac{3}{4}$	= Tret.
Neat	359	2	7	

291. Cloff being always 2 lb *per* every 3 C, take $\frac{1}{3}$ of the Cs, the Quotient will give double lbs.

292.

292. *Example.* Groſs 410C. 2Qrs. 12 ℔, Tare 10 ℔ *per* C, Tret 4 ℔ *per* 104 ℔, Cloff 2 ℔ *per* 3C; what is the Neat?

Solution. By *Art.* 290, the ſecond Suttle is 359 C. 2Qrs. 7 ℔.

We have here omitted the odd 7 ℔, becauſe, as 3C makes but 2 ℔, it would be inconſiderable.

	C. Qrs.
	359 : 2
	℔
$\frac{1}{3}$	119 $\frac{3}{4}$
	119 $\frac{3}{4}$
	239 $\frac{1}{2}$
	239
$\frac{1}{28}$	8 : 15
$\frac{1}{4}$	2

	C.	Qrs.	℔.
Second Suttle	359 :	2 :	7
Deduct Cloff	2 :	0 :	15
Anſ. Neat	357 :	1 :	20

CHAP. XVII.

COMMISSION.

293. COMMISSION (*Commiſſio* low *Latin*) or Factorage (from *Factor*, from *Facteur French*) is, when one Perſon buys or ſells Goods for another, and is allowed a certain Sum *per Cent.* (ſuch as they agree upon) out of the Price of the Goods, for his Trouble.—In this *Rule*, ſome alſo include Average, Brokerage, Duties, (at ſuch a Rate *per Cent.*) Inſurance, Primage, Stowage, and other Things, which are computed at a certain Rate *per Cent.*

294. Average (*Averagium Latin*) is commonly underſtood for the Quota, or Proportion, which each Proprietor in a Ship, or the Goods therein, is adjudged (on a reaſonable Eſtimation) to contribute towards their Loſſes which are ſuſtained, by ſome

 of

of their Goods being thrown overboard for Preservation of the Ship *.—There is also a small Duty, called *petty Average*, which those Merchants, who send Goods in another Person's Ship, allow the Master for his Care of the Goods, over and above the Freight †: But this more properly comes under Fellowship.

295. Brokerage is the Fee or Reward paid unto a Person called a ‡ Broker, for assisting a Merchant, or Factor, in buying or selling Goods.

296. By Duties, we here mean Taxes laid on Merchandizes, by whatever Name denominated, whether called

* "It is in this Sense called *Average*, because it is proportioned "after the Rate of every Man's Goods carried, Stat. 32 *H.* VIII. "By the Laws of the Sea, when there is an extreme Necessity, "the Goods, Wares, Guns, or whatever else is on board the Ship, "may (consulting the Mariners) be thrown overboard by the Master "for the Preservation of the Ship; and it shall be made good by "Average and Contribution, Stat. 49. *Ed.* III. But if the Master "takes in more Goods than he ought, without Leave of the Owners "and Freighters, and a Storm arises at Sea, and Part of the Freighters Goods are thrown overboard, the remaining Goods are not "not subject to Average, but the Master is to make good the Loss "out of his own Estate. And, if the Ship's Geer or Apparel be "lost by Storm, the same is not within the Average. If Goods "are cast overboard before half the Voyage is performed, they are "to be estimated at the Price they cost: But, if they are thrown over "afterwards, they are then to be esteemed according to the Price "the rest sell for, at the Port of Arrival. Where Goods are given "to Pirates by Way of Composition to save the rest, there shall "be Average by the civil Law. In Case of Average, all those for "whose Interest the Thing was cast into the Sea, are to contribute "to indemnify the Person whose Property it was, and every Thing "to be taxed to this Purpose, even Jewels and Gold, notwithstanding they do not burthen the Ship, and even the Vessel itself, but "not Passengers nor Provisions." Suppl. to *Harris*'s Lexicon.

† "Hence, in Bills of Entry are these Words:—Paying so much "Freight for the said Goods, with Primage and Average accustomed." Suppl. to *Harris*'s Lexicon Technicum.

‡ "Brokers were formerly broken Traders, from whence the "Word came. In *London*, if any Persons shall act as Brokers without a Licence from the Lord Mayor, they shall forfeit 5000*l.* "and such Persons as employ them 50*l.* They are to carry about "them a Silver Medal, having the King's Arms and the Arms of "the City." Suppl. to *Harris*'s Lexicon.

called Duties, Subsidies, Poundages, Imposts, additional Duties, &c. computed at such a Rate *per Cent*.

297. Insurance is a Security given in Consideration of a Sum of Money paid in Hand, called the *Premium* of Insurance, to make Good the Loss of Ships, Houses, Merchandizes, &c. which may happen by Storms, Pirates, Fire, &c. to the Value insured.

298. "Primage is an Allowance (appointed by "a Statute of 32. *H.* VIII.) to be paid to Mariners "at their first Sailing out of Port, for their loading "the Ship." *Harris*'s Lexicon.

299. Stowage, the Money paid for stowing the Goods in a Vessel.

300. Here it is evident, that *Questions* of the Nature here proposed may be stated thus, as 100*l.* : the given Sum :: the given Rate *per Cent* : the Commission required; or if 100 : the given Rate :: the given Sum : the Commission required. Whence any Person, who understands the common *Rule* of Three, is capable of solving the *Questions* belonging to this *Rule*; therefore we shall content ourselves with only giving a few *Examples*, worked by Practice, *Case* the last, and aliquot Parts. *Example* 1. What is the Commission of 310*l.* at 5*l. per Cent?*

£. £. £.
If 100 : 310 :: 5
5 See *Art.* 116 and 168.

£. 15|50
|20

10|00 Ans. 15*l.* 10*s.*

 301.

301. *Example* 2. What is the Brokerage of 210 *l.* 10 *s.* 8 *d.* at 15 Shillings *per Cent?*

£. s. £. s. d.
If 100 : 15 :: 210 : 10 : 8

10 *s.* is $\frac{1}{2}$ 105 : 5 : 4
5 *s.* is $\frac{1}{2}$ of 10 *s.* 52 : 12 : 8

1|57 : 18 : 0
|20

11|58
|12

6|96
|4

3|84

Anf. 1 *l.* 11 *s.* 6 *d.* 3 *qrs.* $\frac{84}{100}$.

Note, 15 Shillings being $\frac{3}{4}$ of a £, we might, instead of working for 10 *s.* and 5 *s*, have multiplied by 3 and took the $\frac{1}{4}$ of that Product; or have took $\frac{1}{4}$ and repeated that $\frac{1}{4}$ three Times. This Method we will shew in one of the following *Examples.*

302. *Example* 3. What *Premium* must be paid for Insurance of 410 *l.* 7 *s.* 7 *d*, at 3 *l.* $\frac{2}{5}$ *per Cent?*

£. £. £. s. d.
If 100 : $3\frac{2}{5}$ :: 410 : 7 : 7
3

1231 : 2 : 9
$\frac{2}{5}$ = 164 : 3 : 0 $\frac{1}{4}$
£. 13|95 : 5 : 9 $\frac{1}{4}$
|20

19|05
|12

0|69
|4

2|77 Anf. 13 *l.* 19 *s.* 0 *d.* 2 *qrs.* $\frac{77}{100}$.

£. s. d.
410 : 7 : 7
2

820 : 15 : 2
$\frac{1}{5}$ 164 : 3 : 0 $\frac{1}{4}$

303. *Example* 4. What is the Commission of 210 *l.* 10 *s.* 11 *d*, at 17 *l.* $\frac{1}{2}$ *per Cent?*

```
        £.    £.      £.    s.   d.
If 100 : 17½ :: 210 : 10 : 11
                                 8
                    ---------------
                   1684 :  7 :  4
                                2
                    ---------------
                   3368 : 14 :  8
              Add   210 : 10 : 11
                ½   105 :  5 :  5½
                    ---------------
                £. 36|84 : 11 :  0½
                     |20
                    ---------------
                   16|91
                     |12
                    -------
                   10|80
                     | 4
                    -------
                    3|22
                    -------
```

Anſ. 36 *l.* 16 *s.* 10 *d.* 3 *qrs.* $\frac{22}{100}$.

304. Before we proceed to any more *Examples*, it may not be improper to hint, that if the Rate *per Cent.* be 1 *l*, 1 $\frac{1}{4}$ *l*, 2 *l*, 2 $\frac{1}{2}$ *l*, 5 *l*, 10 *l*, 20 *l*, 25 *l*, or 50 *l*, the Commiſſion, &c. may be very readily found by taking $\frac{1}{100}$, $\frac{1}{80}$, $\frac{1}{50}$, $\frac{1}{40}$, $\frac{1}{20}$, $\frac{1}{10}$, $\frac{1}{5}$, $\frac{1}{4}$, or $\frac{1}{2}$ reſpectively, becauſe 1 *l.* is $\frac{1}{100}$ of 100 *l*; 1 $\frac{1}{4}$ *l.* is $\frac{1}{80}$ of 100 *l*, &c.

305. *Example* 5. What is the Commiſſion of 4 *l.* 3 *s.* 4 *d*, at 25 *l.* *per Cent?*

```
                     £.  s.   d.
                     4 : 3 :  4
                    -----------
By laſt Art. 25 l. is ¼ = 1 : 0 : 10  the Commiſſion.
                    -----------
                                          Q. E. I.
```

306. *Example* 6. What is the Commiſſion of 83 *l.* 6 *s.* 8 *d*, at 5 *l.* *per Cent?*

Solution. By *Art.* 286, $5l.$ is $\frac{1}{20}$, therefore we may take $\frac{1}{2}$ and $\frac{1}{10}$ of that Half. It may also be done by taking the $\frac{1}{10}$ (which is done by cutting off 1 Figure each Time, at the right Hand) and taking $\frac{1}{2}$ of that $\frac{1}{10}$ at the same Time as we are taking $\frac{1}{10}$; thus, cutting off the $3l$, *viz.* dividing by 10, and taking $\frac{1}{2}$ of the $8l$, there is $4l$; which being written down, carry the $3l. = 60s.$ to the 6 Shillings, and there will be 66 Shillings, and, supposing in our Minds the last Figure cut off, there will be 6 Shillings on the left Hand, $\frac{1}{2}$ of which is 3 Shillings to be put down; and the 6 Shillings, or 72 Pence, being added to the 8 Pence, give 80 Pence, from which the right Hand Figure (here 0) being cut off, we have 8 Pence on the left Hand, $\frac{1}{2}$ of which is 4 Pence; whence, the required Commission is $4l.\ 3s.\ 4d.$ See *Art.* 117.

	£.	s.	d.
	83	6	8
$\frac{1}{2}$	41	13	4
$\frac{1}{10}$	4	3	4

	£.	s.	d.
	8|3	6	8
$\frac{1}{2}$	4	3	4

308. *Example* 7. By a Book of Rates (such as *Crouch*'s) we find Tobacco of the *British* Plantations is rated at $1s.\ 8d.$ *per* Pound (Weight) and that, if the Duties be paid down at Entry, there is $25l.$ *per Cent.* Discount allowed out of all the Duties, in Consideration of paying ready Money; and that the Duties are, old Subsidy, which is $5l.$ *per Cent.* of the Value at the Rate; additional Duty at $1d.$ *per* ℔ Weight; new Subsidy or $5l.$ *per Cent.* of the Value at the Rate; one Third Subsidy, being $\frac{1}{3}$ of the new Subsidy; Impost on Tobacco, being $3d.$ *per* ℔ Weight; and Subsidy of 1747, which is $5l.$ *per Cent.* of the Value at the Rate. From whence, it is required to find the ready Money that will pay the Duty of 1000 ℔ of *British* Plantation Tobacco imported?

Solution. First we must find the Value of 1000 ℔ at $1s.\ 8d.$ *per* ℔; which by Practice or the Golden *Rule* will be found to be $83l.\ 6s.\ 8d.$ five Pounds *per Cent.* of which is the Gross old Subsidy by the *Question*, which

which is * 4 *l.* 3 *s.* 4 *d.* from which we are to deduct * 288.
25 *l. per Cent.* or † 1 *l.* 0 *s.* 10 *d.*; and ∴ the Neat old † 287.
Subsidy is 4 *l.* 3 *s.* 4 *d.* — 1 *l.* 0 *s.* 10 *d.* = 3 *l.* 2 *s.* 6 *d.* The next Duty to be allowed, being 1 *d. per* ℔, is = 1000 *d.* or 4 *l.* 3 *s.* 4 *d*; which being equal to the Gross of old Subsidy, the Neat of this must also be = the Neat of that, or 3 *l.* 2 *s.* 6 *d.* The new Subsidy being 5 *l. per Cent.* as well as the old Subsidy, the Neat new Subsidy must be = the Neat old Subsidy = 3 *l.* 2 *s.* 6 *d.* One Third Subsidy being ⅓ of the new Subsidy, the Neat of this Duty must be = ⅓ of the Neat new Subsidy, and ∴ = 1 *l.* 0 *s.* 10 *d.* Impost on Tobacco being 3 *d. per* ℔, the Neat Impost must be three Times the additional Duty, = 3 *l.* 2 *s.* 6 *d.* × 3 = 9 *l.* 7 *s.* 6 *d.* Lastly, the Subsidy of 1747 being 5 *l. per Cent.* the Neat of this must be equal to the Neat new Subsidy; whence, by adding these several Duties together, as is here annexed, the whole Duty = 22 *l.* 18 *s.* 4 *d.*

	£	s.	d.
Neat Old Subsidy	3 :	2 :	6
Neat add. Duty	3 :	2 :	6
New Subsidy	3 :	2 :	6
One Third Subsidy	1 :	0 :	10
Imp. on Tobacco	9 :	7 :	6
Neat Subsidy 1747	3 :	2 :	6
Whole Duty paid	22 :	18 :	4

309. *Example* 8. The old Subsidy is always to be paid at Entry, but the other Duties may be bonded, (that is, Bonds may be given to pay the other Duties in 18 Months to commence at the End of 30 Days from the Date of the Entry) and this is what Merchants generally do; but in this *Case* they have but 15 *l. per Cent.* discounted out of the Gross of those Duties which they bond. Now in this *Case* let it be required to find the Duties of 1000 ℔ of *British* Plantation Tobacco imported?

Solu-

Solution. Comparing the Operation here annexed with the last *Article*, the Nature of the Whole will plainly appear.

	£.	s.	d.	
Neat old Subsidy	3	2	6	paid
Neat additional Duty	3	10	10	
Neat new Subsidy	3	10	10	
Neat one Third Subsidy	1	3	7 $\frac{1}{4}$	
Neat Imp. on Tobacco	10	12	6	
Neat Subsidy 1747	3	10	10	
Duties secured	22	8	7 $\frac{1}{4}$	

	£.	s.	d.
	4	3	4
5 *l.* is $\frac{1}{20}$	0	4	2
			3
For 15 *l. perCent.*	0	12	6
Neat new Subsidy, *&c.*	3	10	10

CHAP. XVIII.

SIMPLE FELLOWSHIP.

310. FELLOWSHIP is the *Rule* by which, when two or more Merchants trade together, we divide the Gain or Loss, in such Proportion that each Merchant may have, or bear, such Part of the Gain or Loss, as is consistent with right Reason.

311. Fellowship is divided into two Parts, simple and compound. Simple is that which we intend to treat of in this Chapter; and is, when either the Stocks, or Times

Times of their respective Continuance in Trade, are equal; and, therefore, may be properly divided into two *Cases*.

312. *Case* 1. When the Stocks are unequal, but the Time of their Continuance equal.

In this *Case*, as the whole Stock is to the whole Gain or Loss, so is each Man's particular Stock to his respective Gain or Loss.

For, the Times being equal, we have nothing to do with them, but must divide the Gain or Loss in Proportion to their Stocks. For it is evident, that, as the Times the Stocks are in Trade are equal, if I put in $\frac{1}{2}$ of the whole Stock, I ought to have $\frac{1}{2}$ of the whole Gain; if my Part of the whole Stock be $\frac{1}{3}$, my Share of the Gain or Loss is $\frac{1}{3}$; and generally, if I put in $\frac{1}{n}$ of the Stock, I ought to have $\frac{1}{n}$ Part of the whole Gain (or Loss) that is, the same Ratio, that the Stock has to the whole Gain or Loss, must each Merchants particular Stocks have to his respective Gain or Loss; whence, * as the whole Stock : whole Gain :: each particular Stock : its respective Gain. *Q. E. D.* * 184.

313. *Example* 1. Two Merchants *A* and *B* trade in Company; *A* put in 100*l*, *B* 150*l*; they gained 80*l*: What is each Man's Part of the Gains?

Solution. *A* put in 100*l*, *B* put in 150*l*, ∵ 100 + 150 = 250*l*. = whole Stock; ∵ as the whole Stock 250*l*. : the whole Gain 80*l*. :: *A*'s Stock 100*l*. : *A*'s Part of the Gain = (by the Golden *Rule*) 32*l*. Then the whole Gain 80*l*. — *A*'s Gain 32*l*. = 48*l*. † = *B*'s Part of the Gain. Or *B*'s Part of the Gain may be found by another Stating, *viz*. as the whole Stock 250*l* : the whole Gain 80*l*. :: *B*'s Stock 150*l* : *B*'s Share of the Gain = 48*l*; and then *A*'s Gain + *B*'s Gain = 32*l*. + 48*l*. = 80*l*. = the whole Gain for Proof. *Note*, There are other Kinds of *Questions* solvable by the same Method, and such for *Example* is the following. † 48.

314. *Question* 2. Suppose 3 Butchers, *A*, *B*, *C*, hire a Piece of Ground for 10*l*. a Year; in which

A keeps 4 Oxen, *B* 3, and *C* 2; what ought each to pay of the Rent?

Solution. Here each Butcher ought to pay such Part of the Rent as is in the same Ratio to the whole Rent that his Number of Oxen is to all the Oxen; whence, the Statings are, if $(4+3+2=)$ 19 : 10*l.* :: 4 : 4*l.* 8*s.* 10*d.* 2*qrs.* $\frac{2}{3}$ = what *A* ought to pay; and if 9 : 10*l.* :: 3 : 3*l.* 6*s.* 8*d.* = the Part of the Rent *B* must pay; again, if 9 : 10*l.* :: 2 : 2*l.* 4*s.* 5*d.* 1*qr.* $\frac{1}{3}$ = the Money *C* ought to pay. And *A*'s, *B*'s, and *C*'s Parts of the Rent, being added up, will make up the whole Rent 10*l.* for Proof.

315. *Case* 2. When the Stocks are equal, but the Times unequal; in this *Case* the *Rule* is, as the whole Time is to the whole Gain or Loss, so is each particular Time to its respective Gain or Loss. For it is evident, that, the Stocks being equal, their respective Gains ought to be in the same Proportion, or Ratio, as their respective Times. Now it is evident further, that it is the same Thing in Effect, whether we suppose these several equal Stocks to be put in Trade these several Times, or one Person to put in the same Stock a Time = all the Times, for his Gain ought to be equal to all theirs; therefore, as the whole Time, &c. as above. *Q. E. D.*

316. *Example.* Two Merchants *A* and *B* put into Trade equal Stocks; *A*'s Stock continued in Trade 3 Months, and *B*'s 7 Months. The whole Sum gained was 3*l.* 18*s.* it is required to find, how, in Reason, the Gains ought to be divided betwixt them?

Solution. Here the Statings may be, if $3+7=10$ Months : 3*l.* 18*s.* :: 3 Months : 1*l.* 3*s.* 4*d.* 3*qrs.* $\frac{2}{10}$ = *A*'s Share. And if 10 Months : 3*l.* 18*s.* :: 7 Months : 2*l.* 14*s.* 7*d.* 0*qrs.* $\frac{8}{10}$ = *B*'s Gain. Now, for Proof, 1*l.* 3*s.* 4*d.* 3*qrs.* $\frac{2}{10}$, and 2*l.* 14*s.* 7*d.* 0*qrs.* $\frac{8}{10}$, being added together, will bring out 3*l.* 18*s.* = the whole Gain.

CHAP.

CHAP. XIX.

COMPOUND FELLOWSHIP, *or* FELLOWSHIP *with* TIME.

317. COMPOUND Fellowſhip, or Fellowſhip with Time, is when both the Stocks and their reſpective Times of Continuance are unequal.

318. In this *Caſe* the *Rule* is, multiply each Man's Stock by the Time it continued in Trade; then ſay, as the Sum of all theſe Products is to the whole Gain or Loſs, ſo is each particular Product to its reſpective Gain or Loſs. The Reaſon of this *Rule* may be thus ſhewn: It is reaſonable, that the Shares of Gain or Loſs ſhould be in the ſame Proportion as the Intereſt, which might be gained by the ſeveral Stocks, put out to Intereſt their reſpective Times, at a certain equal Rate *per Cent. per Annum.* Now that the above *Rule* is agreeable to this Suppoſition may eaſily appear; for the multiplying the Stocks by their reſpective Times may be conſidered as tranforming the *Queſtion* into another, in which the Gains or Loſſes ſhall continue the ſame, but in which the ſeveral Products will be the Stocks put into Trade for an equal Time, *viz.* an Unit of the Denomination of the Time multiplied; (for Inſtance, ſuppoſe 5 *l.* was put into Trade 3 Months, 5 *l.* × 3 = 15 *l*; now it is evident 15 *l*, being 3 Times 5 *l*, ought to gain as much in 1 Month, as 5 *l.* would in 3 Months) therefore, by *Art.* 293, as the Sum of theſe new Stocks (*viz.* the Sum of the ſeveral Products of each Stock into its reſpective Time) is to the whole Gain or Loſs, ſo is each new Stock (*viz.* any Stock into its Time) to its reſpective Gain or Loſs. *Q. E. D.*

319. *Example* 1. Two Men, *A* and *B*, trade in Company; *A* put in 200 *l.* for 1 Year, *B* put in 100 *l.* for 2 Years; they gained 240 *l.* what is each Man's Share?

Solu-

Solution. Here $200l. \times 1 = 200l.$ and $100l \times 2 = 200$, the Sum of these Products $200 + 200 = 400l$; therefore, as $400l. : 240l. :: 200l. : 120l. = A$'s Part $= B$'s Share. And, that this *Solution* is just, is evident; for, though *B* has but half the Stock that *A* has, yet, if his Money be twice so long in Trade, which is the present *Case*, he ought to have as much of the Gain as *A*.

320. *Question* 2. Two Merchants, *A* and *B*, trade in Company; *A* for a Year put in $120l$, but at the End of 4 Months (wanting Money to pay of a Bill) took out $100l$; *B* put in at the Beginning 6 Pieces of Cloth, each Piece containing 20 Yards, at 5 Shillings *per* Yard: They gained at the End of the Year $80l$. What is each Man's Part thereof?

Solution. First find the Value of the six Pieces of Cloth, which by the *Rule* of 3, or Practice, will be found to be $30l$. Now it must be observed that *A*'s Stock must be parted in two, *viz.* $120l.$ for 4 Months, and $120 - 100 = 20l$, for $(12 - 4 =)$ 8 Months; and, therefore, the Work will be the same as for 3 Merchants. Hence, since $120 \times 4 = 480$, $20 \times 8 = 160$, and $30 \times 12 = 360$, we have, as $480l. + 160l. + 360l. = 1000l. : 80l. :: 480l. : 38l.\ 8s. =$ a Part of *A*'s Gains; and as $1000l. : 80l. :: 160l. : 12l.\ 16s. =$ the other Part of *A*'s Gains; whence, $38l.\ 8s. + 12l.\ 16s. = 51l.\ 4s. = A$'s whole Share of the Gains. Lastly, as $1000l. : 80l. :: 360l. : 28l.\ 16s. = B$'s Part; and $51l.\ 4s. + 28l.\ 16s. = 80l. =$ the whole Gains for Proof.

Note, *A*'s Part might have been found by one Stating by adding $480l.$ and $160l.$ together; which we shall leave for the Learner's Exercise.

321. *Questions* of the Nature of the following may be solved by this *Rule*. *Question* 3. Two Butchers, *A* and *B*, rented a Piece of Land for a Year, the Rent of which was $10l$. Now it is required to find how much of the Rent each Butcher ought to pay, *A* having kept 6 Oxen in it two Months, and *B* 2 Oxen the whole Year?

Solution. By considering the Oxen as their respective Stocks, we have $6 \times 2 = 12$, and $2 \times 12 = 24$, and thence these Statings: If $12 + 24 = 36 : 10l. :: 12 : 3l.\ 6s.\ 8d. =$ *A*'s Part; and if $36 : 10l. :: 24 : 6l.\ 13s.\ 4d. =$ *B*'s Part; and $3l.\ 6s.\ 8d. + 6l.\ 13s.\ 4d. = 10l.$ for Proof.

Note. There are other Kinds of *Questions* given by some Authors in this *Rule*, but, as they more properly belong to single Position, we shall refer thither.

322. *Scholium.* Though we have in *Art.* 299. already given the Reason of this *Rule*, yet, for the Sake of our young Algebraist, we will give an algebraical Demonstration, when there are two Partners in Trade, and after the same Manner it may be demonstrated, when there are 3 or more.

Demonstration. Let the two Partners be *A* and *B*; let $a =$ *A*'s Stock, $t =$ the Time of its Continuance in Trade; $b =$ *B*'s Stock, and $c =$ the Time of its Continuance; put $x =$ *A*'s Gain, $g =$ the whole Gain; then, by the *Rule* of Three Direct, as * $a : x :: b : \frac{bx}{a} =$ *B*'s Gain in the Time t; again, as † $t : \frac{bx}{a} :: c : \frac{bcx}{ta} =$ *B*'s Gain in the Time c; $\therefore x + \frac{bcx}{ta} = g$; and, multiplying by ta, we have $tax + bcx$ ‡$= tag$, $\therefore$, dividing by gx, we have $\frac{ta + bc}{g}$ ‖ $= \frac{ta}{x}$, $\therefore$ as § $ta + bc : g :: ta : x$. *Q. E. D.*

* 293.
† 296.
‡ 56.
‖ 108.
§ 184.

CHAP. XX.

The DOUBLE RULE *of* THREE, *or* RULE *of* FIVE NUMBERS.

323. THIS *Rule* is called *Five Numbers*, from its resolving such *Questions* as have 5 Numbers given to find a sixth Proportional; which may be

be solved by two Statings of the *Rule* of Three (whence it is also called the double *Rule* of Three, or *Rule* of Compound Proportion) which Statings are sometimes both direct, and sometimes one direct and one reverse. It ought to be hinted to the Learner, that this *Rule* is not designed to solve all *Questions* which can be solved by two Statings of the *Rule* of Three, but only such as have 3 of the 5 given Terms conjoined together as conditional or supposed; and that the other two are Terms of Demand, or upon them the *Question* is formed; and these two Terms, with the Number sought, must be related, or depend on each other as the 3 Terms of Supposition. Whence, and by casting his Eye on the following *Questions*, he will soon be able to determine at Sight, what *Questions* are resolvable by this *Rule*. *Note*, In solving *Questions* belonging to this *Rule* by 2 Statings of the *Rule* of Three, make the Answer of the first Stating the middle Term in the second Stating.

324. *Example* 1. If 100*l.* in 12 Months gain 5*l.* what will 200*l.* gain in 9 Months?

Solution. First state as 100*l.* : 5*l.* :: 200*l.* : 10*l.* = what 200*l.* would gain in the same Time that 100*l.* gains 5*l*; ∴ say, if 12 Months : 10*l.* :: 9 Months : 7*l.* 10*s.* the Answer. Or we might have worked first by the Time, stating, if 12 Months : 5*l.* :: 9 Months : the Money that could be gained by 100*l.* in 9 Months; and then saying, as 100*l.* : the Sum found by the first Stating :: 200*l.* : the Money required.

We shall hint to the young Arithmetician one Thing more, as it does not appear to have been taken Notice of by Authors, and it is this: *Questions* of this Kind may be transformed into others, which may be solved by a single Stating of the *Rule* of Three. For *Example*, in this *Question*, it is evident, that 12 Times 100*l.* = 1200*l.* ought to gain as much in 1 Month, as 100*l.* in 12 Months; also 9 Times 200*l.* = 1800*l.* ought to gain as much in 1 Month, as 200*l.* in 9 Months; and, therefore, this *Question* must have the

same Anſwer as the following, If 1200 *l.* in 1 Month gain 5 *l.* what ought 1800 *l.* to gain in the ſame Time? Here the Stating would be, by the Rule of Three Direct, as 1200 *l* : 5 *l* :: 1800 *l* : 7 *l.* 10 *s.* the Anſwer as before.

325. *Queſtion* 2. If 10 Men do a Piece of Work in 6 Days, how many Men could do twice ſo much in 3 Days? Firſt ſay, if 6 Days : 10 Men :: 3 Days inverſely : 20 Men, that is, 20 Men could do as much Work in 3 Days as the 10 could in 6 Days; then ſay, if 1 Work : 20 Men :: 2 Works : Men, the Anſwer.

326. From a proper Conſideration of theſe *Queſtions*, the following *Rule* is deduced, which ſolves all *Queſtions* of this Nature by a ſingle Stating, and without taking any Notice whether the *Queſtion* belongs to what is uſed to be called five Numbers direct, or five Numbers reverſe. The *Rule* is, of the three Terms of Suppoſition let that which is the principal Cauſe of Gain or Loſs, Increaſe or Decreaſe, Action or Paſſion, be put in the firſt Place; and that which denotes the Space of Time or Diſtance of Place, &c. be put in the ſecond Place; and the remaining Number, or that which denotes the Gain or Loſs, Increaſe or Decreaſe, Action or Paſſion, be placed in the third Place: Then place the two Terms of Demand, underneath thoſe of the ſame Name.

Having thus placed the Terms, obſerve the following *Theorems*: 1. When there is no Term under the firſt or ſecond Place, multiply the firſt, ſecond, and laſt Terms together for a Dividend, and the other two Terms for a Diviſor; the Quotient will be the Anſwer.

2. When there is no Term under the third Place, multiply the three laſt Terms together for a Dividend, and the two firſt for a Diviſor; the Quotient gives the Anſwer *.

M 327.

* To demonſtrate, or ſhew the Inveſtigation of theſe Rules, let *p*, *t*, *g*, be the three Terms of Suppoſition, *viz.* *p* = the principal Cauſe, *t* = the Time, or Diſtance of Place; *g* = the Gain or Loſs, &c.

317. *Example* 1. *Question* the first, being worked according to the Method just directed, will stand thus:

£.	Months	£.
100 —	12 —	5
200 —	19	

Hence, by the *Rule* the second, $200 \times 9 \times 5 = 9000$ = the Dividend, and $100 \times 12 = 1200$ = the Divisor; now $9000l. \div 1200l. = 7\frac{1}{2}l.$ the Answer.

328. *Question* the second may be solved thus:

The Numbers, being placed according to *Art.* 326, will appear, as we have here annexed:

Men	Days	Work
10 —	6 —	1
	3 —	

Hence,

&c. x, y, and z = the 3 other Terms, *viz.* x = the Principal, y = the Time, &c. z = Gain or Loss, &c. Then, these Quantities, being placed as above directed, will stand thus, $\left\{\begin{matrix} p — t — g \\ x — y — z. \end{matrix}\right.$

Now if the *Question* is of the Nature of *Question* the first, that is, if both Statings are direct, we shall have $p : g :: x : \frac{gx}{p}$ = what x would gain, &c. in the Time t; ∵ as $t : \frac{gx}{p} :: y : \frac{gyx}{tp} = z$. Hence when there is no Term under the third Place, that is, when z is required, $z = \frac{gyx}{tp}$ which is *Theorem* the second.

If y be required, that is, if there be no Term under the second

* 56. Place, by $\frac{gyx}{tp} = z$, we get, by multiplying by tp, $gyx =$ * tpz; and,

† 108. dividing by gx, we have $y =$ † $\frac{tpz}{gx}$, which is one Part of *Theorem* the first.

Again, if x be required, that is, when there is no Term under

|| 108. the first Place, then from $gyx = tpz$ by dividing by gy we get $x =$ || $\frac{tpz}{gy}$, which is the other Part of *Theorem* the first.

Having thus proved the *Rules* to be true, and shewn their Investigations, when both Statings are direct, we ought now to prove them true also, when one Stating is in reciprocal Proportion, and the other direct; such, for Instance, as are *Questions* of the Nature of *Question* the second.

First then, as $t : p :: y$ inversely : $\frac{tp}{y}$ = the principal Cause that could do as much in the Time y, as p could in the Time t; then, as $g : \frac{tp}{y} :: z : \frac{tpz}{gy} = x$. Which, being the same as above, proves the *Theorems* to hold good in all Cases.

Hence, by *Theorem* 1, 10 × 6 × 2 = 120 the Dividend, and 3 × 1 = 3 the Divisor, and 120 ÷ 3 = 40 = the Number of Men, which was required.

329. *Question* 3. If the Carriage of 1 C. 40 Miles cost 9 Shillings, what ought the Carriage of a Tun to cost for 20 Miles?

	C.	Miles	
The Numbers rightly placed	1 —	40 —	9
will stand thus:	20 —	20	

Here is no Term under the third Place; hence, by *Theorem* second, 9 × 20 × 20 = 3600, and 1 × 40 = 40; are the Dividend and Divisor respectively; ∴ 3600 ÷ 40 = 90 = the Shillings required = 4*l.* 10*s.* for Answer.

330. *Question* 4. If 36 Acres of Grass be mowed by 6 Men in 8 Days, how many Acres may be mowed by 36 Men in 38 Days?

	Men	Days	Acres
Solution. The Numbers placed	6 —	8 —	36
in proper Order appear thus:	36 —	38	

Now, by *Theorem* second, 36 × 36 × 38 = 49248 = the Dividend, and 6 × 8 = 48 = the Divisor, and $\frac{49248}{48}$ = 1026 the required Number of Acres.

331. *Question* 5. If 56 Gallons of Drink serve 25 Persons 120 Days, how many Days will 100 Gallons serve 12 Persons?

	Persons	Days	Gallons
Solution. Here the Numbers	25 —	120 —	56
will stand thus:	12 —		— 100

Wherefore, by *Theorem* 1, 25 × 120 × 100 = 300000 = the Dividend, and 56 × 12 = 672 = the Divisor, ∴ 300000 ÷ 672 = 446 $\frac{288}{672}$ Days. *Q. E. I.*

332. *Scholium.* Mr. *Stonehouse*, in his Arithmetic, hints, that the *Rule* we have now been explaining of, is not universal; because, as he says, it will not solve *Questions* which require two inverse Statings of the *Rule of Three*; and that, tho' it is generally supposed

 such

such a *Question* cannot be proposed in the double *Rule* of Three, yet he thinks the following *Question* cannot be otherwise truly answered, *viz.* "Suppose "2000 Men besieged in a Town, with Provision for "three Weeks, allowing each Man 20 Ounces *per* "Day, were to be reinforced with 2000 Men more, "and were resolved to make the aforesaid Provision "last them 6 Weeks, how many Ounces must each "Man have *per* Day?" Here the Numbers, being placed according to the Directions in *Art.* 326, will stand thus:

Men Weeks Oz.
2000 — 3 — 20
4000 — 6

Which being read, according to the Meaning of this *Rule*, will signify thus much, If 2000 Men in 3 Weeks eat 20 Oz. what will 4000 Men eat in 6 Weeks? But this is not the Meaning of the *Question*, and therefore this *Question* does not belong to this *Rule*; consequently, this is no real Objection to it.——Further, this *Question* may, by the Method hinted in *Art.* 324, be tranformed into another, solvable by one Stating of the *Rule* of Three inverse, and to which therefore it might have been referred; for Instance, 2000 Men 3 Months is = 2000 x 3 = 6000 Men 1 Month, and 4000 Men 6 Months = 4000 x 6 = 24000 Men 1 Month; whence the *Question* may stand thus: Suppose there is in a Town Provision sufficient to serve a Garrison of 6000 Men 1 Month, at 20 Ounces each *per* Day; but, it being thought necessary to have 24000 Men in it 1 Month, it is required to find how much each Man must eat *per* Day. Here, by the inverse *Rule* of Three, we have, if 6000 Men : 20 Oz. :: 24000 Men inversely : 5 Ounces.—*Note*, The above *Question* might also have been solved after this Manner: 6 Months being twice 3 Months, if the Men were equal, it is plain they must eat but one Half, *viz.* 10 Oz. each, *per* Day; but, the Men being double in Number, in Order to eat but the same Quantity of Provision, they must eat but $\frac{1}{2}$ of 10 Oz. *viz.* 5 Oz. each Man *per* Day.

CHAP.

CHAP. XXI.

Of Simple Interest.

333. WHEN any Sum of Money is lent out, at so much *per Cent. per Annum*, (that is, so much upon 100*l.* for 1 Year, which, according to Law, must not exceed 5*l.*) the Money which the Lender may demand as his Right, at the End of any Time, for the Lending of the above-mentioned Sum (called the *Principal*) is denominated the *Interest*.

334. The *Rule* of *Interest* is divided into two Parts, Simple and Compound. Simple Interest is that which arises only from the Principal first lent, and the Time it continues lent: Thus, if the Interest of 100*l.* be 5*l.* for 1 Year, the simple Interest of 100*l.* for 2 Years will be 10*l*; 5*l.* being for the Interest of 100*l.* for the first Year, and the other 5*l.* for the Interest of 100*l.* the second Year.

We have already shewn, in Five Numbers, the Method of finding by that Rule the Interest of any Sum, for any Time; for which Reason, we shall here only lay down a few Rules after the Practice Fashion; and these we shall comprehend in two *Cases*.

335. *Case* 1. When the Time is Years, or Years and Months, work for 1 Year as in Commission; then (since for 2 Years the Interest must be twice so much as for 1 Year, and for 3 Years 3 Times as much, &c.) multiply the Interest for one Year by the Number of Years, and work for the Months, if any, by taking the Parts the Months are of a Year.

336. *Question* 1. What is the Interest of 250*l.* for 2 Years, at 5*l. per Cent. per Annum*?

£.

250

		l.	*s.*
250	Interest for 1 Year =	12 :	10
5	× by		2
£. 12|50			
20	Answ. Interest for 2 Years =	25 :	0
s. 10|00			

337. *Question* 2. What is the Interest of 284 *l.* 7 *s.* 6 *d.* for 3 Years 7 Months, at 4 *l.* 10 *s.* per *Cent. per Annum?*

		l.	*s.*	*d.*
		284 :	7 :	6
				4
		1137 :	10 :	0
10 *s.* is ½		142 :	3 :	9
	£. 12	|79 :	13 :	9
		|20		
	s. 15	|93		
		|12		
	d. 11	|25		
		|4		
	Farthing 1	|00		

	l.	*s.*	*d.*	*qr.*
Interest for 1 Year =	12 :	15 :	11 :	1
Interest for 3 Years =	38 :	7 :	9 :	3
4 Months is ⅓ of a Year =	4 :	5 :	3 :	3
3 Months is ¼ of a Year =	3 :	3 :	11 :	3¼
Sum = Interest required	45 :	17 :	1 :	1¼

338. *Case* 2. When the Time is in Days, the Interest may be found by first finding the Interest for 1 Year by Case the first, and then the required Interest by the Rule of Three Direct, stating, if the Days in 1 Year be the Interest for 1 Year, the Days the Money is at Interest, will give the required Interest.

339. *Question* 3. What is the Interest of 410 *l.* 10 *s.* for 1 Year and 40 Days, at 5 *l.* *per Cent per Annum*?

See *Art.* 306. $\left\{\begin{array}{l} \text{£. } \quad \text{s.} \\ 410 : 10 \\ \hline \frac{1}{20} \\ 20 : 10 : 6 = \\ \text{Interest for 1 Year.}\end{array}\right.$

Now, if 365 Days : 20*l.* 10*s.* 6*d.* :: 40 Days : 2*l.* 4*s.* 11*d.* 3*qrs.* $\frac{125}{365}$ = the Interest for 40 Days.

∴ 20*l.* 10*s.* 6*d.* + 2*l.* 4*s.* 11*d.* 3*qrs.* $\frac{125}{365}$ = 22 *l.* 15*s.* 5*d.* 3*qrs.* $\frac{125}{365}$ = the Interest for 1 Year and 40 Days, which was required.

Note, We think Five Numbers as good a Method of solving this Case as any by common Arithmetic; but the Operation by that Rule we shall leave to the Learner for his Exercise.

CHAP. XXII.

Of COMPOUND INTEREST.

340. COMPOUND INTEREST, or *Interest upon Interest*, is that which arises both from the Principal and Interest taken together as it becomes due; *i. e.* when the Interest, at the End of the Year, becomes due, but is not paid, that Interest is made a Part of the Principal; and so the first Principal and Interest of the first Year, added together, becomes the Principal for the second Year; and the Interest for the second Year, added to the Principal for the second Year, gives the Principal for the third Year, *&c. ad infinitum.* One Question will be sufficient to shew the Method of solving this Rule.

341. What will 210*l.* 7*s.* 6*d.* amount to in 3 Years, at 5*l.* per Cent. per Annum?

£. s. d.		£. s. d. qr.
210 : 7 : 6	Principal first Year	210 : 7 : 6 : 0
5	Interest first Year	10 : 10 : 4 : 2
10|51 : 17 : 6	Principal second Year	220 : 17 : 10 : 2
20		5
10|37		11|04 : 9 : 4 : 2
12		20
4|50		0|89
4		12
2|00		10|72
		4
		2|90

	£. s. d. qrs.
Principal of the second Year	220 : 17 : 10 : 2
Interest for the second Year	11 : 0 : 10 : 3 *fere*
Principal for the third Year	231 : 18 : 9 : 1
	5
	11|59 : 13 : 10 : 1
	20
	11|93
	12
	11|26
	4
	1|05

Principal

	£.	s.	d.	qr.
Principal for the third Year	231 :	18 :	9 :	1
Interest for the third Year	11 :	11 :	11 :	1 nearly
Answ. The whole Amount is	243 :	10 :	8 :	2

Note, These Interests might have been found more compendiously by *Art.* 306.

342. *Scholium.* Though the Law does not allow compound Interest to those Persons who lend Money, yet it is used in purchasing Annuities, freehold Estates, &c. which we intend to explain hereafter in its proper Place; and shall here only observe with Mr. *Malcolm*, that, "abstracting from the Reason of the Law "(which may be the Encouraging of Trade, by employing Money that Way rather than upon Interest) "if taking Interest be at all just, compound Interest "cannot be unreasonable. For, if I can demand my "Interest, when it is due, I may take that Interest-"Money, and lend it out again upon Interest to "any other Person; why then may I not lend it out "also to the Person who has my principal Sum? "And, in Point of Right and Justice, it is the same "Thing if I continue or leave that Interest in his "Hands: Therefore, there is the same Reason that "it should bear Interest after it becomes due, as that "the original Sum should do so."

CHAP. XXIII.

Of REBATE, *or* DISCOUNT.

343. REBATE, or DISCOUNT, is the *Rule* by which we find either what present Money will pay a Debt, which cannot be demanded till some Time to come, or what must be deducted or abated out

ed; because, if either the Person who is to receive, or he who is to pay the Money, was to put it to Interest for so long a Time, he would have just as much to receive as the Debt. Whence it is evident the following *Rule* will find the present Worth, *viz.* Find the Amount of 100*l.* for the given Time; then say, as that Sum : 100*l.* :: the given Debt to the present Worth. Or, in other Words, as 1 Year, 12 Months, or 365 Days : the Rate proposed :: the Time proposed : a fourth Number : Then as 100 + that fourth Number : 100*l.* :: given Debt : the present Worth.

351. Let it be here required, for Example, to find the present Worth (or what ready Money will pay the Debt mentioned in *Question* the first, *viz.*) of 210*l.* 10*s.* due at the End of 2 Years, Rebate being made at the Rate of 5*l. per Cent. per Annum?*

Solution. Say first, if 1 Year : 5*l.* :: 2 Years : 10*l.* = the Interest of 100*l.* for 2 Years; hence 100*l.* is the present Worth (or which is the same Thing, would discharge a Debt) of 100*l.* + 10*l.* = 110*l.* payable at 2 Years End. Consequently, as 110*l.* : its present Worth 100*l.* :: 210*l.* 10*s.* : 191*l.* 7*s.* 3*d.* 1*qr.* nearly = its present Worth, *Q. E. I.* And, if the Discount be required, 210*l.* 10*s.* — 191*l.* 7*s.* 3*d.* 3*qrs.* = 19*l.* 2*s.* 8*d.* 3*qrs.* very near, = the Discount as before.

352. *Scholium.* As this *Rule* is of Use in discounting Bills paid before they are due, it may not be improper to observe in this Place, (though something foreign to our present Design) that, according to Mr. *Clare*, in his *Youth's Introduction to Trade and Business*, "the Usance or Usage of Merchants, with Respect to foreign Bills of Exchange, to and from "*London* to *Rotterdam*, *Antwerp*, or any Part of the "*Low Countries*, is one Calendar Month after the "Date of the Bill; double Usance two Months, "*&c.* Usance from *Hamburgh*, *Copenhagen*, *Stockholm*, *Lubeck*, *Strasburgh* to *London*, and *contra*, is "also one Month; tho' Bills from those and other "distant Places are commonly drawn payable after

"Sight,

" Sight, because of the Uncertainty of their Arri-
" val. Usance from *London* to *Lisbon*, or *Madrid*, is
" two Months; to *Leghorn*, *Venice*, or any Part of
" the *Levant*, is three Months, and *contra*.

" After Bills of Exchange become due, whether inland or foreign, payable at Sight or otherwise, there are, by Custom of Merchants, certain Days of Grace allowed the Accepter over and above the Time prescribed by the Bill, which are more or less, according to the Usage of the Country wherein they are to be paid; as, in *Rotterdam*, they allow three Days; *Rouen*, five; *Paris*, ten; *Hamburgh*, twelve; *Antwerp* and *Madrid*, fourteen; and *London* always three: And, on the third Day, before Sun-set, Payment must be demanded on the Part of the Presenter; and, if not complied with, the Bill must that very Day (being the utmost Time allowed by the Law for that Purpose) be noted, in Order to be protested for Non-payment.

" If a Bill fall due on a *Sunday*, or other great Holiday, it is to be demanded and paid, or protested, the Day before."

353. *Question* 2. Suppose a Bill of Exchange for 600*l*. dated at *Antwerp* the 19th of *September* 1752, at double Usance, is accepted at *London*, and Payment offered the Second of *November*, 1752: What Money must be then received, Rebate being made at the Rate of 6*l*. *per Cent*. *per Annum*?

Solution. In *September* there remain 11 Days, *October* 31, *November* 19, Days of Grace 3, the Sum of those Numbers is 64 Days = the Time the Bill is due; but Payment is offered in 11 + 31 + 2 = 44 Days, ∵ 64 − 44 = 20 Days = the Time the Bill is paid before due, for which Time the Discount is to be computed; and therefore the first Stating is, If 365 Days : 5760 Farthings (= 6*l*.) :: 20 Days : 315 Farthings nearly, and the second, as 96315 Farthings (= 100*l*. + 315 Farthings) : 96000 Farthings (= 100*l*.) :: 576000 Farthings (= 600*l*.

: 589*l*.

: 589*l.* 0*s.* 9*d.* nearly, = the present Worth, or Money to be received. Q. E. I.

354. We have here only treated of simple Interest, because, as has been already hinted, the Law does not allow of compound Interest herein. We have only one Thing farther to hint before we put an End to this Chapter, and that is, that, in paying a Bill before it falls due, the Payer ought to be well satisfied (in his Mind) that the Receiver is not likely to fail shortly, because, if the Receiver "should fail before "it falls due, he will be liable to pay it to the Re-"mitter's Order a second Time."

CHAP. XXIV.

EQUATION of PAYMENTS.

355. EQUATION of Payments is a *Rule* by which when several Debts are payable at different Times (and are not supposed to bear Interest till after the Time when they become due) we may be able to determine such a Time for Payment of all the Debts, that neither the Debtor nor Creditor may be wronged thereby; and the finding such a Time is called equating the Times of Payment, or reducing them to one.

356. The common Method of working this *Rule*, is, to multiply the several Sums by their respective Times, and to divide the Sum of all these Products by the whole Debt, and the Quotient, thence arising, is called the equated Time, for Payment of the whole Debt.

357. *Example.* Suppose *A* owes *B* 205*l.* to be paid in 2 Years, and 200*l.* to be paid in 1 Year; what is the proper Time for paying the whole Debt (to prevent the Trouble of two Meetings) at the Rate of 5*l. per Cent. per Annum*, simple Interest?

Solution. In working this by the above common Method, we have nothing to do with the Rate of Interest,

tereft, but work thus, 205 × 24 (the Months in 2 Years) = 4920, and 200 × 12 (the Months in 1 Year) = 2400; then 4920 + 2400 = 7320, and 205 + 200 = 405. Laftly, 7320 ÷ 405 = 18 $\frac{30}{405}$ Months = 1 Year 6 Months and $\frac{30}{405}$ of a Month, for the Anfwer.

358. *Scholium.* Though the above Method is that which is commonly ufed, and taught, in moft arithmetical Books and Schools, it is not true; however, we fhall not here tire the Reader's Patience, by ftaying to demonftrate the Fallacy of this, and fome other Methods, given by arithmetical Writers, for folving this *Rule*; for, when we give the true *Solution*, the Demonftrating of that to be true will be a fufficient Demonftration, that the above, and all other Methods which do not agree with that, are falfe. Farther, amongft all the Treatifes of Arithmetic which have come to our Hand, (and thefe not a few) we do not remember to have met with any true *Solution*, fave one, by the laborious and learned Mr. *Malcolm*; but as neither this Gentleman's, nor our own *Solution*, could be underftood by the Learner in this Place, and as an accurate *Solution* is but of little Moment in Bufinefs, we fhall defer giving a true *Solution*, till we treat of Decimal Arithmetic.

CHAP. XXV.

BARTER, *the first* SORT.

359. BARTER is the *Rule* by which Merchants proportion the Prices, or Quantities of their Goods, in fuch a Manner, that in exchanging them neither may fuftain a Lofs by fuch Traffic.

360. Barter is divided into two Parts, called by Authors the firft and fecond Sort. Sort the firft is when

when the Rate and Quantity of any Kind of Goods, and the Rate of any other Kind of Goods, is given to find the Quantity of these Goods that must be exchanged. *Note*, Both these *Rules* are solved by the *Rule* of direct Proportion.

361. *Example* 1. Two Men *William* and *Thomas* barter; *William* hath 45 Yards of Cloth at 4*s*. 6*d*. *per* Yard; and *Thomas* has Sugar at 7*d*. *per* ℔; how much Sugar ought *Thomas* to give *William* in Exchange for his Cloth?

Solution. First by the *Rule* of Three direct or Practice, find what is the Value of 45 Yards (of Cloth) at 4*s*. 6*d*. *per* Yard, which is = 2430*d*; then, we are to find what Quantity of Sugar is equal in Value to 2430*d*, and ∵ we say by the Golden *Rule*, If 7*d* : 1 ℔ :: 2430*d*. : 347 $\frac{1}{7}$ ℔ = by Reduction 3 C. 0 *qr*. 11 ℔ and $\frac{1}{7}$ of a ℔, for the Answer.

362. *Question* 2. *John* and *James* barter thus: *John* sells *James* 100 Dozen of Rabbit Skins, at 1*s*. *per* Dozen; and is to have in Return, of *James*, Hats of two Sorts, *viz*. of 5 Shillings and 9*s*. each, and of each Sort an equal Number. It is required to find the Number of each Sort?

Solution. First say, if 1 Dozen : 1*s*. :: 100 Dozen : 100*s*. = the Value of the Rabbit Skins. Now, 5*s*. + 9*s*. = 14*s*. = the Value of 2 Hats, *viz*. 1 of each Sort. Whence, we have this Stating, if 14*s*. : 1 Hat of each Sort :: 100*s*. : 7 $\frac{2}{14}$ or 7 $\frac{1}{7}$ = the Number of Hats of each Sort; but, since there is no such Thing as selling a Part of a Hat, *John* ought to have of *James* 7 Hats of 5 Shillings each, and 7 Hats of 9*s*. each, and ($\frac{1}{7}$ of 5*s*. and $\frac{1}{7}$ of 9*s*. or $\frac{1}{7}$ of $\overline{5+9}$ or of 14*s*. =) 2 Shillings in Money.—But if the Number of each Kind was not to have been equal, but in a given Proportion to each other, then we work as in the following *Question*.

363. *Question* 3. Suppose two Men *A* and *B* barter; *A* has 210 Yards of Cloth at 4 Shillings *per* Yard, which he exchanges with *B* for Spoons at 5*s*. and Tea Spoons at 2*s*. each; and being willing, that the Number

ber of large Spoons shall be, to the Number of Tea-Spoons, as 2 to 5, that is, for every 2 large Spoons he is willing to have 5 Tea-Spoons: It is required to find how many Spoons of each Sort *A* must have for his Cloth?

Solution. First 210 Yards, at 4*s.* *per* Yard, are equal in Value to 210 x 4 = 840 Shillings; and 2 Spoons at 5*s.* = 2 x 5 = 10 Shillings; and 5 Spoons, at 2*s.* each, = 5 x 2 = 10 Shillings, whence 10*s.* x 10*s.* = 20*s.* = the Value of 7 Spoons, *viz.* 2 Spoons of the large Sort, and 5 Tea-Spoons; whence, as often as the Value of the 7 Spoons, *viz.* 20*s*, is contained in 840 Shillings, so many 2 Spoons of the larger, and also 5 Spoons of the smaller Sort, must be taken, ∵ 840 ÷ 20 = 42 = the Times that 7 Spoons, *viz.* 5 of the lesser, and 2 of the greater, is contained in the whole Number of Spoons; ∵ 42 x 2 = 84 Spoons of the large Sort, and 42 x 5 = 210 Spoons of the lesser Sort; and, for Proof, 84 Spoons at 5*s.* each = 420*s.* and 210 Spoons at 2*s.* each = 210 x 2 = 420*s*, and and 420*s.* + 420*s.* = 840*s.* the Value of the Spoons = the Price of the Cloth as above.

The Operation for the Number of Spoons, perhaps, if found by the following Method, may appears something clearer to the Learner. By the above, 840 Shillings are = the Value of the Cloth, and 20*s.* = the Value of 7 Spoons; ∵, by the Golden *Rule*, if 20*s.* : 7 Spoons :: 840*s.* : 294 Spoons. But these Spoons are to be divided into two Numbers, in the Proportion of 2 to 5; whence, by Fellowship, as 2 + 5 = 7 : 294 :: 2 : 84 = the Spoons of the larger Kind; and as 7 : 294 :: 5 : 210 = the Number of Spoons of the lesser Sort. And 84 + 210 = 294 Spoons as above, for Proof.

Corollary. Hence follows the Reason of the first Method of *Solution*. For here in one Stating we are to multiply by 7, and divide by 20; and, in the other two Statings, we are to multiply the Number that comes out by the above Stating by 2 and 5 respectively, and divide by 7; whence, as we are both to

 mul-

multiply and divide by 7, it is plain the Answer will be the same if we entirely omit the Seven, and only divide by 20, and multiply the Quotient by 2 and 5 respectively.

CHAP. XXVI.

BARTER the second SORT.

364. BARTER the second Sort shews, if one Man raises the Price of his Goods, how much another ought to raise the Price of his Goods, to barter with the former.

365. *Question* 1. *A* hath Sugar at 6 *d. per* ℔ ready Money, but in Barter will have 7 *d. per* ℔; and *B* hath Corn at 3 *s.* 6 *d. per* Bushel ready Money: What ought *B* to have *per* Bushel to barter with *A*?

Solution. Here it is evident, that as 6 *d.* : 7 *d.* :: 3 *s.* 6 *d.* : 4 *s.* 1 *d*, the Barter Price of *B*'s Corn. *Q. E. I.*

366. *Scholium.* It is common amongst Authors to find the Barter Price, in Order to find the Quantity of Goods that must be given in Exchange; but this is certainly going a round-about Way to no Purpose; for, if the Prices are raised proportionably, there can nothing be gained by either Party by raising them, and therefore it will be the same Thing to compute the Quantity by the ready Money Prices; whence, for what may be done by one Stating, it is common for most Authors to make two. For *Example*, if in the above *Question* it had been demanded, how much Corn *B* ought to give *A* for a given Quantity of Sugar, most Authors would say, first, as we have done in the above *Solution*, and then, if 1 ℔. : 7 *d.* :: given Quantity : the Barter Price of *A*'s Sugar; and then, if 4 *s.* 1 *d.* : 1 Bushel :: the Barter Price of *A*'s Sugar : the Bushels of Corn required; making in all 3 Statings, whereas there are but two necessary; *viz.* first, if

if 1 ℔. : 6*d*. :: the given Quantity : the ready Money Value of *A*'s Sugar; and then, if 3*s*. 6*d*. : 1 Bushel :: the ready Money Value of *A*'s Sugar : the Bushels required.

367. *Question* 2. *A* and *B* barter; *A* hath 1000 ℔ of Sugar at 6*d*. *per* ℔ ready Money, but in Barter 7*d*. *per* ℔, which he sells to *B*, for which he will have $\frac{1}{3}$ of the Barter Price in ready Money, and the rest in Corn; now *B*'s Corn is 3*s*. 6*d*. *per* Bushel ready Money: It is required to find how much in Money, and also in Corn, *B* must give *A* for his Sugar?

Solution. First, find, by the *Rule* of Three direct or Practice, the Value of 1000 ℔ at 6*d*. and 7*d*. *per* ℔, = 25*l*. and 29*l*. 3*s*. 4*d*. respectively; $\frac{1}{3}$ of which last, or the Barter Price, is = 9*l*. 14*s*. 5*d*. 1 *qr*. nearly = the ready Money to be paid by the *Question*. Now, since * it is the same Thing in Effect, whether we consider the Goods sold by the ready Money or Barter Price, we shall chuse the ready Money Price; and ∴ 25*l*. — 9*l*. 14*s*. 5*d*. 1 *qr*. = 15*l*. 5*s*. 6*d*. 3 *qrs*. = the Value of the Corn, and ∴ if 3*s*. 6*d*. : 1 Bushel :: 15*l*. 5*s*. 6*d*. 3 *qrs*. : 349 Bushels and $\frac{9}{14}$ of a Bushel.

* 366.

Note, The *Solutions* given to *Questions* of the Nature of this, by some Authors, are not true. For they take the proposed Part, here $\frac{1}{3}$, and subtract it both from the ready Money and Barter Price, and then say, as the first Remainder : the second :: *B*'s ready Money Price : his Barter Price : But this Proportion is false; for *B*'s ready Money, to his Barter Price, ought to be in the same Ratio as *A*'s ready Money to his Barter Price, and not in the Ratio of the above Differences.

CHAP. XXVII.

LOSS and GAIN.

368. THIS is the *Rule* by which Merchants and others compute their Loss or Gain. It is divided into two Parts; in one the Continuance of the Money or Goods in Trade

is not taken into Consideration, and for that Reason is called Loss and Gain without Time; the other, because in it we also consider the Time, is called, Loss and Gain with Time. Both these *Rules* being easily performed by such as understand the Golden *Rule*, we shall be very compendious in illustrating them.

369. *Example* 1. A Man bought 384 Yards of Cloth at 4*s.* 6*d.* *per* Yard, and sells it at 5*s.* *per* Yard; what did he gain by it?

Solution. This may be solved by two Statings, *viz.* by finding what 384 Yards come to at 4*s.* 6*d.* *per* Yard, and also at 5*s*; for it is evident the Difference of the Values thence arising must be the Gain required. However, it is better solved by one Stating thus: If it be bought at 4*s.* 6*d.* *per* Yard, and sold at 5*s.* *per* Yard, there must be gained 5*s.* — 4*s.* 6*d.* = 6*d.* on each Yard, and ∴, if 1 Yard : 6*d.* :: 384 Yards : 9*l.* 12*s.* = the Sum gained on all the Yards. *Q. E. I.*

370. *Question* 2. If a Person buys 50 Yards of Cloth for 12*l*, how must he sell the Cloth *per* Yard, to gain after the Rate of 10*l.* *per Cent?*

Solution. Say first, as 100*l.* : (100*l.* + 10*l.* =) 110*l.* :: 12*l.* : 13*l.* 4*s.* = what he must sell the 50 Yards for, ∴ say, if 50 Yards : 13*l.* 4*s.* :: 1 Yard : 5*s.* 3*d.* $\frac{18}{50}$ = the Price *per* Yard. *Q. E. I.*

Note, This *Question* might also have been solved by first finding the Value of 1 Yard, as bought; and then finding how that Price *per* Yard must be raised, to gain 10*l.* *per Cent.*

☞ See Miscellaneous *Questions* at the End of this Volume.

CHAP. XXVIII.

LOSS and GAIN with TIME.

371. QUESTION 1. If I buy Cloth at 2*s.* 8*d.* *per* Yard, and sell it at 2*s.* 10*d.* *per* Yard, to be paid in 4 Months; what is gained *per Cent.* *per Annum?*

This

This may be solved by two Statings of the *Rule* of Three direct, *viz.* by saying first, as 2*s.* 8*d.* : (2*s.* 10*d.* — 2*s.* 8*d.* =) 2*d.* :: 100*l.* : 6*l.* 5*s.* = the Money gained by 100*l.* in 4 Months; ∵ say, if 4 Months : 6*l.* 5*s.* :: 12 Months : 18*l.* 15*s.* the Answer. It may also be solved by 5 Numbers thus:

d.	$\frac{1}{3}$ of a Year	d.
32 ——	1 ——	2
24000 ——	3	

Here the Blank falls under the third Place, and therefore, by the second *Theorem* of that *Rule*, we have $2 \times 24000 \times 3 = 144000$ for a Dividend, and $32 \times 1 = 32$ for a Divisor, and ∵ $144000 \div 32 = 4500$*d.* = 18*l.* 15*s.* for the Answer, as before.

372. *Corollary.* Hence it appears, that, since 100*l.* and one Year are always two of the given Terms, any Three of the other four Terms being given, the other may be found by the *Rule* of five Numbers; which we shall leave for the Learner's Amusement.

CHAP. XXIX.

Of EXCHANGE.

373. UNDER this Head, we propose to shew the Method of computing, what Sum of Money ought to be received in one Country for a certain Sum of a different Species paid in another, with other *Questions* relating to such Exchanges.

374. The current Rate of Exchange betwixt any two Countries rises and falls, upon every Occasion depending, in a great Measure, on the Plenty or Scarcity of the Coin, *&c.* but the *Par* of Exchange, that is, the real Value of any foreign Piece or Sum, being always according to the Weight and Fineness of the Coin, remains fixed, unless a new Kind of Coin be struck.

375. The chief Places, with which *England* exchanges, being *France*, *Italy*, *Portugal*, *Spain*, and *Hol-*

Holland, it will be convenient to give a short Account of their Money, and,

I. Of *France*.

At *Paris*, *Lyons*, *Rouen*, &c. they keep their Accompts in Livres, Sols, and Deniers, which are thus divided;

12 Deniers	=	1 Sol
20 Sols	=	1 Livre
3 Livres	=	1 Crown old, or Crown *Turnois*
5 Livres	=	1 Crown new.

The *French* exchange with the *English* by the *French* Crown; the *Par* of the Crown *Turnois* is 4 *s*. 6 *d*. * Sterling.

The Course of Exchange between *London* and *Paris*, *October* 26, 1756, was 30 *d*. *per French* Crown.

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II. Of *Italy*.

376. In *Genoa*, *Leghorn*, &c. they keep their Accompts in Livres, Sols, and Deniers.

At *Genoa* 5 Livres	=	1 Piece of Eight.
At *Leghorn* 6 Livres	=	1 Piece of Eight.

They exchange upon the Dollar, or Piece of Eight, the *Par* of which with *England* is 4 *s*. 6 *d*. Sterling.

The Course of Exchange, between *London* and *Genoa*, *October* 26, 1756, was 46 *d*. $\frac{3}{4}$ *per* Piece of Eight, or between *London* and *Leghorn* 47 *d*. $\frac{5}{8}$.

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III. Of *Portugal*.

377. At *Lisbon*, *Oporto*, &c. they keep their Accounts in Rees, of which 1000 = 1 Mill-Ree. They exchange on the Mill-Ree, the *Par* of which is 6 *s*. 8 *d*. 2 *qrs*. Sterling. The Course of Exchange, between *London* and *Lisbon*, *October* 26, 1756, was 5 *s*. 4 *d*. *per* Mill-Ree; and the same between *London* and *Oporto*.

se p. 372

IV.

* *English* lawful Silver Coin is called *Sterling*; concerning the Derivation of this Appellation there are various Conjectures, some deriving from the *Esterlings*, some of whom were employed by King *Richard* the First, on Account of their Abilities; others from the *Saxon* Word *Ster*, a Rule or Standard, &c.

IV. *Spain.*

378. In *Madrid*, *Seville*, &c. they keep their Accompts in Maravedies, Rials of Plate, and Pieces of Eight, otherwise called current Dollars.

Note.

34 Maravedies	=	1 Rial of Plate
8 Rials of Plate	=	1 Piece of Eight.

They exchange on the Piece of Eight, the *Par* of which is 4*s.* 6*d.* Sterling. The Course of Exchange between *London* and *Madrid*, *October* 26, 1756, was 38 *d. per* Piece of Eight. see p. 372

V. *Holland*, *Flanders*, and *Germany*.

379. In *Amsterdam*, *Antwerp*, *Brussels*, *Rotterdam*, *Hamburgh*, &c. they keep also their Accompts in *Flemish* Pounds, Shillings, and Pence, or in Guilders, Stivers, and Pennings. The *Flemish* Pound, Shilling, and Pence, are divided like ours, *viz.*

12 Pence	=	1 Shilling, or, as they call it, *Schelling*.
20 Shillings	=	1 Pound.

Note also that,

16 Pennings	=	1 Stiver.
6 Stivers	=	1 *Flemish* Shilling.
20 Stivers	=	1 Guilder.
6 Guilders	=	1 *Flemish* Pound.
30 Stivers	=	1 common Dollar.
50 Stivers	=	1 Specie Dollar.
63 Stivers	=	1 Duccatoon.

Hence 1 Stiver = 2 Pence *Flemish*.

They exchange with the *English* on the Pound *Flemish*, the *Par* of which is 38*s.* 5*d. Flemish* = 1 *l.* Sterling. The Course of Exchange between *London* and *Amsterdam*, the 26th of *October*, 1756, was 36*s.* 5*d.* at 2 $\frac{1}{2}$ Usance, or 36*s.* 2 $\frac{1}{2}$ at Sight. It ought also to be observed, that in the *Low-Countries*, *Holland*, *Germany*, &c. their Money goes under three Denominations, *viz.* common Money, Current or Cash

 Money,

Money, and Exchange Money, or Money of the Bank. Their common Money, consisting of their coarse Pieces, is chiefly used by the poor People, But the current Money is a better Sort, and is the lawful Money for paying Debts, *&c.* As for the Exchange Money, or Money *de Banco*, it is the finest and best of their Gold and Silver Coins; and on this it is, that the *Par* of Exchange betwixt Nations is fixed, and with this Bills are generally paid; but, if, at any Time, it happens that there is not a sufficient Sum to pay the Bills, then Merchants are obliged to receive current Money; and, in Consideration of its being worse than Exchange Money, they are allowed to receive so much *per Cent.* more than they would of Bank Money, as the current Money is worse than the Bank Money, which is usually from $4\frac{1}{2}$ to 5 *per Cent.*

380. In Order the better to distinguish the different Kinds of Questions which are of most Use, or frequently occur in Business, we shall treat of them by Way of Cases.

Case 1. To find the Number of one Species of Money, that is equal to any given Number of another Species.

Example, or *Question* 1. What Sterling Money must be paid in *London* to receive in *Paris* 500 Crowns, Exchange at 4*s.* 4*d. per French* Crown?

Solution. By the Rule of Three direct, it will be, if 1 Crown : 4*s.* 4*d.* :: 500 Crowns : 108*l.* 6*s.* 8*d.* *Note*, It may also be worked by Practice.

381. *Question* 2. Change 400*l.* 10*s.* Sterling into Pounds *Flemish*, Exchange at 33*s. Flemish per* Pound Sterling?

Say, as 20 Shillings *English* : 33 Shillings *Flemish* :: 8010 Shillings *English* (= 400*l.* 10*s.*) : 13216*s.* 6*d.* = 660*l.* 16*s.* 6*d. Flemish.* Q. E. I.

382. *Question* 3. Change 500 Guilders, 10 Stivers, into *English* Money, Exchange at 33*s.* 6*d. Flemish per l.* Sterling?

Solu-

Solution. Because 20 Stivers = * 1 Guilder, 500 × 20 + 10 = 10010 Stivers, and ∴, 1 Stiver being = † 2 Pence *Flemish*, we have 10010 × 2 = 20020 the *Flemish* Pence in the given Guilders and Stivers; whence we have this Stating, if 402 *Flemish* Pence (*viz.* the Pence in 33 *s.* 6 *d.*) : 20 Shillings *English* :: 20020 *Flemish* Pence : 996 and $\frac{8}{402}$ Shillings *English*, = 49 *l.* 16 *s.* and $\frac{8}{402}$ of a Shilling. * 379. † 379.

382. Hitherto we have supposed the Proportion betwixt the two Species is given directly; however, sometimes it may happen, that the Proportion betwixt the two Species is not given directly, but with Reference to a third Species, whence will arise such *Questions* as the following: *Question* 4. How many Crowns, at 4 *s.* 6 *d.* Sterling each, ought to be paid in Exchange for 100 Moidores, at 27 Shillings Sterling each?

Solution. If 1 Moidore : 27 *s.* :: 100 Moidores : 2700 *s.* = their Value in Shillings Sterling; ∴ say, if 4 *s.* 6 *d.* : 1 Crown :: 2700 *s* : 600 Crowns. *Q.E.I.*

But, if the direct Rule of Exchange betwixt Moidores and Crowns had been demanded, then we should say, if 4 *s.* 6 *d.* : 1 Crown :: 27 *s.* : 6 Crowns = a Moidore. *Q. E. I.*

383. *Question* 5. The Rate of Exchange, betwixt *London* and *Paris*, being 4 *s.* 6 *d.* Sterling for 1 *French* Crown, and, betwixt *Paris* and *Amsterdam*, 4 *French* Crowns for 20 Shillings *Flemish*; what is the Exchange betwixt *London* and *Amsterdam*?

Solution. Say, if 1 Crown : 4 *s.* 6 *d.* :: 4 Crowns : 18 *s.* = 4 *French* Crowns = 20 *s.* *Flemish*, ∴ the Rate of Exchange, according to the above Supposition, is 18 Shillings Sterling for 20 Shillings *Flemish*.

384. Sometimes the Proportion or Rate of Exchange may be given betwixt one Country and another, betwixt this other and a Third, betwixt this Third and a Fourth, &c. to find the Exchange betwixt any two of these Countries directly. This may be done by the *Rule* of Three, and the Number of Statings will be 2 fewer than there are different Species of Money; an *Example* will make

make this plain. *Question* 6. If the Exchange betwixt *London* and *Amsterdam* be at 1 *l.* Sterling for 32 Shillings *Flemish*; and betwixt *Amsterdam* and *Lisbon* 8 Shillings *Flemish* for 1 Mill-Ree, and betwixt *Lisbon* and *Paris* 500 Rees for 1 Crown; what is the Rate of Exchange betwixt *London* and *Paris*?

Solution. Say first, if 8 Shillings *Flemish* : 1 Mill-Ree :: 32 Shillings *Flemish* (= 1 *l.* Sterling *per Question*) : 4 Mill-Rees = 1 *l.* Sterling; ∴ say again, if 500 Rees : 1 Crown :: 4000 Rees (= 4 Mill-Rees = 1 *l.* Sterling by the first Stating) : 8 Crowns = 1 *l.* Sterling, *i. e.* the Rate of Exchange betwixt *London* and *Paris*, is 8 *French* Crowns for 1 *l.* Sterling. *Q. E. I.*

Note, In solving *Questions* of this Nature, it is common, for the more ready Perception, to place the Numbers of the *Question* as in the Margin. And then, under each Column to place the Number of each Species equal in Value to the given Number of the first, as they come out by the above Statings; and then may be seen, by Inspection, not only the Rate of Exchange betwixt the first and last mentioned Place, but also betwixt any two of them. Hence also it is evident, that the Number of Statings is one fewer than the Number of given Equations.

London	*Amsterdam*	*Lisbon*	*Paris*
If 1 *l.* =	32 *s.*		
	8 *s.* =	1000 Rees	
		500 Rees =	1 Crown
1 *l.* =	32 *s.* =	4000 Rees =	8 Crowns.

385. From what has been said, with a little Consideration, may be easily deduced a *Rule* for finding, by only one Division, how many of the last Species are equal to the given Number of the first, *viz.* Multiply continually all the Consequents, *i. e.* the Numbers on the Right-hand of each Equation, for a Dividend; and the continual Multiplication of all the Antecedents, (*viz.* all the Numbers on the Left-hand of the Sign of Equality) except the first, will give the Divisor.

For

For *Example*, in the above *Question*, $32 \times 1000 \times 1 = 32000 =$ the Dividend, and $8 \times 500 = 4000 =$ the Divisor, and $32000 \div 4000 = 8$ Crowns as above.

Note, If it be required how many of the last Species are equal to 1 of the first, then it is evident we must divide by the Number of the first, unless as in this *Example* it be an Unit.

The Reason of this *Rule* will be sufficiently shewn by hinting that it appears, from the above Statings, that the Consequents are the middle Terms, and therefore to be multiplied; and the Antecedents are in the first Terms of the Statings, and consequently by the Golden *Rule* Divisors.

386. Since, as often as one Number is contained in another, so often must the $\frac{1}{2}$ of that Number be contained in that other, and so often must $\frac{1}{3}$ of that Number be contained in that other, and generally so often must $\frac{1}{n}$ Part of that Number be contained in $\frac{1}{n}$ Part of that other, it follows, that, if both the Divisor and Dividend can be divided by one and the same Number, the taking their respective Quotients for a Divisor and Dividend will produce the same Number for Answer; and hence the *Rule* in *Art*. 385. will frequently admit of considerable Contractions; for, if any of the Antecedents are the same as some of the Consequents, those Numbers which are the same in both, may be omitted, and only the Products of the remaining Numbers be taken for the Divisor and Dividend. Or if any two Numbers, one in the Antecedents, the other in the Consequents, can be divided by one and the same Number, they may, and their Quotients be taken for Factors, instead of the Wholes, in finding the Divisor and Dividend.

For *Example*, in the above *Question* the Consequents are 32, 1000, and 1, and the Antecedents 8 and 500; here both 32 and 8 can be divided by 8, which being done, the Numbers will be 4, 1000, and 1 for the Consequents, and 1 and 500 for the Antecedents; but, here being 1 in each, we may omit the 1, and then the Num-

Sterling. Or the Reason of this may appear by only reflecting, that, since the *Paris* Merchant receives more than his Order at *London*, he ought to pay proportionally more than his Order in Return at *Amsterdam*; and ∵ we may state as above, or, which is the same Thing, as 4*s.* Sterling : 4*s.* 6*d.* Sterling ∷ 34*s.* *Flemish* : the Answer as above. Or if 4*s.* Sterling = what he should receive *per* his Order : 34 Shillings *Flemish* = what he would pay *per* his Order ∷ 6*d.* = what he receives *per* Crown more than his Order : 4*s.* 3*d.* *Flemish* = what he must pay more than by his Order, for 1*l.* Sterling; ∵ 34*s.* + 4*s.* 3*d.* = 38*s.* 3*d.* *Flemish*, for 1*l.* Sterling, as by the above Method.

394. And if the Course of Exchange had been less than the Order at *London* for 1 *French* Crown, as suppose 3*s.* 6*d.* Sterling for 1 *French* Crown, then it is reasonable, that, as the *Paris* Merchant receives less than his Order at *London*, there ought to be proportionally less paid in Return at *Amsterdam*; ∵, supposing other Things as in the above *Question*, the Stating would be, as 4*s.* Sterling : 3*s.* 6*d.* Sterling ∷ 34 Shillings *Flemish* : 29*s.* 9*d.* *Flemish*, to be paid at *Amsterdam per* 1*l.* Sterling.

Note, If we had given at what Rate *London* drew on *Amsterdam*, to find whether the *London* Merchant had followed his Order, then we find by the above Method what Rate *London* ought to have drawn on *Amsterdam*; which if it be the same as what the *London* Merchant did draw on *Amsterdam* for, he has followed his Order; otherwise, he has not; and the Difference will be Gain or Loss.

395. The following Tables, on Account of their Usefulness, deserve a Place here.

A TABLE

The Ma[illegible]										9.	9
—	W.	0 0¼	3	11		3	10	16	13	5.	1
—	W.	0 0¼	1	17	½	1	17	18	6	8.	5
—	B.	1 2	2	5	½	2	9	7	9	3.	6
—	B.	1 2	2	5	½	2	9	3	9	3.	2
—	B.	1 2	2	5	½	2	9	3	9	3.	2
—	B.	1 2	2	5	½	2	9	3	9	3.	2
—	B.	1 2	4	10	½	4	17	9	18	4.	8
—	B.	1 2	2	5	¼	2	8	18	9	2.	7
—	B.	1 2	2	5	½	2	9	3	9	3.	2
—	B.	1 2	2	5	½	2	9	3	9	3.	2
—	B.	1 2	2	5	½	2	9	3	9	3.	2
—	B.	1 2	2	5		2	8	12	9	2.	1
—	B.	1 1½	2	4	¾	2	7	6	8	11.	6
—	B.	1 3½	2	5	¾	2	10	7	9	5.	7
—	W.	0 0¼	4	6	¾	4	16	11	16	7.	6
—			8	14	½						
—			2	4							
—			8	10							
—			8	16							
—			8	13	½						
—			4	6	¾						
—			4	8	½						
	B.	1 2½	4	11		4	18	18	18	7.	7
—	B.	1 2½	2	5	½	2	9	9	9	3.	8
—	B.	1 1	4	11		4	17	1	18	4.	
—	B.	1 1	2	5	½	2	8	5½	9	2.	
—	B.	1 2	4	11		4	18	6	18	6.	5
—	B.	1 2	2	5	½	2	9	3	9	3.	2
[illegible]}	W.	0 0½	4	6		4	5	17	16	6.	7
s,}	W.	2 1½	2	16	¼	2	9	6	9	3.	

a 6*d*. they are here reduced to the present Stand
on there is but a single Figure on the right Han
pFigures on the right Hand of that Dot, they ex
ISAAC NEWTON; we had it from the *Gentleman*'

	d.	Value.
The half Ducat. —	20.	21
The Tarin or Fifth of the Ducat.	8.	9
The Carlin or tenth Part of the Du	4.	4
The Eſcudi Ecu, or Crown of *R*		
Bayſches. —		
The Teſton of *Rome*, or Piece of	18.	32
The Ducat of *Florence* and *Legho*	64.	62
Julio's. —		
The Julio of *Rome*. —		
The Piaſter Ecu, or Crown of *Fera*	54.	
The Piaſter Ecu, or Crown of *Coſ*		
Monies are about 4 *per Cent*. li	51.	69
Piece is 8 ½ Julio's. —		
The Croiſat of *Genoa*, or Piece of	78.	74
The Ecu *d'Argent* of *Genoa*, or Pi		
The Piaſter Ecu, or Crown of *Mila*		
The *Philip* of *Milan*, a Piece of 7		
The Livre or 20 Sols Piece of *Sa*		
The 10 Sols Piece of *Savoy*.		
A Roupee. —	24.	7
A Gout Gulden or Florin *d'Or*, a	26.	26
Another Gout Gulden. —	26.	72
Another. —	29.	15

Gold C

		s.	d.
The old Louis d'Or. —	16	9.	3
The Half and Quarter in Proportio	8		
The new Louis d'Or. —	20	0.	6
The Half and Quarter in Proportio	0		
The old *Spaniſh* double Doubloon.	7	1.	4
The old *Spaniſh* double Piſtole.	3	6.	7
The old *Spaniſh* Piſtole.	6		
The new *Seville* double Piſtole.			
The new *Seville* Piſtole.			
The Half and Quarter in Proport			
The Doppia Moeda, or double M	6	10.	4
The Doppia Moeda as they come in	6		

396. A *Table* of the Coins of ſeveral Countries.

DENMARK. *Silver*; Danſch, Ebrew, Gluckſtadt, Hor, Rix-Mark, Rix-Ort, Scheſdale.

FRANCE. *Gold*; Crown, Lewidore, Lis, Sol. *Silver*; Crown, Groſs, Lis, Petite, Piece, Teſtoon. *Billon*; Cavalot, Denier, Douzain, Sol. *Copper*; Blank, Carolus, Denier, Double, Liard, Maille, Patac, Pite.

GERMANY. *Gold*; Ducat, Florin, Obolus, Rix-Gould. *Silver*; Florin, Hongre, Izellotte, Rix-Dollar. *Billon*; Blaze, Ratze. *Copper*; Albus, Kreuxer, Pfenin, Plappert, Sexling, Rappen, Swaar, Tryling. *Vid.* NETHERLANDS.

ITALY. *Gold*; Piſtole. *Silver*; Carline, Croiſate, Derlingue, Ducatoon, Florin, Julio, Philip, Scudi, Teſtoon, Zecchin. *Billon*; Cavale, Papirole, Pignatelle. *Copper*; Bayoco, Quatrine.

MUSCOVY. *Gold*; Copec. *Silver*; Copec. *Copper*; Muſkofske, Poluſke.

NETHERLANDS. *Gold*; Albert, Crown, Ducat, Ducatoon, Florin, Imperial, Ride, Sovereign. *Silver*; Florin, Gulden, Patagon, Philip, Schelling. *Billon*; Stuyver. *Copper*; Blanc, Duyt, Grooch, Penning, Stooter.

POLAND. *Silver*; Abra, Groch, Ort, Roup.

PORTUGAL. *Gold*; Joannes, Milree, Moidore, Three-pound-twelve Piece. *Silver*; Cruzada, Pataca, Vintain. *Billon*; Vintain. *Copper*; Rez, Vintain.

SPAIN. *Gold*; Caſtellan, Doblon, Piſtole. *Silver*; Dollar, Piaſtre, Real. *Copper*; Blanca, Cornado, Ochavo, Quarto, Real.

SWEDEN. *Silver*; Caroline, Cavaliere, Chriſtin, Marc. *Copper*; Alleuvre, Dollar, Farthing, Marc, Mony, Rouſtique, Whitten.

O 397.

397. A *Table* of Synonymes of Coins and Money.

Alphonsi, *Maravedi*
Abras, *Brummer*
Aslani, *Abouquel*
Bolognies, *Bayoco*
Byzantine, *Bezant*
Cassa, *Rixdollar*
Castillian, *Castellano*
Cecchin, *Zecchin*
Cheffin, Chequin, *Zecchin*
Craca, *Grain*
Creux, *Kreuxer*
Croisate, *Crown of* Genoa
Cruisade, Spanish *Crown or Castillan*
Cruitser, *Kreuxer*
Cruzada, Portugal *Ducat*
Dalle, Daller, *Dollar*
Demi-Angel, ½ *Angel*
Demi-Bayoco, ½ *Bayoco*
Demi-Maille, *Pite*
Dieci Tarini, 10 *Tarins*
Denain, *Silver Copec*
Denier Gross, *Penning*
Doublon, *Doblon*
Stuyver-Dollar, Swedish *Rixdollar*
Turkish Dollar, *Abouquel*
Douzain, *Sol*
Gold Ducat, *Hongre*
Duyt, Flemish *Penny*
Easterling, *Sterling*
Ecu, Escu, *Crown*
Escalin, *Shilling*
Fenin, *Penny*
Genouin, Genoa *Crown*
George-Noble, *Noble*
Gilder, *Florin*
Gludstadt, *Gluckstadt*
Gould, German *Florin*
Grievener, Grieve, *Grif*
Gros, Groch, Grosch, *Groat*
Bohemia Gros, *Blaphace*
Guilder, Gulden, *Florin*
Harper, Irish *Shilling*
Justine, Venice *Ducatoon*
Justus Judex, *Ebrew*
Kapeke, *Copec*
Laureat, *Carolus*
White Lewis, French *Crown*
Livre de Gros, Dutch *Pondt*
Louis d'Or, *Lewidor*
Lub, *Stuyver*
Lundress, *Sterling*
Malvedis, *Maravedis*
Mancos, Mancusa, *Mark*
Marabitini, *Maravedi*
Mearc, *Marc*
Moeda d'Oro, *Moidore*
Niquet, *Double*
New Noble, *Rial*
Obole, *Maille*
Octavo, *Ochavo*
Para, Parasi, Parat, *Meidein*
Patar, Patard, *Stuyver*
Paullo, *Julio*
Penny, *Pfenin*
Peso, Spanish *Dollar*
Peso d'Oro, *Castellano*
Pezza, Pezzo, *Dollar*
Pfenin, *Penny*
Philip, *Ride*
Philip *of* Milan, *Crown*

Piastre,

Piastre, *Dollar*
Spanish Pistole, *Doblon*
Piece of 8 (Reals) *Dollar*
Pogeria, Poitevin, *Pite*
Pougeoise, *Pite*
Poy, Flemish *Penny*
Pundt, *Pound*
Quadrine, Quartrin, *Quartile*
Double Quarto, *Ochavo*
Quilo, *Julio*
Ree, Rez, *Rea*
Real, *Rial*
Real of 8, *Dollar*
Double Rial, *Sovereign*
Riser, *Half Purse*
Royal, *Rial*
Runstick, *Roustique*

Scalin, Schelling, *Shilling*
Sceptre, Unite
Scherif, *Zecchin*
Scudi, *Crown*
Seguin, Sequin, *Zecchin*
Sheckeen, *Zecchin*
Semipite, $\frac{1}{2}$ *Pite*
Seventeener, German *Florin*
Sixain, $\frac{1}{2}$ *Douzain*
Sol, *Bayoco*
Sol de gros, Dutch *Schelling*
Sol-lub, *Schelling*
St. Stephen, *Millrea*
Sultanin, *Zecchin*
Thaler, *Dollar*
Testao, *Testoon*
Xerif, *Zecchin*.

398. A *Table* of the SUB-DIVISIONS of COINS.

$\frac{1}{2}$ $\frac{1}{4}$ Angel, $\frac{1}{4}$ Angelet, $\frac{1}{2}$ Bayoco, $\frac{1}{2}$ (*English*) Crown, $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{12}$ $\frac{1}{16}$ (*French*) Crown, $\frac{1}{2}$ $\frac{1}{4}$ Dollar, $\frac{1}{2}$ Douzain, $\frac{1}{2}$ $\frac{1}{4}$ *(Gold)* Ducatoon, $\frac{1}{2}$ $\frac{1}{4}$ *Florence*, $\frac{1}{2}$ $\frac{1}{4}$ Guinea, $\frac{1}{2}$ $\frac{1}{4}$ Lewidore, $\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{10}$ $\frac{1}{12}$ Moidore, $\frac{1}{2}$ $\frac{1}{4}$ Noble, $\frac{1}{2}$ Ochavo, $\frac{1}{2}$ $\frac{1}{4}$ Pistole, $\frac{1}{2}$ Pite, $\frac{1}{2}$ Real, $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{12}$ $\frac{1}{24}$ $\frac{1}{48}$ Shilling, $\frac{1}{2}$ Sovereign, $\frac{1}{2}$ Sterling, $\frac{1}{2}$ Testoon, $\frac{1}{2}$ $\frac{1}{4}$ Vintain.

Note. For *Example* of the Meaning of this Table, take the first *Article*, which signifies that there are $\frac{1}{2}$ Angels and $\frac{1}{4}$ Angels, as well as Angels.

399. A *Table* of Multiples of COINS.

2 *British* Crown Piece (*&c.*) 2, 4, 6, 15, 30 Deniers. 2 Ducat, 1 $\frac{1}{4}$, 1 $\frac{1}{2}$, 2, 4, 8 (*German*) Groat. 2, 4, Lewidore. 2 Livre. 2, 2 $\frac{1}{2}$, 4 Moidore. 2 (*Elizabeth's*) Noble. 3, 4, 6, 8, 9 (*German*) Penning. 2, 4 Pistole. 2, 2 $\frac{1}{2}$, 4, 5, 10, 250, 400, 500 Ree. 2, 4, 6, 10, 20, 25 Sol. 2 Sovereign. 10 Tarin. 2 $\frac{1}{2}$, 5, 12 Vintain.

The Meaning of this *Table* may be illustrated by one Instance; as, the first Article of it; which signifies, that we have not only Crown Pieces, but also double Crown Pieces, or a Piece equal in Value to 2 Crowns.—These last four *Tables* were first published in the *Gentleman's Magazine*, for *March* 1740; from whence, they were printed in Mr. *Lowe*'s Arithmetic.

CHAP. XXX.

Of Alligation Medial.

400. ALLIGATION, (from the Verb *Alligo*, *Latin*) is the *Rule*, in which we shew the Method of solving *Questions*, relating to the Mixing of several Simples, of different Prices or Qualities. It is divided into two Parts, Medial and Alternate.

401. *Alligation Medial* (which is that we shall treat of in this Chapter) is, when having given the several Quantities of the several Simples, and the respective Rate (*i. e.* the Price or Quality) we are to find the mean Rate of the Compound. And this, it is evident, may be found by the following *Rule*.

402. The *Rule*. Multiply each Quantity by its respective Rate, and find the Sum of these Products (which will be the Value of the whole Mixture; therefore) now say, by the Golden *Rule*, as the whole Quantity is to the Sum of the Products, so is any given Quantity to its Rate.—Two *Examples* will be sufficient to illustrate this *Rule*.

403. *Question* 1. Admit a Grocer would mix 10 ℔. of Currants at 6*d*. *per* ℔, with 12 ℔ at 4*d*. *per* ℔, and 14 ℔ at 5*d*. *per* ℔; what is the Value of 1 ℔ of the Mixture?

Solution. First, $10 \times 6 = 60d.$ = the Value of 10 ℔ at 6*d*. *per* ℔; and $12 \times 4 = 48d.$ = the Value of 12 ℔ at 6*d*; also $14 \times 5 = 70d.$ = the Value of 14 ℔

at

at 5*d.* *per* ℔; consequently $60+48+70=178d.$ $=$ the Value of $10+12+14=36$ ℔; ∵ state, if 36 ℔ : $178d.$ ∷ 1 ℔ : $4d\frac{34}{36}=4d.\frac{17}{18}=$ the Value of 1 ℔ of the Mixture. *Note*, For Proof we may state, if 1 ℔ : $4d.\frac{17}{18}$ ∷ 36 ℔ : $178d.=$ the Value of the 36 ℔ as before.

404. *Question* 2. Suppose a Goldsmith melts 5 Ounces of Gold, of 23 Caracts fine, with 6 Ounces of Gold, 20 Caracts fine; what Quality would the Mixture be of, that is, how many Caracts fine would it be?

Solution. Here $5\times 23=115$, and $6\times 20=120$; hence, $120+115=235$, $\div$ by 11 ($=5+6$) gives * $21\frac{4}{11}$ Caracts fine; or, by stating, as 11 : 235 ∷ 1 : $21\frac{4}{11}$ as before.

CHAP. XXXI.

Of ALLIGATION ALTERNATE.

405. ALLIGATION ALTERNATE is, when several Things of different Prices are not to be mixed together, and it is required to find what Quantity may be taken of each Sort, so as the Mixture may be sold at a given Rate, without either Loss or Gain.—In the common Method of working this *Rule* (which we shall now explain) there are three Varieties or *Cases*.

* The Reason of this Method, when applied to Metals, will plainly appear, by considering that the Value of any Metal must be as the Quantity into its Fineness; ∵, putting $a=$ Quantity of any Metal, $m=$ its Fineness, $c=$ any other Quantity, $n=$ its Fineness, $x=$ the Fineness of the Mixture, then the Value of a is as am, and the Value of c as cn, the Value of the Whole must be as $am+cn$; but the Value of the Whole must be also as $\overline{a+c}\times x$, ∵ $am+cn=\overline{a+c}\times x$, ∵, dividing by $a+c$, we get $\frac{am+cn}{a+c}=x$. *Q.E.D.*

406. *Case* 1. In this *Case*, the Price of each Simple is given, to find what Quantity may be taken of each Sort, so that the Mixture may be sold at a given Rate; neither the whole Quantity of the Mixture, nor any Part thereof being limited.

407. In Order to solve this *Case*, observe this *Rule*: The several Prices, if they are not all of one Denomination, being made so by Reduction, write them down severally one under the other, and to the left Hand it is convenient to place the mean Price: Then join them two and two together, in such a Manner, that a greater may always be chained, or linked, with a less than the main Rate: Having gone thus far, find the Difference betwixt each particular Price and the mean Rate, which, being set opposite to those with which they are respectively linked, will be one Answer to * the *Question*. *Note*, they admit of many Answers.

408. *Example*. Suppose a Tobacconist would mix 3 Sorts of Tobacco, at 5 *d*, 7 *d*, and 8 *d*. *per* ℔; what Quantity of each Sort may he take, to sell the Compound at 6 *d*. *per* ℔?

The

* We will demonstrate this *Rule* to be true, when three Things of different Prices are to be mixed; (and, by the same Method, it may be demonstrated, for any other Number of Things mixed) in Order to which, let a, b, c, be the three Prices, such that a is less, but b and c each greater than the mean Price, which call m; then, these Prices being duly placed, and their Differences found, according to the Directions in the last *Article*, will stand thus:

$$m\left\{\begin{array}{l} a \\ b \\ c \end{array}\right. \quad \begin{array}{l} \text{Differences} \\ b-m+c-m \\ m-a \\ m-a \\ \hline \end{array} \left.\vphantom{\begin{array}{l} a \\ b \\ c \end{array}}\right\} \text{The Quantities to be taken of each Price.}$$

Sum $b+c-2a=$ the whole Quantity mixed; this, at m Price, comes to $bm+cm-2am$ (found by multiplying by m.) But the first Quantity, at a Price, comes to $ba-am+ca-am$

Second at b Price $bm-ba$

Third at c Price $mc-ca$

Sum $bm+mc-2am=$ the Price of the Whole, which is the same Expression as that is found above, by the whole Quantity mixed, at the mean Rate. *Q. E. D.*

∴ The Operation according to the last *Article*, will stand thus:

$$\text{Mean Price } 6d. \begin{cases} \overset{d.}{5} & -2+1=3 \\ 7 & -1 \\ 8 & -1 \end{cases}$$

Explanation. Having linked 5 with 7 and 8 with 5 (the mean Price being 6*d*.) say $6-5=1$, which put opposite to 7, and also opposite to 8, because it is linked with both; then $7-6=1$, which put against 5, it being joined thereto; then, opposite to 5, is $2+1=3$, ∵ one Answer is 3 ℔ at 5*d*. *per* ℔, 1 ℔ at 7*d*. *per* ℔, and 1 ℔. at 8*d*. *per* ℔; and the whole Quantity $=3+1+1=5$ ℔.—Now, for Proof,

		d.
3 ℔ at 5*d*	=	15
1 ℔ at 7*d*	=	7
1 ℔ at 8*d*	=	8
5 Price of the whole		30

And 5 ℔, at the mean Price 6*d*, is also 30 Pence.

Here it may be remarked, that any other Numbers, in the Ratio of 3, 1, 1, will answer this *Question*. As, for *Example*, 6 ℔ at 5*d*, 2 ℔ at 7*d*, and 2 ℔ at 8*d*; or 9 ℔ at 5*d*, 3 ℔ at 7*d*, and 3 ℔ at 8*d*, &c. whence it appears, that these Kind of *Questions* admit of several Answers; but of this more presently; but first we shall endeavour to shew the Reason of the *Rule*, (just now explained) even without Algebra.

409. In Order to shew the Reason of this *Rule*, in an easy and intelligent Manner, we shall only suppose a Mixture of two Simples, *viz.* at *a* Pence, and *b* Pence *per* ℔, to be sold at *m* Pence *per* ℔. This, according to *Article* 407, will stand thus:

$$m \begin{cases} a & b-m \\ b & m-a \end{cases}$$

That is, $b-m$ Pounds, at *a* Price, and $m-a$ Pounds, at *b* Price.

Now we affirm, that, by selling $b-m$ Pounds at *m* Price *per* ℔, there is exactly as much gained as is lost by selling $m-a$ Pounds at *m* Price; and consequently there is nothing at all lost or gained by selling the whole Mixture at the mean Price. For, by selling those Pounds which are valued at *a* Price *per* ℔, for *m* Price *per* ℔, we gain $m-a$ Pence *per* ℔, that is,

$\overline{b-m}\times\overline{m-a}$ Pence, upon $b-m$ Pounds; and, by selling those Pounds, which are valued at b Pence *per* ℔, for only m Pence *per* ℔, we lose $\overline{b-m}$ Pence on every ℔, ∴ we lose, on $m-a$ Pounds, $\overline{m-a}\times$
* 95. $\overline{b-m}$ Pence; but $\overline{b-m}\times\overline{m-a}=$ * $\overline{m-a}\times\overline{b-m}$; therefore, it is as we have above asserted. *Q. E. D.*

Farther, this *Rule* holds good in all Cases; for, whatever be the Number of Simples, and with however many others any one is linked, as it is always a greater with a lesser, there is a Balance of Gain and Loss, upon the Quantities taken from every Linking of two Simples, by the above Demonstration; and, therefore, there must be a Balance on the Whole; and, consequently, the *Rule* is good in all Cases.

410. We have already hinted, that *Questions* solved by this *Rule* admit of many Answers, even infinite; for, having by *Art.* 407 found one Answer, we may find as many others as we please, by only multiplying, or dividing, each of the Quantities found by that *Article*, by 2, 3, or 4, *&c.* And the Reason is evident, for, if two Quantities, of two Simples, make a Balance of Loss and Gain, with Respect to the mean Price, so must also double, or triple, or $\frac{1}{2}$, or $\frac{1}{3}$ Part, or in any other Ratio of those Quantities; for, if we double the Quantities, we shall make the Gains double in those Quantities which gain, and the Loss double in those Quantities which lose; and consequently, since the Gains and Losses balanced before, by the Supposition, the double Gains must also balance, or be $=$ the double Losses; and, for the same Reason, the triple Gains must be equal to the triple Losses, &c. *ad infinitum.* Also, any Part of the Gains must be $=$ the like Part of the Losses. Whence multiplying the Numbers found by the common *Rule*, by any Number at Pleasure, will also give a true Answer; and therefore, since there is an Infinity of Numbers, this *Rule* is capable of an Infinity of Answers; and, for that Reason, these Kind of *Questions* are by Algebraists called indeterminate, or unlimited Problems.

411.

411. We may obtain as many Anſwers as we pleaſe, by only multiplying, or dividing, the two Quantities belonging to any Pair of Simples that are linked together by one and the ſame Number, and leaving the other Pairs to remain as at firſt. Whoever underſtands the Reaſons in the laſt Anſwers, cannot be ignorant of the Reaſons of this.

412. *Caſe* 2. In this *Caſe* the mean Price, alſo the particular Prices of all the Kinds to be mixed, and the Quantity of one Kind, are given, to find the Quantities of the other Ingredients. This *Caſe* is by ſome Authors called Alligation Partial.

413. This *Caſe* is ſolved by finding what Quantities would ſolve the Queſtion, if there was no particular Quantity given, by *Caſe* the firſt; and then, (ſince any other Numbers in the ſame Proportion * * 408.
would alſo anſwer it) we have by the Golden Rule 410.
this Analogy, *viz.* as the Quantity found by the firſt *Caſe*, belonging to the Price whoſe Quantity is given, in the *Queſtion*, is to that Quantity given, ſo is any other of the Quantities found by the firſt *Caſe* to the Quantity ſought belonging to the ſame Price. Or, in other Words, as the Difference belonging to that Price whoſe Quantity is given, is to the Quantity given, ſo is each of the other Differences to its reſpective Quantity ſought.

414. *Example.* Suppoſe a Grocer, with 10℔ of Sugar at 6*d.* *per* ℔, would mix other Sorts of 4*d.* 5*d.* and 8*d.* *per* ℔, and ſell the Compound at 7*d.* *per* ℔? What Quantity may he take of each Sort?

Solution. The Operation will appear thus:

	d.		℔ ℔ ℔ ℔
7*d.*	10	— 3	As 3 : 10 :: 3 : 10 at 4*d.* *per* ℔.
	4	— 3	As 3 : 10 :: 1 : $3\frac{1}{3}$ at 5*d.* *per* ℔.
	5	— 1	As 3 : 10 :: 2 : $6\frac{2}{3}$ at 8*d.* *per* ℔.
	8	— 2	

For

For Proof.

lb.		d.			d.	
10	at	10	*per* lb is	=	100	And 30 lb at 7*d.* *per* lb = 210 Pence, also.
10	at	4		=	40	
$3\frac{1}{3}$	at	5		=	$16\frac{2}{3}$	
$6\frac{2}{3}$	at	8		=	$53\frac{1}{3}$	
Sum 30				=	210	

415. Here it may be remarked, that this *Rule* will give as many different Answers (yet all true) as the Prices can be linked different Ways. Thus the above Prices may be linked thus:

7*d.*	10	2	Now as 2 lb : 10 lb :: 1 lb : 5 lb at 4*d.* *p.* lb,
	4	1	And as 2 : 10 :: 3 : 15 lb at 5*d.*
	5	3	Also as 2 : 10 :: 3 : 15 lb at 8*d.*
	8	3	

For Proof.

10 lb	at 10*d.* *per* lb is		=	100*d.*	And 45 lb at 7*d.* = 315*d.* as before.
5	at 4	— —	=	20	
15	at 5	— —	=	75	
15	at 8	— —	=	120	
Sum 45 lb	its Value		=	315	

416. *Case* 3. In this *Case*, the mean Price, with the particular Prices of each Simple, and the Quantity of the whole Mixture, are given, to find the particular Quantity of each Simple of the Mixture. This *Case* is by some Authors called Alligation Total.

417. This *Case* may be solved by finding, by *Case* the first, what Quantities would solve the *Question*, if the Quantity of the whole Mixture was not limited; and then saying, by the *Rule* of direct Proportion, as the Sum of the Quantities, found by *Case* the 1st, is to the whole Quantity given, so is each particular Quantity, found by the first *Case*, to its respective Quantity sought. Or, in other Words, as the Sum of all the Differences is to the whole Quantity given, so is each particular Difference to its particular Quantity sought.

418.

418. *Example.* Admit a Man would mix three Sorts of Wine together, viz. White Wine, at 3*s.* 4*d. per* Gallon, *Canary* at 4*s*, and *Malaga* at 6*s. per* Gallon, so as to make a Mixture of 30 Gallons, to be sold at 4*s.* 6*d. per* Gallon: What Quantity may he take of each Sort?

Solution. Here the Operation will stand as under:

d.	40*d.*	—	18
54	48	—	18
	72	14 + 6 =	20
		Sum	56

Now as 56 : 30 :: 18 : 9 $\frac{36}{56}$ Gallons, at 40 *d. per* Gallon; and also at 48 *d.* and as 56 : 30 :: 20 : 10 $\frac{40}{56}$ Gallons at 72 *d. per* Gallon.

For Proof.

Gallons		*d.*		*d.*
9 $\frac{36}{56}$	at	40	=	385 $\frac{40}{56}$
9 $\frac{36}{56}$	at	48	=	462 $\frac{48}{56}$
10 $\frac{40}{56}$	at	72	=	771 $\frac{24}{56}$
Sum 30		Value		1620

And 30 Gallons, at 54 *d. per* Gallon, is also = 1620 Pence.

419. It is evident that this *Case* will also give as many different Answers, as the Prices can be linked different Ways.

420. What has beeen hitherto said in this Chapter, concerning the Mixing of Simples, of different Prices, is equally applicable to the Mixing of Metals, by taking their Degrees of Fineness, instead of the Prices. *Example.* Suppose a Goldsmith having Gold of 20 Caracts, and of 22 Caracts fine, has Occasion for 14 oz. of Gold 18 Caracts fine; How much may he take of each Sort, and also what Quantity of Alloy may he put therewith?

Solution. Here the Alloy, being of very little or no Value, in Comparison of the Gold, is denoted by (0) nothing. The Work will be as under:

18	20		18
	22		18
	0	2 + 4 =	6
			42

As 42 : 14 :: 18 : 6 oz. of 20 Caracts, and also 6 oz. of 22 Caracts fine. And as 42 : 14 :: 6 : 2 oz. of Alloy.

Now this *Question* may be proved by the same Method,

thod, as that in *Article* 418; however, we believe many will be satisfied of the Truth of the Answers to the Questions belonging this *Case*, by only adding up the Quantities of the Simples, and finding they bring out the same Sum as the whole Quantity given. For *Example*, in this *Question*, 6 + 6 + 2 = 14 oz. = the whole Quantity to be mixed, as by the *Question*.

421. There is a *Rule* in some ancient Treatises of Arithmetic, called, The *Rule* of *Ceres et Virginum*, which might have been here added; but we have chosen to omit it, in this Place; not only, because it is of no Use in Business, but chiefly because both this and Alligation will be much better solved, when we treat of unlimited Questions in Algebra: For, whereas the common Methods of working Alligation Alternate, *&c.* find many Times only a few Answers, and those frequently in broken or fractional Numbers, Algebra discovers all the possible Answers in whole Numbers; for which Reason, we have omitted several other Things and Methods, which might have been here given.

CHAP. XXXII.

Of COMPOUNDING MEDICINES.

422. IN this Chapter, we intended to shew the Use of Arithmetic in compounding Medicines; and the Problem which we propose to solve, is, to augment or diminish a Medicine in Quantity, but, at the same Time, to retain the Proportions which the several Simples of which it is compounded, have to each other. This may be resolved by the Golden *Rule*, after the same Manner as Fellowship.

423. *Example. Balsamum Viride* (the green Balsam) according to Dr. QUINCEY, is thus made:

℞ Linseed

		℥	ʒ		ʒ	
℞ Linſeed Oil	—	6	0	=	48	Mix, and boil them together, over a gentle Heat, ſo as to make them into a Balſam.
Gum Elemi	—	2	0	=	16	
Verdigreaſe in Powder		0	2	=	2	
				Sum	66	

Now, ſuppoſe it was required to make only 3 ℥ of the ſaid Balſam, what Quantity of each Simple muſt be taken?

Solution. 3 ℥ = 3 × 8 = 24 ʒ; hence, theſe Statings, as 66 ʒ : 24 ʒ :: 48 ʒ — ʒ $17\frac{30}{66}$ = ℥ 2 : ʒ 1 $\frac{30}{66}$. And as 66 : 24 :: 16 : ʒ $5\frac{54}{66}$; alſo as 66 : 24 :: 2 : $\frac{48}{66}$ of a Dram.

		℥	ʒ
Hence ℞ Linſeed Oil	—	2	1 $\frac{30}{66}$
Gum Elemi	—	0	5 $\frac{54}{66}$
Powder of Verdigreaſe		0	$\frac{48}{66}$
Proof		3	0

424. Perhaps it may be expected by ſome, that, as we have ſhewn the Method of augmenting or diminiſhing a Medicine in Quantity, ſo as to retain the ſame Quality, that we ſhould now proceed to ſolve by Alligation Alternate, as ſome ingenious Authors have thought they have done, Problems of this Nature; *viz.* "An Apothecary hath "4 Sorts of Simples, A, B, C, D, whoſe Qualities "are as follows, *viz.* A is hot in the fourth Degree, "B is hot in the Second, C is temperate, and D is "cold in the third Degree; the *Queſtion* is to know, "what Quantity of each ought to be taken, to make "a Medicine, whoſe Quantity may be 12 ℥, and "the Quality in the firſt Degree of Heat?" Theſe Kind of Queſtions they ſolve by AlligatAion lternate, grounding their *Solutions* on this Principle, that Heat, Cold, &c. depend on a Mixture of Ingredients only; thus, for Inſtance, they ſuppoſe, that if equal Quantities

Quantities of two Simples, one cold in one Degree, the other cold in three Degrees, that the Compound would be cold in two Degrees. But this Hypothesis is not true in Fact, (which is our Reason for not giving a *Solution* by it;) for, if this Hypothesis be true, then, any two cold Things being mixed, the Compound would always be cold, and the Degree of Cold of the Compound would be between the two Degrees of the Ingredients. But Dr. Desaguliер, and others, have found by Experiments, that two cold Things, *viz.* Oil of Tartar *per Deliquium*, poured on Oil of Vitriol, will produce Heat, by causing the Compound to boil, fume, &c. Also, that Salarmoniac, dissolved in Water, makes the Mixture colder, than each singly. And hence it must follow, that Heat, Cold, &c. cannot depend entirely on a Mixture of the Simples, but perhaps on their different attractive Powers, Size, Figure, and Motion of their Parts; but this is not a proper Place to discourse more largely on these Things. We have only now to add, that neither does the Efficacy of most Medicines depend so much on the different Degrees of Heat and Cold, as on some other Properties peculiar to them; and that our Design in this *Scholium* was only to produce sufficient Reason for omitting what some ingenious Authors have thought they have usefully inserted, and to shew that such Questions do not admit of an Arithmetical Solution.

CHAP. XXXIII.

Of Arithmetical Progression, *or* Arithmetical Proportion *continued.*

425. WHEN a Rank, or Series of Numbers, increases, or decreases, by the continual

al Addition or Subtraction of one common Number, such Rank, or Series, is called an Arithmetical Progression; thus, for Instance, 1, 2, 3, 4, 5, &c. is an Arithmetical Progression, increasing by the continual Addition of Unity.

426. The Number continually added or subtracted is called by some the common Difference, by others the Arithmetical Ratio.

427. To find any Term in an Arithmetical Progression, the least Term, common Difference, and Number of Terms, being given. Multiply one less than the Number of Places by the common Difference, to which add the last Number, and the Sum will be the required Term*.

428. *Example.* A Gentleman, relating his Travels in Company, was asked how many Miles he travelled the first Day; to which, to avoid a direct Answer, he said, that his Journey took up 12 Days, that the second Day he travelled 3 Miles fewer than he did the first, and the third 3 Miles less than on the second, and so continued each Day, to the End of his Journey, to ride 3 Miles fewer than on the preceding Day, and the last Day he rode 4 Miles. Whence, says the Gentleman, it is easy to compute how many Miles I rode the first Day?

Solution. By the above *Rule*, the Operation is thus: The least Number of the Series being 4, the common Difference 3, and Number of Places 12, we have $12 - 1 = 11, \times 3 = 33, + 4 = 37$ the greatest Term = the Miles travelled on the first Day; as the Learner may be easily satisfied in, by adding 3 to 4 which gives 7, and $7 + 3 = 10$, &c. till the 12th Term.

429.

* Let $a =$ the least Term, $d =$ the common Difference, $n =$ the Number of Terms, $g =$ the required Term; then the Terms of the Series are, a, $a + d$, $a + 2d$, $a + 3d$ $a + 4d$, &c. to n Terms. Now it is evident by Inspection, that d is repeated as many Times (in any Term) as the Number of Terms from the least; and ∵ the nth Term is $a + \overline{n-1} \times d = g$. *Q. E. D.*

429. To find the Sum of the whole Progression, (called the total Sum) having the two Extremes and Number of Terms given; the *Rule* is: The Sum of the two extreme Terms, being multiplied by the Number of Terms, will be equal to twice the Sum of the whole Progression*.

430. *Example.* Suppose there are 10 Pieces of Cloth, the first containing 2 Yards, the second 4 Yards, the third 6 Yards, and so increasing by the continual Addition of 2: How many Yards are there in all the Pieces?

Solution. The Number of Terms 10, — 1 = 9,
† 427. and 9 x 2 = 18, + 2 = 20 = † the last Number; hence, by the above *Rule*, 20 + 2 = 22, x 10 = 220 Yards = twice the Sum of the whole Progression; and ∴ 220 ÷ 2 = 110 Yards (or 22 x 5, half the Number of Terms) = the Sum of the whole Progression, which was required.

CHAP. XXXIV.

Of GEOMETRICAL PROGRESSION, *or* GEOMETRICAL PROGRESSION *continued.*

431. WHEN a Rank, or Series of Numbers, increases, or decreases, by the continual Multiplication or Division of one common Number, such

*Any Series may be expressed by the continual Addition of the common Difference to the least Term, or the continual Subtraction of the same from the greatest Term; thus a being = least Term, d = common Difference, n = Number of Terms, g = greatest Term, s = Sum of all the Terms, we may express the Series two Ways, *viz*, the Sum of $\left\{ \begin{array}{l} a \,.\, a+d \,.\, a+2d \,.\, a+3d \,.\, \&c. = s. \\ g \,.\, g-d \,.\, g-2d \,.\, g-3d \,.\, \&c. = s. \end{array} \right.$ And therefore, if we add these two Series together, their Sum must be = 2s. But it is evident that the Sum of any two corresponding Terms is constantly the same, *viz.* = a + g, and ∵ the Sum of all the Terms in both = a + g taken n Times; that is, $\overline{a+g} \times n = 2s$. *Q. E. D.*

ſuch Rank, or Series, is called a Geometrical Progreſſion. As, for Inſtance, 2 . 4 . 8. 16 . 32 . . &c. is a Rank or Series in Geometrical Progreſſion increaſing, the common Multiplier being 2. This Series may be expreſſed in a decreaſing Order thus, 32 . 16 . 8 . 4 . 2 , and here the common Diviſor is 2. Now this common Multiplier, or Diviſor, is called the Geometric Ratio.

432. The Ratio multiplied by the Ratio is called the ſecond Power of the Ratio; and the Ratio multiplied by the Ratio, and that Product by the Ratio, is called the third Power of the Ratio; and the third Power of the Ratio, multiplied by the Ratio, is called the fourth Power of the Ratio, &c. &c.

433. Any Term, in an increaſing Geometrical Progreſſion, is equal to the firſt Term multiplied by that Power of the Ratio which is denoted by its Diſtance from the firſt Term*. (Whence if over the Terms of the Geometrical Progreſſion we put an Arithmetical Progreſſion whoſe firſt Term is 0, and common Difference 1, the Terms of the Arithmetical Series (called the Indices) will denote the Power of the Ratio, in any of the correſponding Terms, of the Geometric Progreſſion.)

434. *Example.*

A Gentleman, as he did ride
Near to a pleaſant Common-Side,
Ten Shepherdeſſes chanc'd to meet,
Driving their Flocks, whom he did greet,
God ſpeed you well; and may you be
As happy as you're fair (ſaid he:)
Proſper your Flocks, and may they thrive;
Tell me how many Sheep you drive?
One of the Damſels ſtraight reply'd,
Sir, you ſhall ſoon be ſatisfy'd:

P For,

* This will plainly appear, by only repreſenting a Geometric Progreſſion by Letters, *viz.* putting a = leaſt Term, r = common Multiplier, the Indices are 0 . 1 . 2 . 3 . 4 . &c.
And the Geometric Progreſſion is a. ar. arr. arrr. arrrr. &c.

For, if for one of us you do
Count one Sheep, for the next count two,
For the third four, for the fourth eight,
So doubling at each Maid aright,
At the last Maid the Sum will be
As many as the Sheep you see.

Quere the Number of Sheep?

Solution. Here, the Distance of the last Term from the first being $= 10 - 1 = 9$, we have, by *Art.* 433, the last Term $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 1 = 512$ the Number of Sheep which was required.

435. But since this Method of finding the Power of the Ratio, if the Number of Terms be many, is very troublesome, it is many Times convenient to make Use of the following Contrivance, *viz.* It plainly follows from the latter Part of *Article* 433, that the Product of the Powers of the Ratio under any two Indices will be equal to the Power of the Ratio corresponding to that Index, which is equal to the Sum of the two Indices; hence the above *Question* may be solved, by setting down some of the Powers of the Ratio with their corresponding Indices,

as, $\begin{cases} \text{Indices} & 1 . 2 . 3 . 4 . 5 . \&c. \\ \text{Powers of the Ratio} & = 2 . 4 . 8 . 16 . 32 . \&c. \end{cases}$

Here, the Distance of the first and last Term being 9 by the *Question*, we are to take two such Indices as being added together shall be equal to 9, *viz.* $5 + 4 = 9$; now under 4 is 16, and under 5 is 32, $\because 16 \times 32 = 512 =$ the 9th Power of the Ratio, which, being multiplied by the first Term of the Series (here 1) gives $512 \times 1 = 512$ Sheep as before.

436. It may also be solved by the Addition of 3 or more Indices, as, for *Example*, by the Indices, 4, 3, and 2, for $4 + 3 + 2 = 9$; and under 4 is 16, under 3 is 8, and under 2 is 4. And $16 \times 8 \times 4 = 512$ = the 9th Power of the Ratio as before.

437. But, perhaps, the most compendious Method is, not to write down any Terms of the Series at all, but to imagine the Indices fixed over the Terms in your Mind, and work as follows, *viz.* The Index

dex 1 + (itself) 1 = 2, ∴ the Power of the Ratio under that Term whose Index is 2, is $2 \times 2 = 4 =$ the Ratio × by the Ratio; and Index 2 + (itself) 2 = 4, ∴, the Power of the Ratio belonging to the third Term, *viz.* 4, being multiplied by itself, that is, $4 \times 4 = 16 =$ the Power of the Ratio belonging to the fifth Term; but the Index 4 + (itself) 4 = 8 = the Index belonging to the 9th Term, ∴ $16 \times 16 = 256 =$ the Power of the Ratio belonging to the ninth Term; lastly, the Indices $8 + 1 = 9 =$ the Index of the 10th Term, ∴ $256 \times 2 = 512 =$ the Power of the Ratio belonging to the tenth or last Term as before.

438. In any Geometrical Progression, as any Antecedent is to its Consequent, so is any other Antecedent to its respective Consequent. This is evident; for by the Nature of such Progressions, if *r* denote the common Multiplier, any Consequent is *r* Times its Antecedent.

439. Therefore, in a continued Proportion, all the Terms, except the last, are called Antecedents, and all the Terms, except the first, Consequents.

440. Whence it follows *, that as the least (or any other) Term is to its Consequent (the Term next following) so is the Sum of all the Antecedents of the whole Progression to the Sum of all the Consequents.

441. And from hence is deduced the following *Rule*, to find the Sum of all the Terms, the two extreme Terms and common Multiplier being given,

* Let *a. ar. arr. arrr. arrrr. arrrrr*, &c. be the Series; $A =$ all the Antecedents, $C =$ all the Consequents, then, $a + ar + arr + arrr + arrrr$, &c. $= A$; and $ar + arr + arrr + arrrr + arrrrr$, &c. $= C$; Hence it is evident by a bare Inspection, that the Sum of all the Consequents is *r* Times the Sum of all the Antecedents; but each particular Consequent is also *r* Times its respective Antecedent, and, consequently, any Antecedent, its Consequent, the Sum of all the Antecedents, and Sum of all the Consequents, are four Quantities in direct Proportion, *viz.* as any Antecedent : its Consequent :: $A : C$. *Q. E. D.*

given, *viz.* Multiply the greateſt Term by the common Multiplier, from which Product ſubtract the firſt Term, and divide the Remainder by one leſs than the common Multiplier, and the Quotient will be the Anſwer *.

442. *Example*, or *Queſtion* 2. Suppoſe *A* agrees with with *B* to ſell him a Houſe, which has 12 Windows, if he will put 2 Pence in the firſt Window, 6 Pence in the Second, 18 Pence in the Third, and ſo on, multiplying by three each Time, thro' all the Windows: What would the Houſe coſt?

Solution. Here, the Number of Terms being 12, we are firſt to raiſe 3, the Ratio, to the eleventh Power; to do which, the ſhorteſt Method is, $3 \times 3 = 9 =$ the ſecond Power, and $9 \times 9 = 81 =$ the fourth Power, then $81 \times 81 = 6561 =$ the eighth Power; and, $8 + 2$ being $= 10$, $6561 \times 9 = 59049 =$ the tenth Power, and, $10 + 1$ being $= 11$, we ſhall have $59049 \times 3 = 177147 =$ the eleventh Power, $\because$ $177147 \times 2 = 354294 =$ the twelfth or greateſt Term; now, to find the Sum of all the Terms, we have, firſt, $354294 \times 3 = 1062882$, from which ſubtracting the firſt Number 2, we get 1062880 for a Dividend, which, divided by $3 - 1 = 2$, gives 531440 Pence $= 2214$ *l.* 6 *s.* 8 *d.* for the Anſwer.

443. The Reaſons for our having been ſo ſhort on Arithmetical, and Geometrical Progreſſion are, firſt, that they are of no Uſe in common Arithmetic; ſecondly, that we ſhall have Occaſion to diſcourſe more largely of them, when we treat of Algebra: And, for

* Let $s =$ the Sum of the Series, $a =$ the leaſt Term, $d =$ the common Multiplier, then $da =$ the next Term to the leaſt; alſo put $g =$ the greateſt Term, then $s - g =$ the Sum of all the Antecedents,
* 440. and $s - a =$ the Sum of all the Conſequents; hence, as * $a : da :: s - g : s - a$. $\because$ $das - gda =$ † $sa - aa$, $\because$, dividing by a,
† 185. we have $ds - gd =$ ‡ $s - a$, hence, by adding gd to each Side of
‡ 108. the Equation, we ſhall get $ds =$ ‖ $gd + s - a$, and, by ſubtracting s
‖ 22.
§ 36. from this, $ds - s =$ § $gd - a$; but $ds - s = \overline{d - 1} \times s$, $\because$ $\overline{d - 1}$
¶ 108. $\times s = gd - a$; which, divided by $d - 1$, gives $s =$ ¶ $\frac{gd - a}{d - 1}$. *Q. E. D.*

for the ſame Reaſons, we ſhall only obſerve concerning Harmonical or Muſical Proportion, that, when 3 Numbers are ſo related, that the firſt hath the ſame Ratio to the Third, as the Difference between the firſt and ſecond hath to the Difference between the ſecond and third; or when 4 Numbers are ſuch, that the firſt hath the ſame Ratio to the fourth, as the Difference between the firſt and ſecond hath to the Difference between the third and fourth; theſe Numbers are ſaid to be in the Harmonical Proportion.

CHAP. XXXV.

Of Involution.

444. INVOLUTION (*Involutio Lat.*) is the Raiſing of Powers from any given Root; and is performed like Multiplication, with only this Limitation, that in Involution the Factors or Multipliers continue the ſame; whereas Multiplication admits of different Factors.

445. The Number to be raiſed is called the Root.

446. When we ſay a Number is raiſed to ſuch a Power, we only mean, that it is multiplied into itſelf a certain Number of Times.

446. A Square Number, or a Number of the ſecond Power, is compoſed of two equal Numbers, (*i. e.*) produced by their Multiplication. *E. G.* 4 is a Square Number, and compoſed of 2 and 2, for $2 \times 2 = 4$.

447. A Cube Number, or a Number of the third Power, is compoſed of three equal Numbers, *viz.* produced by their continual Multiplication; thus 27 is a Cubic Number, for $3 \times 3 \times 3 = 27$.

448. A Number of the fourth Power is compoſed of 4 equal Numbers; as $2 \times 2 \times 2 \times 2 = 16$ is a Number of the fourth Power. And after the ſame Manner Numbers of any other Powers are to be underſtood.

449. *Example.* Let it be required to raiſe 12 to the ſixth Power?

The Root or ſingle Power	12
× by	12
The Square =	144
×	12
The Cube =	1728
×	12
The fourth Power =	20736
×	12
The fifth Power =	248832
×	12
The ſixth Power =	2985984

and, after this Manner, we may proceed to what Power we pleaſe. And, by this Method, the following Table was calculated.

A

A *TABLE* of Powers.

R. or 1st Power	1	2	3	4	5	6	7	8	9
2d Power	1	4	9	16	25	36	49	64	81
3d Power	1	8	27	64	125	215	343	512	729
4th Power	1	16	81	256	625	1296	2401	4096	6561
5th Power	1	32	243	1024	3125	7776	16807	32768	59049
6th Power	1	64	729	4096	15625	46656	117649	262144	531441
7th Power	1	128	2187	16384	78125	279936	823543	2097152	4782969
8th Power	1	256	6561	65536	390625	1679616	5764801	16777216	43046721
9th Power	1	512	19683	262144	1953125	10077696	40353607	134217728	387420489
10th Power	1	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
11th Power	1	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059609
12th Power	1	4096	531441	16777216	244140625	2176782336	13841287201	68719476736	282429536481
13th Power	1	8192	1594323	67108864	1220703125	13060694016	96889010407	549755813888	2541865828329
14th Power	1	16384	4782969	268435456	6103515625	78364164096	678223072849	4398046511104	22876792454961
15th Power	1	32768	14348907	1073741824	30517578125	470184984576	4747561509943	35184372088832	205891132094649

450. But the fourth, fifth, or any other higher Power, may be found without the Trouble of finding all the inferior Powers, by considering, that the several Powers form a Geometrical Series, in the same Manner as the Ratio in Geometrical Progression; and consequently any Power may be found by this *Rule*. Find two or more such Powers of the Root (by the Method shewn in the last *Article*) as that the Sum of their Indices may be equal to the Index of the required Power; then the continued Product of these Powers will be equal to that Power of the Root which was required. Or, if the Index of any one of the inferior Powers is an aliquot Part of the Index of the required Power, we may find that Power of the Root, whose Index is that aliquot Part, and, taking this Power as a Root, involve it to that Power whose Index is the Denominator of that aliquot Part, and we shall have the required Power which was to be found.

Note, The Index of the Root is 1, of the second Power is 2, of the Third 3, of the Fourth, 4, *&c.*

451. For an Illustration of this *Rule*, let the *Example* be the preceding one.

Solution. First, $12 \times 12 = 144 =$ the second Power, or that Power whose Index is 2; and $144 \times 12 = 1728$ $=$ the third Power, the Index is 3; now the Indices $3 + 2 + 1 = 6 =$ the Index of the required Power; whence $1728 \times 144 \times 12 = 2985984 =$ the sixth Power, which was required. Or, according to the latter Part of the *Rule*, thus: The Index of the third Power, *viz.* 3, is $\frac{1}{2}$ of 6, the Index of the required Power; therefore, the third Power raised to the second Power, *viz.* $1728 \times 1728 = 2985984$ as above. *Q. E. I.*

452. An *Axiom*. The like Powers of equal Numbers in what Manner soever expressed are equal. Thus, for Instance, the second Power of 5 is $=$ the second Power of $\overline{2 + 3}$. Also, the like Roots of equal Numbers are equal.

453. The Names given by the Ancients to the several Powers are much more complex and burthensome

fome to the Memory, than thofe now in Ufe; for, whereas the Moderns fay, the firft, fecond, third, fourth, fifth, fixth, feventh, eighth, ninth, tenth, eleventh, twelfth, thirteenth, fourteenth, fifteenth, &c. Power; the Ancients diftinguifhed the Powers by thefe Terms, the Root, the Square, Cube, Biquadrate, Surfolid, Square-cubed, Second-Surfolid, Biquadrate-fquared, Cube-cubed, Surfolid-fquared, Third-Surfolid, Square-cubed-fquared, Fourth-Surfolid, Second-Surfolid-fquared, Surfolid-cubed, &c. refpectively.

454. In *Involution* of Algebra, if r denote the Root, then by the Definitions $rr =$ the fecond Power, $rrr =$ the third Power, &c. but, to exprefs the Powers more compendioufly, we generally put the Index of the Power over the Root; as for rr we write r^2 and for rrr we put r^3, and for $rrrr$ we write r^4, &c. and the Powers of compound Quantities are expreffed after the fame Manner, by putting the Index over the *Vinculum*; as, for Inftance, for the Square of $\overline{r+x}$ we write $\overline{r+x}|^2$, and for the Cube of $\overline{r+x}$ we place down $\overline{r+x}|^3$, &c.

455. Sometimes it will be convenient to exhibit the Powers of compound Quantities, without the *Vinculum*, by the actual *Involution* of the Quantity; thus, for, $\overline{r+x}|^2$ we may write $r^2 + 2rx + {}^2$, and alfo $\overline{r+x}|^3 = r^3 + 3r^2x + 3rx^2 + x^3$; both thefe Expreffions were found as under:

$$
\begin{array}{lr}
\text{Multiply} & r+x \\
\text{by} & r+x \\
\hline
 & r^2+rx \\
 & +rx+x^2 \\
\hline
\text{Product} & r^2+2rx+x^2 = \overline{r+x}|^2 \\
\text{Multiplied by} & r+x \\
\hline
 & r^3+2r^2x+rx^2 \\
 & +r^2x+2rx^2+x^3 \\
\hline
\text{Product} & r^3+3r^2x+3rx^2+x^3 = \overline{r+x}|^3
\end{array}
$$

And

And by this Method any compound Root may be raiſed to any given Power.

CHAP. XXXVI.

Of the EVOLUTION *of the* SQUARE ROOT.

456. EVOLUTION (*Evolutus Lat.*) is the Extracting of Roots, and is the Inverſe of *Involution*; for, as *Involution* ſhews how to raiſe any Root to a given Power, *Evolution* teaches to find the Root of any given Power; it is divided into the Square Root, Cube Root, *&c.*

457. The Square Root, (from *Yſgwâr*, *Welch*, or *Quadratus Lat.* and *Rôt Swediſh*, is by having the Square, or a Number of the ſecond Power given, to find its Root.

458. A ſurd or irrational Number is the Root of a Number, whoſe Root cannot be exactly found: As the Square Root of 2 is a ſurd Number, for we cannot find a Number, which, being multiplied by itſelf, will produce exactly 2.

459. To extract the Square Root of a given Power.

The *Rule*. The firſt Thing to be done is to point the given Number, that is, beginning at the right Hand, put a Dot over the Figure in the Unit's Place, and, proceeding towards the Left, over every other Figure put a Dot (.) *i. e.* paſſing over one each Time, and proceeding to the next. And here it ought to be remarked, that, as many Dots as there are, ſo many Figures the Root conſiſts of. —Then look in the Table of Powers, in *Art.* 449, for a Number equal to, or the next leſſer than the Number in the firſt (left Hand) Point of the given Number, which ſubtract out of the firſt Point; or, in other Words, ſubtract the greateſt Square poſſible out of the firſt Point, and its Root place in the Quotient, for the firſt

first Figure of the required Root; and to the Remainder bring down the Figures in the next Point, (proceeding towards the right Hand) for a Dividend, (or, as some call it, a Resolvend.) Then double the Quotient, and try how often you can take it out of the Dividend, supposing the Figure denoting the Times you go (which never can exceed 9) placed on the right Hand of that double, to make up the whole Divisor; *i. e.* such a Fgure must be sought (by Trials) as, when it is annexed on the right Hand of the double Quotient, this being taken as a Divisor, and multiplied by the Number of Times you go, the Product may be the greatest (found by this Method) that can be deducted from the Dividend.—Place the Figure expressing the Times you go in the Quotient, and, having deducted the abovementioned Product, to the Remainder bring down the next Point, and, taking the Whole as a Dividend, double the Quotient (which now consists of two Figures) and proceed as before; and thus continue to form new Dividends and Divisors, till all the Points are taken down by the abovementioned Method. A few *Examples* will explain the Meaning of this *Rule.*

460. What is the Square Root of 144?

Solution. The Number being pointed, the greatest Square in the first Point is 1, which put under the first Point, and its Root (1) place in the Quotient; now, subtracting the greatest Square 1 from the first Point 1, there remains 0; ∵, taking down the next Point, the Dividend is 44; then the Quotient, being doubled, is 1 x 2=2 the left Hand Figure of the Divisor; now say, how many Times 2 in 4 the left Hand Figure of the Dividend, which is 2 Times; then this 2, being placed to the right Hand of the other, gives the whole Divisor=22; put the 2 (Times) in the Quotient; then 22 x 2 = 44, which being taken from the Dividend, the Remainder is 0, and ∵ the required Root is 12. Which may be easily proved, for 12 x

```
    144( 1 Root
    1 =greatest Square
22) 44
    0
```

12 =

12 = 144. Or, perhaps, it may to the Learner appear plainer thus: Assume 10 = the Root (because 20 x 20 = 400 would be too great) then 10 x 10 = 100, which deducted from 144, there will remain 44 for the Dividend; then 10 x 2 = 20; now seek how many Times 20 in 44, which is 2 Times; then 20 + 2 = 22 = the Divisor, and 22 x 2 = 44, which being subtracted from the Dividend, the Remainder is 0; and therefore the Root = 10 + 2 = 12.

```
        144(10 = assumed
        100(      Root
       -----
  20 } 44 }
 + 2 } 44 } 2
Di.22}  — }____
       Sum 12=trueRoot.
```

461. Take another *Example*. What is the Square Root of 119025?

Operation.

```
       119025(Root
        9     345
      ------
  64 ) 290
       256
      ------
 685 ) 3425
       3425
      ------
         0
```

Operation explained.

```
                119025(assumed Root,
                90000 300
300 x 2 = 600 ) 29025(40
        + 40  ) 25600
1st Divisor 640          340 Sum of these two.
 340 x 2 = 680 ) 3425(
     +       5 ) 3425  5
2d Divisor 685      0  345 = Root
```

462. As the Application of the Square Root to Affairs of Business is grounded on Geometrical Principles, we shall only give one *Example, viz.* Suppose it is required to find how long a Ladder must be, to reach the Top of a Wall that is 40 Feet high, one End of the Ladder standing 30 Feet off from the Bottom

Bottom of the Wall? *Note*, the Plane on which the Wall is built is ſuppoſed to be truly level.

Solution. If we may believe Geometricians, the Square of 30, added to the Square of 40, will be equal to the Square of the Ladder's Length. But the Square of $30 = 30 \times 30 = 900$, and the Square of $40 = 40 \times 40 = 1600$, $\because 900 + 1600 = 2500 =$ the Square of the Ladder's Length; and conſequently its Square Root 50 (ſee the Margin) = the Length of Ladder. *Q. E. I.*

```
 2500(50
 25
100)0
```

Note, Many Times the Number to be extracted does not admit of an integral Root, and in ſuch Caſe there will be a Remainder; for finding the Value of which, Recourſe muſt be had to Decimals.

463. This *Rule* being applicable to many Purpoſes of Buſineſs and Pleaſure, a Table that will ſhew any Root from 1 to 1000, by a bare Inſpection, muſt be very uſeful; for which Reaſon we will here give one, but will firſt ſhew the eaſieſt Method of making it; for the Method of continual *Involution* is very tedious. And, in Order to this, let us firſt obſerve, that the Difference of any two Square Numbers whoſe Roots differ by an Unit is an odd Number, and equal to twice the leſſer Root more one, or equal to the Sum of their Roots *.

464. Hence it follows, that in a Table of Squares, whoſe Roots increaſe an Unit each Time, the Sum of the Roots of an Antecedent and Conſequent, being added to the Square of the Antecedent, will be equal to the Square of the Conſequent. Thus, *E. G.* in the Roots 4 and 5, the Square of 4 is 16, and $4 + 5 = 9$, then $16 + 9 = 25 =$ the Square of 5. Whence a Table may be eaſily made by this Corollary.—The Reaſon

* Demonſtration. Let r be one Root, and $r + x$, = the other Root x being $= 1$; then the Square of $r =$ * r^2; and the Square * 454.
of $r + x =$ † $r^2 + 2rx + x^2 =$ (becauſe $x = 1$) $r^2 + 2r + 1$; but $r^2 + 2r + 1$ Minus $r^2 = 2r + 1 =$ the Difference of their Squares. But the Sum of the Roots r and $r + 1 = 2r + 1$ alſo. *Q. E. D.* † 455.

Reason of this Corollary easily follows from the last *Article*; for, by that, the Sum of the Roots of the Antecedent and Consequent is = the Difference of their Squares; and therefore, if to the Square of the Antecedent be added the Sum of their Roots (which is equal to the Difference of their Squares) the Sum will be
* 50. equal to * the Square of the Consequent. This will be useful to correct any Error of the Press.

465. It follows by *Art.* 463, that, if the Roots be 1 and 2, the Difference of their Squares is 3; if 2 and 3, the Difference of their Squares is 5; if 3 and 4, the Difference of their Squares is 7, and so on in a Series of odd Numbers, 3, 5, 7, 9, 11, &c. *ad infinitum.* Consequently, a Table of Squares may be easily made by the constant Addition of these odd Numbers. *E. G.* $1 + 3 = 4 =$ the Square of 2; $4 + 5 = 9$ = the Square of 3; $9 + 7 = 16 =$ the Square of 4; &c. whence the following Table is easily constructed by (an easy) Addition only.

466. A TABLE of the Square Numbers of all the integral Roots from 1 to 1000 inclusive.

The Use of this Table is very plain; for, if the Root be given, right over-against the Root, in the Column of Squares, you will find the Square which was required. And on the contrary, if the Root of any Number less than 1000000 be required, look for the given Number in the Column of Squares, and, in the Column of Roots, you will find the corresponding Root. But, if the given Number cannot be found in the Column of Squares, find the nearest to it, and the Root belonging to the next lesser will be less than the true Root, and that belonging to the next greater Square will be more than the true Root; but the exact Root cannot in such *Case* be found by the Table, because its Root is not an integral Number. *E. G.* If the Root of 876054 be required, the next lesser Square in the Table is 874225, whose Root is 935, and the next greater Square is 876096, whose Root is 936; whence the required Root is more than 935, but not so great as 936.

A

R.	Square	R.	Square	R.	Square
1	1	40	1600	79	6241
2	4	41	1681	80	6400
3	9	42	1764	81	6561
4	16	43	1849	82	6724
5	25	44	1936	83	6889
6	36	45	2025	84	7056
7	49	46	2116	85	7225
8	64	47	2209	86	7396
9	81	48	2304	87	7569
10	100	49	2401	88	7744
11	121	50	2500	89	7921
12	144	51	2601	90	8100
13	169	52	2704	91	8281
14	196	53	2809	92	8464
15	225	54	2916	93	8649
16	256	55	3025	94	8836
17	289	56	3136	95	9025
18	324	57	3249	96	9216
19	361	58	3364	97	9409
20	400	59	3481	98	9604
21	441	60	3600	99	9801
22	484	61	3721	100	10000
23	529	62	3844	101	10201
24	576	63	3969	102	10404
25	625	64	4096	103	10609
26	676	65	4225	104	10816
27	729	66	4356	105	11025
28	784	67	4489	106	11236
29	841	68	4624	107	11449
30	900	69	4761	108	11664
31	961	70	4900	109	11881
32	1024	71	5041	110	12100
33	1089	72	5184	111	12321
34	1156	73	5329	112	12544
35	1225	74	5476	113	12769
36	1296	75	5625	114	12996
37	1369	76	5776	115	13225
38	1444	77	5929	116	13456
39	1521	78	6084	117	13689

R.	Square	R.	Square	R.	Square
118	13924	157	24649	196	38416
119	14161	158	24964	197	38809
120	14400	159	25281	198	39204
121	14641	160	25600	199	39601
122	14884	161	25921	200	40000
123	15129	162	26244	201	40401
124	15376	163	26569	202	40804
125	15625	164	26896	203	41209
126	15876	165	27225	204	41616
127	16129	166	27556	205	42025
128	16384	167	27889	206	42436
129	16641	168	28224	207	42849
130	16900	169	28561	208	43264
131	17161	170	28900	209	43681
132	17424	171	29241	210	44100
133	17689	172	29584	211	44521
134	17956	173	29929	212	44944
135	18225	174	30276	213	45369
136	18496	175	30625	214	45796
137	18769	176	30976	215	46225
138	19044	177	31329	216	46656
139	19321	178	31684	217	47089
140	19600	179	32041	218	47524
141	19881	180	32400	219	47961
142	20164	181	32711	220	48400
143	20449	182	33124	221	48841
144	20736	183	33489	222	49284
145	21025	184	33856	223	49729
146	21316	185	34225	224	50176
147	21609	186	34596	225	50625
148	21904	187	34969	226	51076
149	22201	188	35344	227	51529
150	22500	189	35721	228	51984
151	22801	190	36100	229	52441
152	23104	191	36481	230	52900
153	23409	192	36864	231	53361
154	23716	193	37249	232	53824
155	24025	194	27636	233	54289
156	24336	195	38025	234	54756

R.	Square	R.	Square	R.	Square
235	55225	275	75625	315	99225
236	55696	276	76176	316	99856
237	56169	277	76729	317	100489
238	56644	278	77284	318	101124
239	57121	279	77841	319	101761
240	57600	280	78400	320	102400
241	58081	281	78961	321	103041
242	58564	282	79524	322	103684
243	59049	283	80089	323	104329
244	59536	284	80656	324	104976
245	60025	285	81225	325	105625
246	60516	286	81796	326	106976
247	61009	287	82369	327	106929
248	61504	288	82944	328	107584
249	62001	289	83521	329	108241
250	62500	290	84100	330	108900
251	63001	291	84681	331	109561
252	63504	292	85264	332	110224
253	64009	293	85849	333	110889
254	64516	294	86436	334	111556
255	65025	295	87025	335	112225
256	65536	296	87616	336	112896
257	66049	297	88209	337	113569
258	66564	298	88804	338	114244
259	67081	299	89401	339	114921
260	67600	300	90000	340	115600
261	68121	301	90601	341	116281
262	68644	302	91204	342	116964
263	69169	303	91809	343	117649
264	69696	304	92416	344	118336
265	70225	305	93025	345	119025
266	70756	306	93636	346	119716
267	71289	307	94249	347	120409
268	71824	308	94864	348	121104
269	72361	309	95481	349	121801
270	72900	310	96100	350	122500
271	73441	311	96721	351	123201
272	73984	312	97344	352	123904
273	74529	313	97969	353	124609
274	75076	314	98596	354	125316

R.	Square	R.	Square	R.	Square
355	126025	395	156025	435	189225
356	126736	396	156816	436	190096
357	127449	397	157609	437	190969
358	128164	398	158404	438	191844
359	128881	399	159201	439	192721
360	129600	400	160000	440	193600
361	130321	401	160801	441	194481
362	131044	402	161604	442	195364
363	131769	403	162409	443	196249
364	132496	404	163216	444	197136
365	133225	405	164025	445	198025
366	133956	406	164836	446	198916
367	134689	407	165649	447	199809
368	135424	408	166464	448	200704
369	136161	409	167281	449	201601
370	136900	410	168100	450	202500
371	137641	411	168921	451	203401
372	138384	412	169744	452	204304
373	139129	413	170569	453	205209
374	139876	414	171396	454	206116
375	140625	415	172225	455	207025
376	141376	416	173056	456	207936
377	142129	417	173889	457	208849
378	142884	418	174724	458	209764
379	143641	419	175561	459	210681
380	144400	420	176400	460	211600
381	145161	421	177241	461	212521
382	145924	422	178084	462	213444
383	146689	423	178929	463	214369
384	147456	424	179776	464	215296
385	148225	425	180625	465	216225
386	148996	426	181476	466	217156
387	149769	427	182329	467	218089
388	150544	428	183184	468	219024
389	151321	429	184041	469	219961
390	152100	430	184900	470	220900
391	153881	431	185761	471	221841
392	153664	432	186624	472	222784
393	154449	433	187489	473	223729
394	155536	434	188356	474	224676

R.	Square	R.	Square	R.	Square
475	225625	515	265225	555	308025
476	226576	516	266256	556	309136
477	227529	517	267289	557	310249
478	228484	518	268324	558	311364
479	229441	519	269391	559	312481
480	230400	520	270400	560	313600
481	231361	521	271441	561	314721
482	232324	522	272484	562	315844
483	233289	523	273529	563	316969
484	234256	524	274576	564	318096
485	235225	525	275625	565	319225
486	236196	526	276676	566	320356
487	237169	527	277729	567	321489
488	238144	528	278784	568	322624
489	239121	529	279841	569	323761
490	240100	530	280900	570	314900
491	241081	531	281961	571	326041
492	242064	532	283024	572	327184
493	243049	533	284089	573	328329
494	244036	534	285156	574	329476
495	245025	535	286225	575	330625
496	246016	536	287296	576	331776
497	247009	537	288369	577	332929
498	248004	538	289444	578	334084
499	249001	539	290521	579	335241
500	250000	540	291600	580	336400
501	251001	541	292681	581	337561
502	252004	542	293764	582	338724
503	253009	543	294849	583	339889
504	254016	544	295936	584	341056
505	255025	545	297025	585	342225
506	256036	546	298116	586	343396
507	257049	547	299209	587	344569
508	258064	548	300304	588	345744
509	259081	549	301401	589	346921
510	260100	550	302500	590	348100
511	261121	551	303601	591	349281
512	262144	552	304704	592	350464
513	263169	553	305809	593	351649
514	264196	554	306916	594	352836

R.	Square	R.	Square	R.	Square
595	354025	635	403225	675	455625
596	355216	636	404496	676	456976
597	356409	637	405769	677	458329
598	357604	638	407044	678	459684
599	258801	639	408321	679	461041
600	360000	640	409600	680	462400
601	361201	641	410881	681	463761
602	362404	642	412164	682	465124
603	363609	643	413449	683	466489
604	364816	644	414736	684	467856
605	366025	645	416025	685	469225
606	367236	646	417316	686	470596
607	368449	647	418609	687	471969
608	369664	648	419904	688	473344
609	370881	649	421201	689	474721
610	372100	650	422500	690	476100
611	373321	651	423801	691	477481
612	374544	652	425104	692	478864
613	375769	653	426409	693	480249
614	376996	654	427716	694	481636
615	378225	655	429025	695	483025
616	379456	656	430336	696	484416
617	380689	657	431649	697	485809
618	381924	658	432964	698	487204
619	383161	659	434281	699	488601
620	384400	660	435600	700	490000
621	385641	661	436921	701	491401
622	386884	662	438244	702	492804
623	388129	663	439569	703	494209
624	389376	664	440896	704	495616
625	390625	665	442225	705	497025
626	391876	666	443556	706	498436
627	393129	667	444889	707	499849
628	394384	668	446224	708	501264
629	395641	669	447561	709	502681
630	396900	670	448900	710	504100
631	398161	671	450241	711	505521
632	399424	672	451584	712	506944
633	400689	673	452929	713	508369
634	401956	674	454276	714	509796

R.	Square.	R.	Square	R.	Square
715	511225	755	570025	795	632025
716	512656	756	571536	796	633616
717	514089	757	573049	797	635209
718	515524	758	574564	798	636804
719	516961	759	576081	799	638401
720	518400	760	577600	800	640000
721	519841	761	579121	801	641601
722	521284	762	580644	802	643204
723	522729	763	582169	803	644809
724	524176	764	583696	804	646416
725	525625	765	585225	805	648025
726	527076	766	586756	806	649636
727	528529	767	588289	807	651249
728	529984	768	589824	808	652864
729	531441	769	591361	809	654481
730	532900	770	592900	810	656100
731	534361	771	594441	811	657721
732	535824	772	595984	812	659344
733	537289	773	597529	813	660969
734	538756	774	599076	814	662596
735	540225	775	600625	815	664225
736	541696	776	602176	816	665856
737	543169	777	603729	817	667489
738	544644	778	605284	818	669124
739	546121	779	606841	819	670761
740	547600	780	608400	820	672400
741	549081	781	609961	821	674041
742	550564	782	611524	822	675684
743	552049	783	613089	823	677329
744	553536	784	614656	824	678976
745	555025	785	616225	825	680625
746	556516	786	617796	826	682276
747	558009	787	619369	827	683929
748	559504	788	620944	828	685584
749	561001	789	622521	829	687241
750	562500	790	624100	830	688900
751	564001	791	625681	831	690561
752	565504	792	627264	832	692224
753	567009	793	628849	833	693889
754	568516	794	630436	834	695556

R.	*Square*	R.	*Square*	R.	*Square*
835	697225	875	765625	915	837225
836	698896	876	767376	916	839056
837	700569	877	769129	917	840889
838	702244	878	770884	918	842724
839	703921	879	772641	919	844561
840	705600	880	774400	920	846400
841	707281	881	776161	921	848241
842	708964	882	777924	922	850084
843	710649	883	779689	923	857929
844	712336	884	781456	924	853776
845	714025	885	783225	925	855625
846	715716	886	784996	926	857476
847	717409	887	786769	927	859329
848	719104	888	788544	928	861184
849	720801	889	790321	929	863041
850	722500	890	792100	930	864900
851	724201	891	793881	931	866761
852	725904	892	795664	932	868624
853	727609	893	797449	933	870489
854	729316	894	799236	934	872356
855	731025	895	801025	935	874225
856	732736	896	802816	936	876096
857	734449	897	804609	937	877969
858	736164	898	806404	938	879844
859	737881	899	898201	939	881721
860	739600	900	810000	940	883600
861	741321	901	811801	941	885481
862	743044	902	813604	942	887364
863	744769	903	815409	943	889249
864	746496	904	817216	944	891136
865	748225	905	819025	945	893025
866	749956	906	820836	946	894916
867	751689	907	822649	947	896809
868	753424	908	824464	948	898704
869	755161	909	826281	949	900601
870	756900	910	828100	950	902500
871	758641	911	829921	951	904401
872	760384	912	831744	952	906304
873	762129	913	833569	953	908209
874	763876	914	835396	954	910116

R.	*Square*	*R.*	*Square*	*R.*	*Square*
955	912025	971	942841	987	974169
956	913936	972	944784	988	976144
957	915849	973	946729	989	978121
958	917764	974	948676	990	980100
959	919681	975	950625	991	982081
960	921600	976	952576	992	984064
961	923521	977	954529	993	986049
962	925444	978	956484	994	988036
963	927369	979	958441	995	990025
964	929296	980	960400	996	992016
965	931225	981	962361	997	994009
966	933156	982	964324	998	996004
967	935089	983	966289	999	998001
968	937024	984	968256	1000	1000000
969	938961	985	970225		
970	940900	986	972196		

467. It is Time now to proceed to the Demonſtration of the *Rule* delivered in *Article* 459, for extracting the Square Root; and, for the more regular doing this, it may be proper to premiſe,

Firſt, that the Number of Figures in the Product of any two Numbers may be equal to, but cannot poſſibly be greater than the Number of Figures in the two Factors: And, though they may be leſs, yet never leſs than one fewer.

Demonſtration. That the Product may have as many Places of Figures as are in both the Factors, one *Example*, for Inſtance, $9 \times 7 = 63$, is a ſufficient *Demonſtration.* But that the Number of Places, in the Product of any two Numbers, cannot be more than the Number of Places in both Factors, we thus ſhew: Let a and b be the two Factors, $p =$ their Product, $c =$ a greater Number than b, *viz.* $=$ 1 with as many 0's on the right Hand as there are Places in b; then it
* 67. is plain, that $a\,c$ muſt be greater than p; but $a\,c =$* a with as many 0's on the right Hand as there are Places in b; ∵ the Number of Places in $a\,c$ is $=$ the Number of Places in $a +$ the Number of Places in b; whence the Number of Places in $a\,c$, which is a greater Product than p, is only $=$ the Number of Places in a and b; and, conſequently, p which is leſs, cannot have more, for that is abſurd. *Q. E. D.*

As to the above Aſſertion, that the Number of Places in the Product may be one leſs than the Number of Places in both Factors, one *Example*, *viz.* $2 \times 3 = 6$, is a ſufficient Proof.

It only remains now to be demonſtrated, that the Number of Places in the Product of any two Numbers cannot, in any Caſe, be leſs than the Number of Places in both Factors *Minus* one. And this may be thus demonſtrated: If one leſs than the Number of Places in each Factor is denoted by d and n reſpectively, the leaſt Numbers, conſiſting of the ſame Number of Places as the Factors, are 1 with d, 0's; and 1 with n, 0's, by the Nature of Notation; and
∴ their Product is † 1 with d, 0's $+$ n, 0's, that is, it

conſiſts

consists of $1 + d + n$ Places; but $d + 1$, and $n + 1$, being the Number of Places in each respective Factor, their Sum, or the Number of Places in both, is $d + n + 2$, which is but one more than the Number of Places in the above Product $(1 + d + n)$; therefore the least Factors possible can have but one Place less in the Product than the Number of Places in both Factors, and consequently no other Factors, for the least Factors must have the least Product. *Q. E. D.*

468. Hence it follows, that, if the Factors are equal, as they are in raising any Root to its Square, then the Product, or Square, can never have more Places, than twice the Number of Places in the Root; nor less than twice the Number of Places in the Root *Minus* 1.

469. And from the last *Article* it follows, that, if a Square be pointed as directed in *Art.* 459, the Number of Points will be equal to the Number of Places in the Root. For if the Root, is but a single Figure, the Square * cannot be more than 2 Places, nor less than one Place, but either 1 or two Places admit of more than one Point; and, if the Root consists of two Places, the Square cannot have more than four Places, nor less than three Places, which, by the Method of Pointing, admits but of two Points: Also, if there be three Places in the Root, the Square cannot have more than six, nor less than five, neither of which admit of more than three Points, &c. *ad infinitum.* For one Place admits of one Point, three Places of two Points, five of three Points, &c. Or generally thus: It is plain, that if the Number of

Places be	1. 3. 5. 7. &c.
The Number of Points are	1. 2. 3. 4. &c.

which are two correspondent Series in Arithmetical Progression; in the first of which Series the common Difference is 2, and in the other 1. Now let n = one less than the Number of Terms in each Series, then, $2n + 1$ = † the last Term in the first Series = the Number of Places in the Square; and $n + 1$ =‡ the corresponding (last Term) in the other Series = the Number of Points in the Square. But, if we put d = the

* 468.

† 427.

‡ 427.

= the Number of Places in the Root, then the Number of Places in the Square cannot be more than * $2d$, nor less than $2d - 1$; let us suppose the Number of Places to be the least, *viz.* $2d - 1$; then, by the above,
† $2n + 1 =$ † $2d - 1$, ∴, subtracting 1 from each Side of the Equation, we get $2n =$ ‡ $2d - 2$; ∴ dividing by 2 we find $n =$ ‖ $d - 1$, to which adding 1, we have $n + 1 =$ ¶ d, but $n + 1 =$ the Number of Points, ∴ d = the Number of Points, when the Square has the fewest Places possible, *viz.* $2d - 1$; and ∴, since the Square can never have but one Place more, *viz.* $2d$ Places, it can have but d Points; because two Places would make but one Point more. *Q. E. D.*

* 468.
† 23.
‡ 36.
‖ 108.
¶ 22.

470. *Lemma* 2. If any Number, which is not a perfect Square, be given to find the greatest Square contained in it; if it be pointed by the Method already delivered, the Number of Points will be the same, as the Number of Points in the greatest Square which is contained in it.

Demonstration. Let a = the Number which is not a perfect Square, s = the greatest Square which is contained in it; suppose a and s pointed, then it is evident, that s cannot have more Points than a, because it is a lesser Number; and that s cannot have fewer, is easily shewn, by assuming the least Square Number that has the same Number of Points as a, *viz.* 1 with twice as many o's as the Number of Places on the right Hand of the first Point, or the Root = 1 with as many o's as the Number of Points less 1; for then its Square is 1 with twice the Number of o's in the Root, or 1 with as many o's as there are Places on the right Hand of the superior Point is a square Number, and is contained in (for, being the least Number having the given Number of Places, it cannot exceed) the Number a; and consequently, since this Square has the same Number of Points, certainly the greatest Square contained in it cannot have fewer. And, since a can neither have more nor less Points than s, it must certainly have equal. *Q. E. D.*

471. Hence it follows, that the Root of the great-

eſt Square contained in any Number a, which is not a Square, hath as many Figures as a hath Points; for it hath as many Figures as its own Square hath Points* which is = the Points in † a. ∵ the Number of Places in the Root of s = the Number of Points in a.

* 469.

† 470.

472. *Lemma* 3. If any Number be pointed according to the Method already ſhewn, then if we conſider the Period in the firſt Point (*viz.* the firſt on the left Hand) as a Number of itſelf, the greateſt Square contained in that Period is equal to the Square of the firſt Figure, (*viz.* that in the higheſt Place) of the Root of the given Number, if it be a perfect Square, or of the Root of the greateſt Square contained in it, if it be not a ſquare Number. Farther, the greateſt Square contained in the two firſt Periods (of the given Number) taken as one Number by themſelves, is equal to the Square of the two firſt Figures of the Root of the given Number, or of the greateſt Square contained in it; and, if in the ſame Manner we compare 3 or 4, &c. firſt Periods, the Square of 3 or 4, &c. firſt Figures of the Root, is equal to 3 or 4 firſt Periods taken in themſelves as one Number, or equal to the greateſt Square contained in them. To illuſtrate our Meaning, let a be any Number, b its Square Root, or the Root of the greateſt Square contained in it. And put c to repreſent the firſt, (*viz.* the ſuperior Period) or two firſt, &c. Periods; (as, for *Example*, if the Number be the ſame as in *Art.* 461; then $c = 1\dot{1}9\dot{0}$) and let r = the firſt, or two firſt, &c. Figures of the Root, according as we conſider the firſt, or two firſt Periods of the given Number, or of the Square b (*viz.* $r = 3$, or $r = 34$;) then r^2 = the Square of the firſt, or two firſt Figures of the Root; now all that we mean is, that r^2 is the greateſt Square contained in c.

Demonſtration. If we can ſhew, that r^2 cannot be greater than c, nor a greater than r^2 be taken from c, then we ſhall have given a proper *Demonſtration*; and this may be done as follows.

1. That r^2 is not greater, or which is the ſame Thing, is contained in c. We

* 469. 471. We have already proved * that the Number of Points (or Periods) in a is $=$ the Number of Places or Figures in b; consequently, in all *Cases* whatever, there are as many Points on the right Hand of c in the whole Number a, as there are Figures on the right Hand of r in the whole Root b. $\therefore$, if we put d to express the Number of Figures on the right Hand of r in the whole Root b, then r, taken in its compleat Value, is r with d, o's, and r^2 taken in its compleat Value $= r^2$ with $2d$, o's; also c in its compleat Value $= c$ with twice as many o's as there are Points on the right Hand of it, in the whole Number a; *viz.* $= c$ with $2d$, o's. Now it is evident that, if r^2 is contained in c, then r^2 with $2d$, o's, must be contained in c with $2d$, o's (the Number of o's in each being the same;) now, if possible, let us suppose r^2 greater than c, then it follows that, taking them in their compleat Values, r^2 with $2d$, o's, is greater than c with $2d$, o's; but, according to this Hypothesis, r^2 with $2d$, o's, is greater than a, for $a = c$ with as many Figures on its right Hand as we have put o's on c's; but these Figures can never exceed the Excess of r^2 above c, if it be but one, because it is in a higher Place; $\therefore$ from this Supposition it follows, that r^2 with $2d$, o's, that is, the Square of Part of the Root, is greater than a, which the Square of the whole Root doth not exceed; $\therefore$ to say that r^2 is greater than c, or, which is the same, that r^2 is not contained in c, is absurd.

2. Now, to prove that r^2 is the greatest Square that is contained in c, let us suppose, that r^2 is not the greatest, but that g^2, a greater Square, is contained in c; then g^2, taken in its compleat Value, is g^2 with $2d$, o's, which by the Supposition is contained in c with $2d$, o's; the square Root of g^2 with $2d$, o's, is g with d, o's, for g with d, o's squared is $= g^2$ with $2d$, o's; but, since g^2 is greater than r^2 by the Supposition, g with d, o's must be greater than r with d o's; but there being as many Figures on the Right Hand of r in the whole Root, as o's on the Right Hand of r taken in its compleat Value, it follows, from the Nature of Notation, that, if g exceed r by only an Unit, then g with

g with d, o's must be greater than the whole Root r with d Figures; and since g with d, o's is greater than the whole Root b, consequently its Square g^2 with $2d$, o's must be greater than b^2, or s; and from hence it follows that, c, a Part of s, contains g^2 with $2d$, o's the Square of g with d, o's, a Number which is greater than b, the square Root of the greatest Square 8, which is contained in a, that is, making a Part to contain more than the Whole; ∵ it is absurd to say, that a greater Square than r^2 is contained in c, and, if a greater cannot be contained, then r^2 must be the greatest. *Q. E. D.*

473. Whence we may observe by Way of Corollary, that if we find the Root of the greatest Square contained in the first Point, or Period (*viz.* the first on the Left Hand) that Root will be the first Figure of the required Root; and, if we find the Root of the greatest Square contained in the two first Periods, it will be the two first Figures of the Root, and so on to any Number of Periods.

474. *Lemma* 4. Let $n =$ any Number whose Root if it be a square Number, or the Root of the greatest Square contained in it, if it be not a square Number, is to be found; then if we put $r =$ any Part of such Root assumed at Pleasure, and $x =$ the other Part of the Root, we affirm, that if such a Number be taken for x as will make $\overline{2r+x} \times x = n - r^2$, if n be a square Number, or $\overline{2r+x} \times x =$ the greatest Product that can be taken from $n - r^2$, if n be not a square Number; such assumed Value of x will be its true Value.

Demonstration. The Demonstration of this may be conveniently parted into two Parts, *viz.* when the Number n is a Square, and when it is not a square Number.

First. When the Number n is a square Number, then, $r+x$ being $=$ the Root, $\overline{r+x}^2$ must be $= n$, ∵ $r^2 + 2rx + x^2 =$ * n, from each Side of this Equation subtracting r^2, we get $2rx + xx =$ † $n - r^2$; but $2rx + xx = \overline{2r+x} \times x$, ∵ $\overline{2rx+xx} \times x =$ ‡ $n - r^2$. *Q. E. D.* * 455. † 36. ‡ 23.

Part

Part 2. When n is not a square Number. To shew the Truth of the *Lemma*, in this *Case*, we put $g =$ the Excess of n above the next lesser square Number,
then $\overline{r+x}|^2 + g = n$; $\because$ subtracting $\overline{r+x}|^2$ from
* 36. both Sides of the Equation gives $g = {}^{*}\, n - \overline{r+x}|^2$;
† 455. but $\overline{r+x}|^2 = {}^{\dagger}\, r^2 + 2rx + x^2$, $\therefore$, by putting this
Value of $\overline{r+x}^2$ instead of $\overline{r+x}|^2$, we have $g = n - r^2 - \overline{2rx + x^2}$; now, since n and r^2 are supposed constant, they being given or known Quantities, it is plain that, the greater $2rx + x^2$ is, the lesser will g be; and consequently in $2rx + x^2$, or, which is the same, in $\overline{2r+x} \times x$, x must be the greatest possible; so that $\overline{2r+x} \times x$ be less than $n - r^2$. *Q. E. D.*

Hence if $g = o$, that is, if n be a square Number then $\overline{2r+x} \times x$ is $= n - r^2$, the same as we have already demonstrated, in the first Part of this Demonstration.

475. Hence it may be easily proved, that if, in extracting the Square Root of any Number, we have a Remainder greater than twice the Root found, there must be some Mistake made in the Operation; for let the Root found be called a, then, if the Remainder g be greater than $2a$, it must be at least $2a + 1$; to which if we add a^2, the Sum will be $a^2 + 2a + 1$, which is equal to $\overline{a+1}|^2$, as will plainly appear by writing a for r, and 1 for x, in *Art.* 455; but the Square of the Root of the greatest Square $+ g$ is $= n$, and, consequently, $a^2 + 2a + 1$ must be contained in n, and $\because$ a cannot be the true Root, because the Square of a greater Number $a + 1$ is contained in n. *Q. E. D.*

476. Having premised the Things necessary, we now come to demonstrate, or shew the Reason and Truth of the *Rule* which we gave in *Article* 459; and illustrated in subsequent ones.

‡ 469. First then. We have already shewn ‡ that the
471. Number of Figures in the Root is equal to the Number of Points in the Number whose Root, or the Root of the greatest Square contained in it, is to be found.

We

We have alſo demonſtrated * that the Root of the greateſt Square contained in the firſt (left Hand) Period or Point, is the firſt Figure of the Root, which is another Part of the *Rule* which we are now demonſtrating; therefore, if we take this firſt Figure of the Root (which may always be found in the Column of the ſecond Power in the Table of Powers †; because a Point can never have more than two Figures, and, therefore, the Root of its greateſt contained Square is but one Figure, for the leaſt Number of two Figures is 10 whoſe Square conſiſts of 3 Figures, *viz.* 100) in its compleat Value, *viz.* the Figure with o on its right Hand; then if we put r for the compleat Value of this firſt Figure, and $x =$ the other Figure of the Root, $n =$ the Number whoſe Root is to be extracted, then x muſt be ſuch a Number ‡ that $\overline{2r + x} \times x$ may be the greateſt Product that can poſſibly be taken from $n - r^2$; but this is, in Effect, the ſame as the *Rule* directs; which it may not be improper to illuſtrate by the *Example* in *Article* 460; here the greateſt Square in the firſt Point is 1, and its Root 1, which is the firſt Figure of the Root; and, as the Root muſt conſiſt of two Figures, there being two Points, r, taken in its compleat Value, is 10; ∵ $r =$ 10, and $r^2 = 100$; ∴, n being $= 144$, $\overline{n - r^2} = \overline{144 - 100} = 44$; ∵ $\overline{2r + x} \times x = \overline{2 \times 10 + x} \times x =$ $\overline{20 + x} \times x$ is the greateſt Product in 44; now x cannot be more than 2, for 20 can be taken from 44 only twice, and $20 + x$ is to be taken x Times from 44; and, if it be tried, it will be found that x is not leſs than 2, for $\overline{20 + 2} \times 2 = 22 \times 2 = 44$ which is contained exactly in $n - r^2 = 44$, and ∴, there being no Remainder, the true Root is 12; and this is exactly the ſame as one of the Methods in *Article* 460, and ∴ agreeable to the *Rule*.

* 473.

† 449.

‡ 474.

Again, if the Root conſiſts of three Figures, the two firſt Figures of the Root may be found by finding the Root of the greateſt Square contained in the two firſt Periods ‖ of the given Number, by the Method

‖ 473.

thod (already proved) for finding a Root of two Figures; and then, if we put $r =$ these two first Figures of the Root taken in their compleat Value, and x for the other Figure, and $n =$ the whole Number,
*474. then $\overline{2r + x} \times x$ must be the greatest Product in * $n - r^2$; but, in finding the two first Figures of the Root, we have found $n - r^2$ (and ∵ need not now square r to deduct it from n) it being equal to what remained after the second Figure of the Root was found; for, if we put a to denote first Figure of the Root taken in its compleat Value, and $b =$ the second Figure of the Root in its real Value, then $a + b =$ the two first Fi-
† 455. gures in their compleat Value $= r$; now $\overline{a + b}^2 =$ † $a^2 + 2ab + b^2 = r^2$; but, in finding the two first Figures of the Root, we first deducted a^2, and there must
‡ 459. remain $n - a^2$; from this again we deducted ‡ $\overline{2a + b} \times b = 2ab + b^2$, and ∵ there must remain $n - a^2 - 2ab - b^2$, or $n - \overline{a + b}^2$, or, since $\overline{a + b}^2 = r^2$, the Remainder must be $= n - r^2$; and ∵ we have only to take what remains after the two first Figures of the Root are found, and to take x such a Figure, that $\overline{2r + x} \times x$ may be the greatest Product that can possibly be taken out of that Remainder; and this is agreeable to the *Rule*; we will illustrate it by an *Example*, for Instance, that in *Article* 461. In that *Example*, a in its compleat Value is 300, and b the second Figure of the Root in its true Value, as yet supposed unknown; then $n - a^2 = 119025 - 90000 = 29025$; now b is to be taken such, that $\overline{2a + b} \times b = \overline{600 + b} \times b$ may be the greatest possible in 29025; ∴ $b = 40$ (taken in its true Value) for $\overline{600 + 40} \times 40 = 640 \times 40 = 25600$ (and, if b was taken $= 50$, it would be too much, for $\overline{600 + 50} \times 50 = 650 \times 50 = 32500$ is greater than 29025, and ∵ cannot be contained in it) now $29025 - 25600 = 3425 = n - r$ by the above; hence $a = 300$, and $b = 40$, and ∵ $a + b = 340 = r$; and $2r = 340 \times 2 = 680$, ∵ $\overline{2r + x} \times x = \overline{680 + x} \times x$ is to be the

the greateſt in 3425, $\because x = 5$, for $\overline{680 + 5} \times 5 = 685 \times 5 = 3425$, which deducted from 3425 leaves 0, and, conſequently, $r + x = 685$ is exactly the true Root.

477. The ſame Method of Reaſoning holds good in any other Number of Points, but we think it unneceſſary to proceed any further on this Head; and therefore ſhall only add, that, as Evolution is the Reverſe of Involution, an Evolution may be proved by involving the Root (and adding to the Involution the Remainder, if any) and the Number thus found, if the Work be right, muſt be equal to the given Number; and from the Nature of Multiplication it follows, that (after having ſubtracted the Remainder, if any, from the given Number, as what then remains muſt be a perfect Square) if out of the ſquare Number we caſt out the Nines, as out of the Product in Multiplication, and alſo caſt the Nines out of the Root, and the Exceſs multiplied by itſelf, and the Nines being caſt out, the Exceſs muſt be = the Exceſs after all the Nines were caſt out of the Square Number. *Example.* The Root of the greateſt Square contained in 150 is 12 and 6 remaining, then $150 - 6 = 144$ the Square Number, out of which the Nines being caſt, the Remainder is 0; and, the Nines (or rather the Nine) being caſt out of 12, the Exceſs is 3; and $3 \times 3 = 9$, out of which the Nine being taken, the Exceſs is 0, the ſame as out of the Square Number; and $\because$ 12 is the true Root of the greateſt Square. And, if we prove it by the Method of Involution, it ſtands thus, $12 \times 12 = 144$ equal to the greateſt Square, to which adding the Remainder 6, the Sum is = 150 the given Number for Proof.

CHAP. XXXVII.

Of the Evolution *of the* Cube Root.

478. THE Cube Root (from Κύβος *Greek, a Die*) is that *Rule*, which ſhews the Method of finding the Root of any given Cube; or, by having any Number which is not a Cube Number, to find the Root of the greateſt Cube which is contained in it.

479. To extract the Cube Root, according to the common Method, mind the following Directions: Firſt put a Point or Dot (.) over every third Figure of the given Number, beginning at the right Hand; and, as many Periods as the Number is thus divided into, ſo many Figures will the Root conſiſt of.

Then in the Row of the third Power in the Table of Powers, *Art.* 449, ſeek the greateſt Cube Number that is contained in the firſt Point, or Period, (*viz.* that laſt dotted;) and put the Root thereof as a Quotient, which will be the firſt Figure of the Root, but ſubtract its Cube from the firſt Period, and on the right Hand of the Remainder bring down the Figures in the next Point, and we ſhall then have what Arithmeticians call the *Reſolvend*, (from *reſolvo Lat.*) under which draw a Line.

Again, ſquare the Quotient, and multiply that Square by 300, which place under the Reſolvend, and its Name is the triple Square.

Then, multiply the Quotient by 30, and put the Product under the triple Square, and call it the triple Quotient; and under this triple Quotient draw a Line.

Then add the triple Square and triple Quotient together, and call the Sum the Diviſor; and under it draw a Line, then try how often this Diviſor is contained in the Reſolvend, which muſt never be taken more than 9 (though ſometimes the Diviſor may be taken

taken out of the Resolvend more than 9 Times; because we are giving Directions to find only one Figure at a Time) and the Number so taken may be, but is not always, the next Figure of the Root; however, take it as the true Figure, and multiply the triple Square by it; then square this Figure, and by its Square multiply the triple Quotient, and to these two Products add the Cube of the said Figure, and, if this Sum, which is called the *Ablatitium*, be not greater than the Resolvend, the Figure taken was the true one; if it does, then take it one less, and find another Ablatitium; then subtract the Ablatitium from the Resolvend, and to the Remainder, if there be any more Points, bring down the next Period; and the Remainder and Period so taken down, considered as one Number, will be a new Resolvend, under which draw a Line; and now begin again (at the Place marked * in these Directions) with squaring the Quotient, which now consists of two Figures; and so proceed after the above Manner, until all the Periods have been taken down, finding at each Step one Figure of the Root.

480. *Example.* 1. What is the Cube Root of 1728?

This, worked according to the Directions just laid down, will stand thus:

$\dot{1}72\dot{8}$ (12
1
728 Resolvend
1 x 1 x 300 = 300 Triple Square
1 x 30 = 30 Triple Quotient
The Sum or Divisor 330 is contained in 728 twice,
300 x 2 = 600
2 x 2 = 4 and 30 x 4 = 120
2 x 2 x 2 = 8
Sum = 728 Ablatitium
Remains 0

Or 1728 may (with a little Alteration in the Directions) be extracted as follows; and, though this Method, and that above, may at first Sight appear different, yet by Comparison they will appear the very same:

$$
\begin{array}{rll}
 & 1728 & (10 = \text{the assumed Root} \\
10 \times 10 \times 10 = & 1000 & \\
\hline
 & 728 & \text{Resolvend} \\
10 \times 10 = 100, \text{and } 100 \times 3 = & 300 & \text{Triple Square} \\
10 \times 3 = & 30 & \\
\hline
 & 330)728(2 & \text{Times} \\
300 \times 2 = & 600 & \\
30 \times 2 \times 2 = & 120 & \\
2 \times 2 \times 2 = & 8 & \\
\hline
 & 728 & \text{Ablatitium} \\
\hline
 & 0 &
\end{array}
$$

For Proof $12 \times 12 \times 12 = 1728$.

481. *Example* 2. What is the Cube Root of 16003008?

Here we can go the Divisor 1260 six Times in the Resolvend 8003; but, if the Reader works it down with 6 for the next or second Figure of the Root, he will find the Ablatitium greater than the Resolvend; and therefore we must put down one less, *viz.* 5. As for the new Divisor, it is contained in the new Resolvend twice; ∵ the third or last Figure of the Root is 2, and so the whole Root = 252. For Proof, $252 \times 252 \times 252 = 16003008$.

$$
\begin{array}{rll}
 & 16003008 & (252 \\
\overline{2}\rceil^3 = & 8 & \\
\hline
 & 8003 & \text{Resolvend} \\
2 \times 2 = 4, 4 \times 300 = & 1200 & \text{Triple Square} \\
2 \times 30 = & 60 & \text{Triple Quotient} \\
\hline
 & 1260 & \text{Divisor} \\
\hline
1200 \times 5 = & 6000 & \\
6 \times 5 \times 5 = & 1500 & \\
5 \times 5 \times 5 = & 125 & \\
\hline
 & 7625 & \text{Ablatitium.}
\end{array}
$$

	378008	New Resolvend
$\overline{25}\rvert^2 \times 300 =$	187500	New triple Square
$25 \times 30 =$	750	New triple Quotient
	188250	New Divisor,
$187500 \times 2 =$	375000	
$750 \times 2 \times 2 =$	3000	
$2 \times 2 \times 2 =$	8	
	378008	New Ablatitium
	0	

482. There are many other Methods contrived for Extracting; but we shall here only give a Method a little different from the foregoing, and shew that there is a perfect Agreement betwixt them; and consequently, if the preceding one be true, this cannot be false. This Method differs from the former, only, in finding the Ablatitium, which is found thus: Put the Square of the Figure, which shews how often the Divisor is contained in the Resolvend under the Divisor; under this put the Product of the triple Quotient multiplied by that Figure; and under these two Numbers place the triple Square; and, having added these three Numbers together, multiply the Sum by the Figure last placed in the Quotient, and the Product will be the Ablatitium *.

* The Agreement of this and the *Rule* in *Art.* 479. may be thus shewn: Let $x =$ the Figure shewing how often the Divisor is contained in the Resolvend; $s =$ the triple Square, $q =$ the triple Quotient; then, according to *Art.* 479, the Ablatitium is $sx + qx^2 + x^3$, but this, it is evident, is $= \overline{s + qx + x^2} \times x = \overline{x^2 + qx + s} \times x$, the Method here delivered. *Q. E. D.*

483. *Example.* Let it be required to xtract 1728 by this Method.

The Learner may compare this Operation with those in *Art.* 480.

The Operation.

$$
\begin{array}{rrl}
 & \dot{1}72\dot{8}\,(\,12 & \\
 & 1 & \\
\hline
 & 728 & \text{Resolvend} \\
1 \times 1 \times 300 = & 300 & \text{Triple Square} \\
1 \times 30 = & 30 & \text{Triple Quotient} \\
\hline
 & 330 & \text{Divisor} \\
2 \times 2 = & 4 & \\
30 \times 2 = & 60 & \\
 & 300 & \text{Triple Square} \\
\hline
 & 364 & \\
\times \text{ by} & 2 & \\
\hline
 & 728 & \text{Ablatitium.} \\
\hline
 & 0 &
\end{array}
$$

Whence it appears, that this Method is something less troublesome than the foregoing.

484. The Application of the Cube Root to Affairs of Business being grounded on Geometrical Theorems, it is necessary to defer its Application to a Variety of Purposes of common Life, till we have treated of such Theorems; and, therefore, shall at present only give one Instance, *viz.* If a Shot whose Diameter is 2 Inches, weighs 3 ℔: What is the Diameter of another Shot of the same Metal, that weighs 24 ℔?

Solution. It is evident, that the Weight of the Shots must be proportionable to the Quantity of Metal they contain; and the Geometricians acquaint us, that the Quantity of Metal or Solidity is as the Cube of the Diameter; ∵ as 3 ℔ : 8 (the Cube of 2 Inches) :: 24 ℔ : (by the Golden Rule) $8 \times 24 \div 3 = 64$ Inches, the Cube of the Diameter of the required Ball; and ∵ the Cube Root of 64, *viz.* 4 Inches = the Diameter required. *Q. E. I.*

485.

485. Since these Methods of extracting the Cube Root are troublesome, it will be very useful to have a Table, which will give the Root of any Number, (whose Root doth not consist of more than three Figures) by Inspection only. But, before we give such a Table, as the Method of Involution would be very tedious, it may not be improper to shew how such Tables may be made by Addition only, from a Table of Squares: Which is done by Help of this Theorem, *viz.* Let there be any two Roots, whose Difference is an Unit; then we affirm, that the Difference of their Cubes is equal to * the Sum of the Square of the greater Root, twice the Square of the lesser Root.

486. Hence, a Table of Cubes may be made from the Table of Squares by Addition only; for *Example*, suppose it was required to find the Cube of 12, the Cube of 11 being given = 1331. First, by the Table of Squares, the Square of 11 = 121, and the Square of 12 = 144. Now twice 121 = 121 + 121, ∵ by the Theorem 144 + 121 + 121 + 11 = the Difference of their Cubes, and, consequently, 1331 + 144 + 121 + 121 + 11 = * 1728 the Cube of * 50.
12 the greater Root.

487. A TABLE of Cube Numbers, or the Cube, or third Power of all the Integral Roots, from one to 1000 inclusive.

The Method of using this Table is the same as that for the Table of Squares.

* Let the Roots be r and $r + 1$, then their Cubes are $r3$ and $r3 + 3r^2 + 3r + 1$ (* x being $= 1$); ∵, subtracting $r3$ from this * 455.
last, the Difference of their Cubes $= 3r^2 + 3r + 1$, which, it is evident, is $= r^2 + 2r^2 + r + 2r + 1 = r^2 + 2r + 1 + 2r^2 + r$; but $r^2 + 2r + 1 =$ † $\overline{r + 1}|^2$, and ∵ by writing $\overline{r + 1}|^2$ † 455.
for it is equal $r^2 + 2r + 1$ in the last Expression for their Difference, we get $\overline{r + 1}|^2 + 2r^2 + r =$ the Difference of the Squares. *Q. E. D.*

R.	Cube	R.	Cube	R.	Cube
1	1	40	64000	79	493039
2	8	41	68921	80	512000
3	27	42	74088	81	531441
4	64	43	79507	82	550408
5	125	44	85184	83	571787
6	216	45	91125	84	592604
7	343	46	97336	85	614125
8	512	47	103823	86	636056
9	729	48	110592	87	648303
10	1000	49	117649	88	681472
11	1331	50	125000	89	705669
12	1728	51	135651	90	729000
13	2197	52	140608	91	753571
14	2744	53	148877	92	778688
15	3375	54	157464	93	804357
16	4096	55	167375	94	830584
17	4913	56	175616	95	857375
18	5832	57	185193	96	884736
19	6859	58	195112	97	915673
20	8000	59	205379	98	941192
21	9261	60	216000	99	970299
22	10648	61	216981	100	1000000
23	12167	62	238328	101	1030301
24	13824	63	250047	102	1061208
25	15625	64	262244	103	1092727
26	17576	65	274625	104	1124856
27	19683	66	287496	105	1157625
28	21972	67	300753	106	1191016
29	24389	68	314432	107	1225043
30	27000	69	329199	108	1259712
31	29791	70	343000	109	1295029
32	32768	71	357911	110	1331000
33	35937	72	373348	111	1367631
34	39304	73	389017	112	1404928
35	42825	74	405224	113	1442897
36	48656	75	411875	114	1481544
37	50653	76	438976	115	1520875
38	54872	77	456533	116	1560896
39	55419	78	474522	117	1601613

R.	Cube	R.	Cube	R.	Cube
118	1643032	157	3869893	196	7529536
119	1685159	158	3944312	197	7645373
120	1728000	159	4019679	198	7762392
121	1771561	160	4096000	199	7880599
122	1815848	161	4173281	200	8000000
123	1860867	162	4251528	201	8120601
124	1906624	163	4330744	202	8242408
125	1953125	164	4410944	203	8365427
126	2000376	165	4492125	204	8489664
127	2048383	166	4574296	205	8615325
128	2097152	167	4657463	206	8741816
129	2146689	168	4741632	207	8869743
130	2197000	169	4826809	208	8999912
131	2248091	170	4913000	209	9129329
132	2299968	171	5000211	210	9261000
133	2352637	172	5088448	211	9393931
134	2406104	173	5177717	212	9528128
135	2460375	174	5268024	213	9663597
136	2515856	175	5359375	214	9800344
137	2570353	176	5451776	215	9938375
138	2628072	177	5545233	216	10077696
139	2685619	178	5639752	217	10218313
140	2744000	179	5735339	218	10360232
141	2803221	180	5832000	219	10503459
142	2864288	181	5929741	220	10648000
143	2924207	182	6028568	221	10793861
144	2985984	183	6128487	222	10941048
145	3027525	184	6229504	223	11089567
156	3112136	185	6331625	224	11239424
147	3176523	186	6434856	225	11390625
148	3241792	187	6539203	226	11543176
149	3307949	188	6644672	227	11697083
150	3375000	189	6751269	228	11852452
151	3442951	190	6859000	229	12008989
152	3511808	191	6967871	230	12167000
153	3581577	192	7077888	231	12326391
154	3652264	193	7189057	232	12487168
155	3723875	194	7301384	233	12649337
156	3796416	195	7414875	234	12812904

R.	Cube	R.	Cube	R.	Cube
469	103161799	508	131096512	547	163667323
470	103823000	509	131872229	548	164566592
471	104487111	510	132651000	549	165469149
472	105154048	511	133432821	550	166375000
473	105823817	512	134217728	551	167284151
474	106496424	513	135005697	552	168196608
475	107171875	514	135796744	553	169112377
476	107050176	515	136590875	554	170031464
477	108531333	516	137388096	555	170953875
478	109215352	517	138188413	556	171879616
479	109902239	518	138991832	557	172808683
480	110592000	519	139798359	558	173741112
481	111284641	520	140608000	559	174676879
482	111980168	521	141420761	560	175616000
483	112678587	522	142246648	561	176558481
484	113379904	523	143055667	562	177504328
485	114084125	524	143877824	563	178453547
486	114791256	525	144703125	564	179306144
487	115501303	526	145531576	565	180262125
488	116214272	527	146363183	566	181221496
489	116930169	528	147197952	567	182154263
490	117649000	529	148035889	568	183150432
491	118370771	530	148877000	569	184220009
492	119095488	531	149721291	570	185193000
493	119823157	532	150568768	571	186169411
494	120553784	533	151419437	572	187149248
495	121287375	534	152273304	573	188132517
496	122023936	535	153130375	574	189119224
497	122763473	536	153990656	575	190109375
498	123505992	537	154854153	576	191102976
499	124251499	538	155720872	577	192100033
500	125000000	539	156590819	578	193100552
501	125751501	540	157464000	579	194104539
502	126506008	541	158340421	580	195112000
503	127263527	542	159220088	581	196122941
504	128024064	543	160103007	582	197137368
505	128787625	544	160989184	583	198155287
506	129554216	545	161878625	584	199176704
507	130323843	546	162771336	585	200201625

R.	Cube	R.	Cube	R.	Cube
586	202230056	625	244140625	664	292754944
587	202262003	626	245314376	665	294079625
588	203297472	627	246491883	666	295408296
589	204336469	628	247673152	667	296740963
590	205379000	629	248858189	668	298077632
591	206425071	630	250047000	669	299418309
592	207474688	631	251239591	670	300763000
593	208527857	632	252435968	671	302111711
594	209584584	633	253636137	672	303464448
595	210644875	634	254840104	673	304821217
596	211708736	635	256047875	674	306182014
597	212776173	636	257259456	675	307546875
598	213847192	637	258474853	676	308915776
599	214921799	638	259694072	677	310288733
600	216000000	639	260917119	678	311665752
601	217081801	640	262144000	679	313046839
602	218167208	641	263374721	680	314432000
603	219256227	642	264609288	681	315821241
604	220348864	643	265847707	682	317214568
605	221445125	644	267089984	683	318611987
606	222545016	645	268336125	684	320813504
607	223648543	646	269586136	685	321419115
608	224755712	647	270840023	686	322828856
609	225866529	648	272097792	687	324242703
610	226981000	649	273359449	688	325660672
611	228099131	650	274625000	689	327082769
612	229220928	651	275854451	690	328509000
613	230346397	652	277167808	691	329939371
614	231475544	653	278445077	692	331373888
615	232608375	654	279726264	693	332812557
616	233744896	655	281011375	694	334255384
617	234885113	656	282300416	695	335702375
618	236029032	657	283593393	696	337153536
619	237176559	658	284890312	697	338608873
620	238328000	659	286191179	698	340068392
621	239483061	660	287496000	699	341532099
622	240641848	661	288804781	700	343000000
623	241804367	662	290117528	701	344472101
624	242970624	663	291434247	702	345948408

R.	Cube	R.	Cube	R.	Cube
703	347428927	742	408518488	781	476379541
704	348913664	743	410172407	782	478211768
705	350402625	744	411830784	783	480048687
706	351895816	745	413493625	784	481890304
707	353393243	746	415160936	785	483736625
708	354894912	747	416832723	786	485587656
709	356400829	748	418508992	787	487443403
710	357911000	749	420189749	788	489303872
711	359425431	750	421875000	789	491169069
712	360944128	751	423564751	790	493039000
713	362467097	752	425259008	791	494913671
714	363994344	753	426957777	792	496793088
715	365525875	754	428661064	793	498677257
716	367061696	755	430368875	794	500566184
717	368601813	756	432081216	795	502459875
718	370146232	757	433798093	796	504358336
719	371694959	758	435519512	797	506261573
720	373248000	759	437245479	798	508169592
721	374805361	760	438976000	799	510082399
722	376367048	761	440711081	800	512000000
723	377933067	762	442450728	801	513922401
724	379503424	763	444194947	802	515849608
725	381078125	764	445943744	803	517781627
726	382657176	765	447697125	804	519718464
727	384240583	766	449455096	805	521660125
728	385828352	767	451217663	806	523606616
729	387420489	768	452984832	807	525557943
730	389017000	769	454756609	808	527514112
731	390617891	770	456533000	809	529475129
732	392223168	771	458314011	810	531441000
733	393832837	772	460099648	811	533411731
734	395446904	773	461889917	812	535387328
735	397065375	774	463684824	813	537367797
736	398688256	775	465484375	814	539353144
737	400315553	776	467288576	815	541343375
738	401947272	777	469097433	816	543338496
739	403583419	778	470910952	817	545338513
740	405224000	779	472729139	818	547343432
741	406869021	780	474552000	819	549353259

R.	*Cube.*	*R.*	*Cube*	*R.*	*Cube*
820	551368000	859	633839779	898	724150792
821	553387661	860	636056000	899	726572699
822	555412248	861	638277381	900	729000000
823	557441767	862	640503928	901	731432701
824	559476224	863	642735647	902	733870808
825	561515625	864	644972544	903	736314327
826	563559976	865	647214625	904	738763264
827	565609283	866	649461896	905	741217625
828	567663552	867	651714363	906	743677416
829	569722789	868	653972032	907	746142643
830	571787000	869	656234909	908	748613312
831	573856191	870	658503000	909	751089429
832	575930368	871	660776311	910	753571000
833	578009537	872	663054848	911	756058031
834	580093704	873	665338617	912	758550528
835	582182875	874	667627624	913	761048497
836	584277056	875	669921875	914	763551944
837	586376253	876	672221376	915	766060875
838	588480472	877	674526133	916	768575296
839	590589719	878	676836152	917	771095213
840	592740000	879	679151435	918	773620632
841	594823321	880	681472000	919	776151559
842	596947688	881	683797841	920	778688000
843	599077107	882	686128968	921	781229961
844	601211184	883	688465387	922	783777448
845	603351125	884	690807104	923	786330467
846	605495736	885	693154125	924	788889024
847	607645423	886	695506456	925	791453125
848	609800192	887	697864103	926	794022776
849	611960049	888	700227072	927	796597983
850	614125000	889	702595369	928	799178752
851	616295051	890	704969000	929	801765089
852	618470208	891	707347971	930	804357000
853	620650477	892	709732288	931	806954491
854	622835864	893	712121957	932	809557569
855	625026375	894	714516984	933	812166237
856	627222016	895	716917375	934	814780504
857	629422793	896	719323136	935	817400375
858	631628712	897	721734273	936	820025856

R.	Cube	R.	Cube	R.	Cube
937	822656953	959	881974079	981	944076141
938	825293672	960	884736000	982	946966168
939	827936019	961	887503681	983	949862087
940	830584000	962	890277218	984	952763904
941	833237621	963	893056347	985	955671625
942	835896888	964	895841344	986	958585256
943	838561807	965	898632125	987	961504803
944	841232384	966	901428696	988	964430272
945	843908625	967	904231063	989	967361669
946	846590536	968	907039232	990	970299000
947	849278123	969	909853209	991	973242271
948	851971392	970	912673000	992	976191488
949	854670349	971	915498611	993	979146657
950	857375000	972	918330048	994	982107784
951	860085351	973	921167317	995	985074875
952	862801408	974	924010424	996	988047936
953	865523177	975	926859375	997	991026973
954	868250664	976	929714176	998	994011992
955	870983875	977	932574833	999	997002999
956	873722816	978	935441352	1000	1000000000
957	876467493	979	938313739		
958	879217912	980	941192000		

488. We shall now proceed to the *Lemma*'s necessary for demonstrating the Method given in *Art.* 479. (and illustrated in the succeeding ones) for extracting the Cube Root.

Lemma 1. The Number of Figures, in the continued Product of any three Numbers, can never be more than the Number of Figures in all three Factors, but may be equal to them; neither can the continued Product be at least but two Figures fewer.

Demonstration. This evidently follows from *Art* 467; for, by the *Lemma* in that *Article*, the Product of any two Factors cannot have more Figures than are in both Factors, nor less than one fewer; and therefore, if we consider this Product as one Factor, and the remaining Number the other, this Product can have at most but as many Figures as are in both the Factors of which it is compounded, and at least but one fewer; now the Number of Places in one of these Factors, *viz.* the first Product by the above, can never exceed the Number of Places in the two first Factors, and at least be but one fewer; consequently, the Number of Places, in this last Product, can never exceed the Number of Places in all three Products, nor be at least but two fewer than the Number of Figures in the three Factors. *Q. E. D.*

489. *Corollary* 1. Hence it is evident, that the Cube of any integral Number cannot have more Figures than three Times the Number of Figures in the Root, nor less than two fewer than triple the Number of Figures in the Root.

490. *Corollary* 2. Whence it follows that, if any Cube be pointed as we have already directed *, then the Number of Points will be equal to the Number of Figures in the Root; for, if the Number of Places be 1. 4. 7. 10, &c. the Number of Points are 1. 2. 3. 4, &c. Which are two correspondent Series in Arithmetical Progression; the first, being the least Number of Places, which will admit the correspondent Number of Points, in the second Series. Now it is evident by a bare Inspection, that, if to any Term in the first

* 479.

we add 2, a Third of that Sum will be equal to the correspondent Term in the second Series; therefore if we put $n =$ one less than the Number of Terms in the Series, as the common Difference is 3, the Term
* 427. which is n Terms distant from the first $=$ * $3n+1$ $=$ the Number of Places in the Cube; to which adding 2 we have $3n+3$, a Third of which is $n+1 =$ the Number of Points. (This might have been found otherwise, by the same Method as shewn in *Art.* 469, for the Square; but we have given the above Method, for Variety Sake.) Now, putting $d =$ the Number of Places in the Root, the least Number of Places in the
† 489. Cube is † $3d-2$; ∵ $3d-2 =$ ‡ $3n+1$; and
‡ 23. ∴, adding 2 to each Side of the Equation, we have $3d = 3n+3$; hence, dividing by 3, we find, that
‖ 22. $d =$ ‖ $n+1$; but $n+1 =$ the Number of Points,
§ 23. by the above; therefore, $d =$ § the Number of Points. *Q. E. D.*

491. *Lemma* 2. If any Number, not a perfect Cube,
¶ 479. be pointed according to the Directions already given ¶, the Number of Points will be equal to the Number of Points in the greatest Cube, which is contained in it.

Demonstration. Let a be a Number; (not a Cube;) $c =$ the greatest Cube Number contained in it; then it is plain, that c cannot have more Points than a, for then it must have more Figures, and so c be greater than a, and then could not be contained in it, which is contrary to the Supposition. And that c cannot have fewer Points than a, may be thus shewn: Let $3n$ be the Number of Figures on the right Hand of the superior Point of a; then, if we take 1 with $3n$ o's which it is evident is contained in a, it is a Cube Number, and its Root $=$ 1 with $\frac{1}{3}$ of $3n$ o's, *viz.* 1 with n o's; for 1 with n, o's cubed $=$ 1 with $3n$, o's; wherefore, 1 with $3n$ o's is a Cube with as many Points as a hath, and, being contained in a, it follows, that the greatest Cube c cannot have fewer

Points;

Points; and, if c can neither have more nor fewer Points than a, the Number of Points in a and c must be equal. *Q. E. D.*

492. *Corollary.* The Root of the greatest Cube, contained in any Number a, which is not a Cube, hath as many Figures as a hath Points; for it hath as many Figures as there are Points in c *; and the Points in c and a are equal †; consequently, the Root of c hath as many Figures as there are Points in a. *Q. E. D.* * 490. † 491.

493. *Lemma* 3. If any Number be pointed according to the Directions already given ‡, and we consider the Period in the first Point (*viz.* the first on the left Hand) as a Number of itself, then will the greatest Cube, contained in that Period, be equal to the Cube of the first Figure of the Root of the given Number, if it be a perfect Cube; or of the Root of the greatest Cube which is contained in it, if it be not a perfect Cube. Again, the greatest Cube contained in the two first Periods (of the given Number) taken as one Number by themselves, is equal to the Cube of the first two Figures of the Root of the given Number, if a perfect Cube; if not a Cube Number, of the greatest Cube contained in it. And in general, if we compare, by this Method, 3 or 4, &c. first Periods, the Cube of 3 or 4, &c. first Figures of the Root, will be respectively equal to the Root of 3 or 4, &c. first Periods of the given Number, taken by themselves as one Number, if the Number be a perfect Cube; or of 3 or 4, &c. first Periods of the greatest Cube contained in it, if it be not a Cube Number. Our Meaning, in this *Article*, may be illustrated in the same Manner as that made Use of in *Lemma* 3. for the Square Root, *viz.* Let a be any Number, b its Cube Root, if a perfect Cube, or the Root of the greatest Cube contained in it: Put c to represent the first, or two first, &c. Periods (as, for *Example*, if the Number be the same as in *Art.* 481, then $a = 16$ ‡ 479.

or 16003, &c.) and let $r =$ the first, or two first, &c. Figures of the Root; according as we consider the first, or two first Periods of the given Number, or of the Cube b^3; (*viz.* $r = 2$, or $r = 25$, &c.) then $r^3 =$ the Cube of the first, or two first Figures of the Root; now all that we affirm is, that r^3 is the greatest Cube contained in c.

Demonstration. We shall first shew that r^3 is contained in c, or, which is the same, that r^3 cannot be greater than c; and then prove that a greater Cube than the Cube of r cannot be taken from c.

*490. 1. We have already demonstrated *, that the Num-
492. ber of Points (or Periods) in a is $=$ the Number of Places or Figures in b; and consequently, in all Cases whatever, there are as many Points on the right Hand of c in the whole Number a, as there are Figures on the right Hand of r in the Root b; ∴, if we put d to express the Number of Figures on the right Hand of r in the whole Root b, then r, taken in its compleat Value, is r with d, o's; and r^3, taken in its compleat Value, is r^3 with $3\,d$, o's; also c, in its compleat Value, $= c$ with three Times as many o's as there are Points on the right Hand of it, in the whole Number a, *viz.* c with $3\,d$, o's; now it is evident that, if r^3 is contained in c, then also, in their compleat Values, r^3 with $3\,d$, o's must be contained in c with $3\,d$, o's; but, if possible, let us suppose r^3 greater than c, then it follows that, taking them in their compleat Values, r^3 with $3\,d$, o's is greater than c with $3\,d$, o's; but, according to this Hypothesis, r^3 with $3\,d$, o's is greater than a; for $a = c$ with as many Figures on its right Hand as we have put o's on the right Hand of c; but these Figures can never exceed the Excess of r^3 above c, if it be but 1, because it is in a higher Place; ∴, from the Supposition that r^3 is greater than c, it follows, that r^3 with $3\,d$, o's, that is, the Cube of Part of the Root, is greater than a, which the Cube of the whole Root doth not exceed; that is, making a Part greater than the Whole, which is absurd, and there-

therefore r^3 cannot be greater than c, and consequently must be contained in it.

2. We are now to prove that r^3 is the greatest Cube Number that is contained in c; we will at present suppose, that r^3 is not the greatest, but that g^3, a greater Cube, is contained in c; then g^3, taken in its compleat Value, is g^3 with $3\,d$, o's; which, by the Supposition, is contained in c with $3\,d$, o's; the Cube Root of g^3 with $3\,d$, o's, is g with d, o's; for g with d, o's cubed, is $= g^3$ with $3\,d$, o's; but, since g^3 is greater than r^3 by the Supposition, g with d, o's, must be greater than r with d, o's; but, there being as many Figures on the right Hand of r in the whole Root, as o's on the right Hand of r taken in its compleat Value, it follows, from the Nature of Notation, that, if g exceed r by only an Unit, then g with d, o's must be greater than the whole Root r with d Figures; and consequently, g with d, o's being greater than the whole Root b, its Cube g^3 with $3\,d$, o's must be greater than the Cube of b; $\therefore$, by the above Supposition, a greater than b^3, *viz.* g^3 with $3\,d$, o's is contained in a less than b^3, *viz.* in r^3 with $3\,d$, o's or c with $3\,d$, o's, which is absurd; and $\therefore r^3$ is the greatest Cube in c. *Q. E. D.*

494. *Corollary.* If we find the Root of the greatest Cube contained in the first Point, or Period (*viz.* the first on the left Hand) that Root will be the first Figure of the required Root; and, if we find the Root of the greatest Cube contained in the two first Periods, it will give the two first Figures of the Root, and so on to any Number of Periods.

495. *Lemma* 4. Let $n =$ any Number, whose Root, if it be a Cube Number, or the Root of the greatest Cube contained in it, if it be not a Cube Number, is to be found; then, if we put $r =$ any Part of such Root, assumed at Pleasure, and $x =$ the other Part of the Root, that is $r + x =$ the Root, we affirm, that, if we make $3\,r^2 + 3\,r$ a Divisor, and seek how often it is contained in $n = r^3$, under this Limitation, the first Part of the Divisor $3\,r^2$ being multiplied

 by

by the Quote, and the second Member $3r$ being multiplied by the Square of the Quote, and to the Sum of these two the Cube of the Quote being added, the whole Sum may be equal to $n - r^3$ if it was a Cube Number; or the greatest that can be so taken from $n - r^3$, if n be not a perfect Cube, then the Quote so found is equal x.

Demonstration. First, when n is a Cube Number, we shew the Truth of this *Lemma* thus: Since $r + x =$
* 455. the Root, $\overline{r + x}|^3 = n$, that is, * $r^3 + 3r^2x + 3rx^2 + x^3 = n$; whence by subtracting r^3 we get
† 36. $3r^2x + 3rx^2 + x^3$ † $= n - r^3$; but by the above *Rule* we take x such, that the Sum of $3r^2x$, $3rx$, and x^3, that is, $3r^2x + 3rx^2 + x^3$ may be equal to $n - r^3$, which must be so by the above; therefore this *Rule* is true in this Case.

Secondly. When n is not a Cube Number, let $g =$ the Excess of n above the next lesser Cube Number, then
‡ 455. $r^3 + 3r^2x + 3rx^2 + x^3$ ‡ $= n - g$, and taking r^3 from both Sides of this Equation, gives $3r^2x + 3rx^2 + x^3 =$
‖ 36. ‖ $n - g - r^3 = n - r^3 - g$; and, to this adding g, it
¶ 22. becomes $3r^2x + 3rx^2 + x^3 + g =$ ¶ $n - r^3$, and from this Equation subtracting $3r^2x + 3rx^2 + x^3$,
* 36. gives $g =$ * $n - r^3 - \overline{3r^2x + 3rx^2 + x^3}$; now since g is the least possible, and $n - r^3$ invariable; x being the only unknown Quantity, $3r^2x + 3rx^2 + x^3$ must be the greatest that can be taken from $n - r^3$; and this is what the above *Rule* directs, and ∴ the Truth of it is plainly shewn.

Hence, if $g = 0$, that is, if n be a Cube Number, then, $n - r^3 - \overline{3r^2x + 3rx^2 + x^3} = 0$, or $3r^2x + 3rx^2 + x^3 = n - r^3$, as was before proved, in the first Part of this Demonstration; and, perhaps, the perfect Agreement of these Demonstrations is pleasing to the young Student, as it shews how the Demonstrations of particular Cases may be deduced from the more general ones.

496. *Corollary.* Whence may be easily shewn the Reason, why, in making $3r^2 + 3r$ the Divisor, we

can

can sometimes go it oftener in the Resolvend $n - r^3$, than we ought; for, in taking the Divisor $3r^2 + 3r$ x Times, we only take $3r^2x + 3rx$, whereas we must deduct * $3r^2x + 3rx^2 + x^3$; and consequently, till we have worked down to the Ablatitium, that is, found what $3r^2x + 3rx^2 + x^3$ would bring out, according to that assumed Value of x, we cannot be certain whether x be taken too much; but then, if $3r^2x + 3rx^2 + x^3$ be greater than $n - r^3$, we must take x less than before, and work again. * 495.

497. *Corollary* 2. When n is not a Cube Number, the Remainder which is g, can never exceed triple the Root found, and triple its Square; that is, g cannot be greater than $3b^2 + 3b$, b being the Root. For, if g be greater, it must be at least 1 greater, that is, $3b^2 + 3b + 1$, to which adding b^3, it is $b^3 + 3b^2 + 3b + 1 = \overline{b+1}^3$; (for $\overline{b+1}^3 = b^3 + 3b^2 + 3b + 1$, as is evident by Writing b for r, and 1 for x, in *Art.* 455.) And since this Cube it is manifest is contained in n (because it is the Sum of the greatest Cube, b^3, and the Remainder $3b^2 + 3b + 1$ added together) it must follow, that $b + 1$ the Root of $b^3 + 3b^2 + 3b + 1$ is the Root of a Number contained in n; and consequently b, according to this, is not the greatest Root, because $b + 1$ is greater; but this is contrary to the Supposition, and $\therefore g$ cannot be greater than $3b^2 + 3b$.

498. We now proceed to demonstrate the *Rule* given in *Art.* 479. for extracting the Cube Root.

Demonstration. We have already proved, † that the the Number of Figures in the Root is equal to the Number of Points in the Number whose Root is to be found; and also that ‡ the Root of the greatest Cube contained in the first (left Hand) Period, or Point, is the first Figure of the Root; which may be found by a bare Inspection of the Column of third Powers, in the Table ‖ of Powers); hence Part of the *Rule* is already demonstrated. Now having found the first Figure of the Root, if the Root consists but of two Figures, the first Figure in its compleat Value is that Figure with 0 on the right Hand † 490. 492. ‡ 494. ‖ 449.

of it; for this compleat Value put r, and x = the other Figure of the Root which is required; n = the Number whose Root is to be found; then, making $3r^2 + 3r$ a Divisor, we must * divide $n - r^3$ by it, so that x, the Quotient, may be such, as $3r^2x + 3rx^2 + x^3$ may not exceed $n - r^3$; and that our *Rule* is agreeable to this, is best shewn by an *Example*, *viz.* Let it be required to extract the Root of 16003, which is Part of the Number in *Art.* 481, with which *Article* compare what follows. The Root of the first Point 16 is 2, for $\overline{2})^3 = 8$; but, as the Number 16003 admits of two Figures in the Root, the first Figure 2 in its compleat Value is = 20 = r; and the subtracting 8 the Cube of 2 from the first Point 16 and bringing down the remaining Point, is in Effect the same as subtracting the r^3, *viz.* $\overline{20})^3 = 8000$ from 16003 or n, $\therefore n - r = 16003 - 8000 = 8003$ the Resolvend; then we took $2 \times 2 \times 300$ for the triple Square, and this is the same as $3r^2$, for this is $3 \times 20 \times 20 = 1200$; then we took 2×30 for the triple Quotient, which is the same as $3r$ or $3 \times 20 = 60$, the Sum of $1200 + 60 = 1260 = 3r^2 + 3r$ the Divisor, which is contained in the Resolvend 8003 six Times; but, according to the above Limitation, only five Times; hence, $x = 5$; then $3r^2x = 1200 \times 5 = 6000$, and $3rx^2 = 60 \times 5 \times 5 = 1500$, and $x = 5 \times 5 \times 5 = 125$; now $3r^2x + 3rx^2 + x^3 = 6000 + 1500 + 125 = 7625$ the Ablatitium; which, taken from the Resolvend, leaves 378, for a Remainder; whence we have shewn the Reason of the *Rule*, when there are but two Figures in the Root; and, in Order to shew it when there are three Figures in the Root, let it be required to extract the Cube Root of 16003008: Now we have found already the Root of the two first Periods, or Points, *viz.* 25; but taken in its compleat Value, as there must be three Figures, it is 250 = r; and we have already found the Resolvend $n - r^3$, n being now = 16003008, for we have already deducted the Cube of 25 from 16003, and taken down the

* 495.

the next Point 008; or, which is the same, the Cube of 250 from 16003008, and there remains 378008 for a Resolvend, or $n - r^3$; now the remaining Work will stand as under, which, compared with the Operation in *Art.* 481, will sufficiently shew the Agreement.

$$n - r^3 = 378008 \text{ New Resolvend}$$
$$3r^2 = 3 \times 250 \times 250 = 187500 \text{ New triple Square}$$
$$3r = 3 \times 250 = 750 \text{ New triple Quotient}$$
$$188250 \text{ New Divisor.}$$

Now take $x = 2$ then $\left.\begin{array}{l} \\ 3r^2x = 187500 \times 2 = \end{array}\right\}$ 375000

$$3rx^2 = 750 \times 2 \times 2 = 3000$$
$$x^3 = 2 \times 2 \times 2 = 8$$
$$378008 \text{ New Ablatitium.}$$
$$0$$

Whence the whole Root $= 252$, and the *Rule* true in 8 Figures; and the same Reasoning, in proceeding from Point to Point, will hold good in any Number of Figures.

499. We have already hinted, that Evolution may be proved by Involution, and also by casting out the Nines; for *Example*, let it be required to prove, that 252, just now found, is the Root of 16003008; first then by Involution $252 \times 252 \times 252 = 16003008$ for Profit; or by casting out the Nines from 16003008 there remains 0, and by casting the Nines out of 252 there remains 0, and ∵ this last Remainder 0×0 will be 0, &c. ∵ right; again, for Proof that 85 is the Cube Root of 164125, the Nines being cast out of 614125, there remains 1, and out of 85 there remains 4, and $4 \times 4 = 16$, out of which the Nine being cast, there remains 7, and $7 \times 4 = 28$, out of which the Nines being cast, the Excess is 1 as before, for Proof.

500. *Scholium.* Besides the Use already shewn of the Tables of the Square, and Cube Root, they will be very useful in finding the Root when of more than 3 Figures; for *Example*, let it be required to extract the Cube Root of 14366628991: This pointed will be

be 14366628991, and, by looking in the Table of Cubes, the Root of the greatest Cube contained in the three first Points 14366628 is found, by Inspection, to be 243, and its Cube 14348907, which, subtracted from 14366628, leaves 17721; to which annexing the next Point 991, the Resolvend will be 17721991; and now the Work will stand thus: By the Table of Squares 243 squared = 59049, which multiplied by 300 = 17714700 the triple Square, and $243 \times 30 = 7290$ the triple Quotient; and $17714700 + 7290 = 17721990$ the Divisor, which is contained in the Resolvend once, ∴ put 1 in the Quotient; and now, to perform the remaining Part of the Work according to *Art.* 482, we first reserve the Square of the Figure last put in the Quotient, *viz.* $\overline{1}|^2 = 1$; then the triple Quotient $7290 \times 1 = 7290$, and the triple Square 17714700; the Sum of these three Numbers = 17721991, which multiplied by the Figure last placed in the Quotient, *viz.* 1, = 17721991 the Ablatitium; which being the same as the Resolvend, the Remainder will be nothing; and ∴ 14366628991 is a perfect Cube, and its Root = 2431.

501. We shall not here give the *Rules* necessary for extracting the Roots of higher Powers, for these two Reasons: 1. Because they are of no Use in common Arithmetic; and, 2dly, because they will be much more intelligible in another Place, as will also a more compendious Method of extracting the Cube Root. We shall, therefore, only at present hint, concerning the Root of higher Powers, that, when the Index of any Power is composed of two or more Indices of the inferior Powers, the Root of the superior Power may be found by the continual Extraction of the Roots of the inferior Powers: To illustrate this by an *Example*, let it be required to extract the Root of the sixth Power of 2985984; now the Index of this Power 6 is compounded of 3 and 2, for $3 \times 2 = 6$; therefore, find the Cube Root of 2985984, which by the Table of Cubes is 144, and the

the Square Root, or Root of the second Power of this, by the Table of Squares, is = 12, the required Root.

CHAP. XXXVIII.

Of Single POSITION.

502. POSITION, or the *Rule* of *False*, as it is called by some Authors, is a Rule for working a particular kind of Questions, which cannot be easily solved by the other Rules of common Arithmetic. The Reason of its being called *Position*, or *Rule* of *False*, is because by the Supposition of one or two false Numbers, by this *Rule*, we are enabled to find the true; and, from the Number of Suppositions necessary to solve the *Question*, we say such *Question* is in *single*, or *double Position*.

503. *Single Position* (which is what we shall confine ourselves to in this Chapter) solves *Questions*, in which there is some Partition of Numbers into Parts proportional.

504. The Method of solving *Questions* in *single Position* is by supposing a Number for the Number sought, and working with this supposed Number, as if it was the true Number, to find the Result, according to the Conditions of the *Question*; and then, since we took the supposed Numbers in the same Proportion as the *Question* directs, that is, in the same Proportion as the true, or required ones, it is manifest, that the Ratio of the false Conclusion and supposed Number must be the same as the Ratio betwixt the true Result, or given Number, and the Number sought: Or, in other Words, as the Result is to the Number thought, so is the given Number to the Number sought. A few *Examples* will illustrate this *Article*, and clearly shew the Reason of this Proportion.

505. *Question* 1st. Suppose that, three Men *A*, *B*, *C*, being at a Tavern drinking, the Reckoning came to 12 Shillings, which was thus agreed to be paid; *viz.* that, as many 3 Pences as *A* paid, so many 2 Pences *B* should pay, and so many Pence *C* should pay: *Quære*, What ought each to pay? *Solution.*

Solution. Suppose C paid 1 $s.$

Then, *per Question*, $\begin{cases} B \text{ must pay } 2 \\ A \text{ must pay } 3 \end{cases}$

Sum $\overline{6}$

Here according to this Supposition they would pay in all but 6 Shillings, whereas they must all together pay 12 Shillings; however, it is manifest, that, if we had supposed twice as much for C, the Result would have been twice as much as it now is; and, if we had supposed 3 Times as much for C, the Result must have been 3 Times as much; and in general, if we had supposed n Times as much, the Result must have been n Times as much; that is, the Result would always increase or diminish in the same Ratio as the Supposition; ∵ the above *Rule* in *Art.* 504 is rational, and the remaining Part of the *Solution* to this *Question* will stand as follows, *viz.* As the Result 6 : the Number thought 1 :: the given Number 12 : the Number sought 2; ∵ C must pay 2 $s.$

Then, by the Conditions of the *Question*, $\begin{cases} B \text{ pays } 4 \\ A \text{ pays } 6 \end{cases}$

Sum $= \overline{12}$ for Proof.

506. *Question* 2. Admit there is 212 $l.$ 14 $s.$ 7 $d.$ to be divided amongst a Captain, 4 Men, and a Boy; the Captain to have a Share and Half, the Men each a Share, and the Boy $\frac{1}{3}$ of a Share: What ought each Person to have?

Solution. Here it will be proper to suppose a Number that we can take the $\frac{1}{2}$ and $\frac{1}{3}$ of, without leaving any Remainder; and 6 is such a Number, ∵ suppose each Man's Share 6 $d.$

The 4 Men would have $6 \times 4 = 24$

The Captain $6 + \frac{6}{2} = 6 + 3 = 9$

The Boy $\frac{1}{3}$ of $6 = 2$

Sum $= \overline{35}$

Now say, as 35 $d.$: 6 $d.$ :: 51055 $d.$ (the Pence in 212 $l.$ 14 $s.$ 7 $d.$) : 8752 $d.$ $\frac{10}{35}$ = 36 $l.$ 9 $s.$ 4 $d.$ $\frac{2}{7}$ = each Man's Share.

Hence

	£.	s.	d.
Hence each Man muſt have	36	9	4 $\frac{2}{7}$
Then 4 Men muſt have	145	17	5 $\frac{1}{7}$
The Captain	54	14	0 $\frac{3}{7}$
The Boy	12	3	1 $\frac{3}{7}$
Sum =	212	14	7 for Proof.

507. *Queſtion* 3. Let us ſuppoſe, that a Man, whoſe Wife was with Child by the firſt, being on his Death-Bed, made his Will in this Manner, *viz.* that his Effects, in Value 1200£. ſhould be divided in the following Manner ; (*i. e.*) if his Wife ſhould be delivered of a Son, the Son ſhould have $\frac{2}{3}$ and the Mother $\frac{1}{3}$; but, if it ſhould prove to be a Daughter, the Mother ſhould have $\frac{2}{3}$ and the Daughter $\frac{1}{3}$: But it happened that, after the Husband's Deceaſe, the Woman was brought to Bed of a Son and two Daughters: *Quære* how muſt the Effects be diſpoſed of, according to the Will of the Teſtator?

Solution. It is manifeſt that it was the Teſtator's Deſire, that the Mother ſhould have twice as much as a Daughter, and a Son twice as much as the Mother:

Hence if we ſuppoſe one Daughter to have	1 *l.*
The other muſt have	1
The Mother	2
The Son	4
Sum	8

∴ As 8*l.* : 1*l.* :: 1200*l* : 150*l.* = one Daughter's Share ; hence the Anſwer is

	£
One Daughter	150
The other	150
The Mother	300
The Son	600
Proof £	1200

CHAP.

CHAP. XXXIX.

Double POSITION.

508. WHEN a *Question* is not divided into Parts proportional, it cannot be solved by one Supposition, and therefore we are obliged to make another Supposition, for which Reason this *Rule* is called *double Position*.

509. The Method of working this *Rule*, is, by assuming two different Numbers for the Number sought, and working with them, separately, according to the Nature of the *Question*, to find their respective Errors, which if they were too great mark with ⊏; if too little with ⊐: Then multiply the first Supposition by the second Error, and the second Supposition by the first Error; and if the Errors are alike, that is both too great, or both too little, divide the Difference of these Products by the Difference of the Errors, and the Quotient will be the Number sought: But if the Errors are unlike, that is, one too great, the other too little, divide the Sum of the Products by the Sum of the Errors, and the Quotient will give the required Number. Or mind this memorial *Rule*:

Unlike Signs Addition doth desire,
Alike Signs Substraction doth require.

510. In the *Solution* of *Questions* by the above *Rule*, it is supposed, that, as the first Error is to the second Error, so is the Difference between the first Supposition and the Number sought to the Difference between the second Supposition and the required Number. For otherwise the *Question* will not admit of a *Solution* by this *Rule*, it being entirely founded on this Supposition: But that any *Question* may be solved by the *Rule* delivered in the last *Article*, when there is such

a Pro-

a Proportion as we have given in this *Article*, is demonstrated in the underwritten Note *.

511, *Question* 1.

† When first the Marriage Knot was ty'd,
Betwixt my Wife and me;
My Age did her's as far exceed,
As three Times three doth three:

But,

* *Demonstration*. We are now to shew that any *Question* which admits of the Proportion mentioned in this *Article*, may be solved by *Art*. 509. In Order to which, let $x =$ the Number sought; a and $b =$ the two Suppositions, m and n $=$ their respective Errors. This admits of three *Cases*.

Case 1. When the Suppositions are both too little. Then,
by the Definition, as $m : n :: x - a : x - b$, ∵ $mx - mb$* $= nx$ * 185.
$- na$; ∵ adding na to both Sides of this Equation gives $mx +$
$na - mb$ † $= nx$; and, subtracting mx from both Sides of † 22.
this Equation, we have $na - mb$‡ $= (nx - mx =) \overline{n - m} \times x$; ∴, ‡ 36.
dividing both Sides by $n - m$, we get $\frac{na - mb}{n - m}$ ‖ $= x$. Which ‖ 108.
is a Demonstration of one *Case*, of the latter Part of this *Article*.

Case 2. When the Errors are both too great.
Let n be $\sqsubset m$; by the Supposition, as $m : n :: a - x : b - x$;
∵ $m \times \overline{b - x}$ ¶ $= n \times \overline{a - x}$, or, which is the same, $mb - mx$ ¶ 185.
$= na - nx$; and, by adding nx to each Side of the Equation,
we shall get $mb + nx - mx$ * $= na$; and from this subtracting * 22.
mb gives $nx - mx$ † $= na - mb$, or, which is the same, $\overline{n - m} \times x =$ † 36.
$na - mb$; whence, dividing by $n - m$ will give x ‡ $=$ ‡ 108.
$\frac{na - mb}{n - m}$. Which is agreeable to the Rule in *Article* 509.
Q. E. D.

Case 3. When the (Suppositions or) Errors are one too great, the other too little.
Let $a \sqsubset b$. By the Supposition as $m : n :: a - x : x - b$;
∵ $mx - mb$ ‖ $= na - nx$; and, if to each Side of this Equa- ‖ 185.
tion we add nx, we shall find that $mx + nx - mb$ § $= na$; § 22.
and to each Side of this Equation adding mb will give this
Equation $mx + nx$ ¶ $= na + mb$; or, which is the same, ¶ 22.
$\overline{m + n} \times x = na + mb$; which last Equation, being divided
by $m + n$, will give x * $= \frac{na + mb}{m + n}$, which is the same as * 108.

the

But, after ten and half ten Years
 We Man and Wife had been,
Her Age came up as near to mine,
 As Eight is to Sixteen.
Now tell me (you who can) I pray,
What were our Ages on the Wedding-Day ?

Solution. Suppose, when first the Marriage Knot was tied,

	Yrs.	Yrs.
The Wife's Age	1 or	2
Then by the *Question* his Age was	3	6
15 Years after, the Wife must be	16	17
15 Years after, the Husband must be	18	21
Twice the Wife's Age then =	32	34
Twice the Wife's Age then is greater than the Husband's	14	13

Here according to these two Suppositions, after they had been married fifteen Years, twice the Wife's Age was greater than the Husband's 14 and 13 respectively, whereas it ought by the *Question* to have been equal; ∵ the first Error is $14 \sqsubset^{-}$; and the second $13 \sqsubset^{-}$; whence, the Signs being both alike, we subtract the Product of 13×1 from 14×2, *i.e.* $28 - 13 = 15$; and the Difference of the Errors is $14 - 13 = 1$ for the Divisor; which being an Unit, the Dividend 15, divided by it, will give 15 in the Quotient; and ∵ the Wife's Age on the Marriage-Day was 15 Years; the Husband's (three Times as many, *viz.*) 45; and, adding 15 Years to each, she would be 30 Years, he 60, just twice her Age for Proof.

512. *Question* 2.

** A Man that was idle and minded to spend
Both Money and Time, went to drink with his Friend:
He

the *Rule* in 509. directs. And consequently we have now compleatly demonstrated, that any *Question* in which there is such a Proportion as is mentioned in *Article* 510, can be solved by the *Rule* laid down in *Article* 509. *Q. E. D.*

† *Question* 1. is from the *Monthly Entertainments*, for *January* 1711, but the Operation is not there inserted.

** By Mr. *Leadbetter*, in the *Monthly Entertainments* 1711. The *Solution* is not there inserted.

He said to his Host, if you'll now to me lend
As much Coin as I have, then my Sixpence I'll spend:
His Host lent the Money, his Sixpence he spent,
And, having so done, to another House went,
Where the same he requested, and the same Sum
[he spent.
He went to a third House, where, Landlord, cries he,
Lend me as much Money as I've left here you see;
Which having received, his Sixpence he spent,
So, all being gone, Home the Fuddle-Cap went,
To cast up his Reck'nings; but, his Head aching sore,
He begs you to do't, and he'll do so no more;
What had he at first, and how much is on Score?

Solution. If we suppose he had at first 6*d*. and 7*d*; then these two Suppositions, worked according to the Nature of the *Question*, will stand thus:

	d.		*d.*
Had first	6	or	7
First lent	6		7
	—		—
Had then	12		14
First spent	6		6
	—		—
Had left	6		8
Second lent	6		8
	—		—
Had then	12		16
Second spent	6		6
	—		—
Had left	6		10
Third lent	6		10
	—		—
Had then	12		20
Third spent	6		6
	—		—
Remains	6		14
	—		—

Here remains 6*d*. and 14*d*, but by the *Question* nothing should remain; ∵ the first Error is 6*d*, the Second 14*d*; both too great. Hence by *Art*. 509. the first Supposition 6 × by the second Error 14 being = 84, and the second Supposition 7 × by the first Error 6 being = 42, we have 84 — 42 = 42 for the Dividend, and the Difference of the Errors, *viz.* 14 − 6 = 8 for the Divisor, ∴ 42 ÷ 8 = 5*d* ∴ the Money he had at first.

	d.
Hence he had at first	$5\frac{1}{4}$
First lent	$5\frac{1}{4}$
Had then	$10\frac{1}{2}$
First spent	6
Had left	$4\frac{1}{2}$
Second lent	$4\frac{1}{2}$
Had then	9
Second spent	6
Had left	3
Third lent	3
Had then	6
Third spent	6
Had left for Proof	0

To cast up what he owes.

	d.
First lent	$5\frac{1}{4}$
Second lent	$4\frac{1}{2}$
Third lent	3
owes	$12\frac{3}{4}$

Hence it appears that he had at first $5\,d.\,\frac{1}{4}$, and has $12\,d.\,\frac{3}{4}$ on Score.

513. *Question* * 3.

A Painter, of Skill and much Fame in the Town,
Had procur'd himself Work for more Hands than his [own.
He employ'd an Assistant, to help him in Part;
A Proficient in every Branch of his Art.
O'er a Glass of good Wine upon Terms they debate,
And the Bottle was drain'd while they state and unstate.
For as Plenty of *Bacchus's* enliv'ning Juice
Does most commonly Projects and Whimsies produce;
So, when that their Spirits grew warm with the Liquor,
Fresh Maggots were started, and Fancies flow'd [quicker.
They were long in contriving what both Sides could [please,
At length the Proposals agreed on were these;

For

* This was proposed Mr. *William Massey*, in the *Ladies Diary*, and answered in the succeeding one by an Algebraic Process.

For a single Year's Service the Man should be ty'd;
And, for every Day that he full was employ'd,
Seven Shillings each Day should his Wages be paid;
And, for all such as those when he rested or play'd,
He should forfeit three Shillings: The Year was com-[pleat,
Neither Master nor Man was in each other's Debt.
Now what Time he neglected, young Artists, is sought,
And how much for his Master in Painting he wrought?

Solution. Suppose the Man worked 100 Days, then he played 365 — 100 = 265 Days; ∴ his Wages would be 700 Shillings, and his Forfeits 795 Shillings; hence the first Error is 795 — 700 = 95 ⊐. Again, suppose he worked 120 Days, then he played 365 — 120 = 245 Days, whence, according to this Supposition, his Wages would be 840*s*, and his Forfeits 735 Shillings; ∴ the second Error will be 840 — 735 = 105 ⊏; then the Operation by *Art.* 509. will be as under:

The 1st Supposition	100	2d Supposition	120
x by the 2d Error	105	x by the 1st Error	95
Product	10500		600
			1080
The Errors { 95 ⊐ / 105 ⊏		Product	11400
Sum	200	Products { 10500 / 11400	
		Sum	21900

21900 ÷ 200 = 109½. Hence the Answer is, the Man worked 109½ Days, and played 365 — 109½ = 255½ Days. For Proof, 109½ Days at 7*s.* is 766*s.* 6*d.*; and 255½ Days at 3*s.* is also = 766*s.* 6*d.*

514. *Question* * 4.

A Gentleman has an Orchard of Fruit Trees, one Half of the Trees bearing Apples, one Fourth Pears,

* This is *Question* 352, in the *Ladies Diary*, 1752.

one Sixth Plums, and fifty of them bearing Cherries; how many Fruit Trees in all grew in the said Orchard?

Solution. Here we may supposed any Numbers, but, to avoid Fractions, such as can be divided without a Remainder by 2, 4, and 6; and such Numbers are 12 and 24, which being taken for the two Suppositions, the Work will be as follows:

	Trees	Trees
Suppose there are in the Orchard	12	24
$\frac{1}{2}$ Apples	6	12
$\frac{1}{4}$ Pears	3	6
$\frac{1}{6}$ Plums	2	4
Sum of the Apples, Pears, and Plums	11	22
Hence according to this the Cherries must be	$1 = 12 - 11$	$2 = 24 - 22$.

But the Cherries should be 50, and ∵ the first Error is $50 - 1 = 49$ ⊐, and the second Error $= 50 - 2 = 48$ ⊐; now the first Supposition multiplied by the second Error $= 12 \times 48 = 576$, and the second Supposition by the first Error $= 24 \times 49 = 1176$; ∵ $1176 - 576 = 600$ = the Dividend, the Errors being both alike; and the Difference of the Errors $49 - 48 = 1$ for the Divisor; ∵ the Number of Trees in the Orchard is 600, as may be easily proved by the *Question*.

515. *Question* 5. Suppose a Maid, carrying Apples to Market, was met by three Boys, and that the First took Half that she had, but returned 10; that the Second took one Third that she then had, but returned two; lastly, that the Third took away Half that she had left, but returned her one; and, when she she was got clear, she had twelve Apples left: What Number of Apples had she at first?

Solu-

Solution. Suppose she had

	Apples		Apples
At first	100	or	70
First took $\frac{1}{2}$	50		35
Returned	10		10
She then had	60		45
Second took $\frac{1}{3}$	20		15
Then she had left	40		30
The Second returned	2		2
She had then	42		32
Third took $\frac{1}{2}$	21		16
Then she had left	21		16
Third returned	1		1
Whence by this she had at last left	22		17

Hence the Errors are 10 ⊏ and 5 ⊏

Now $100 \times 5 = 500$, and $70 \times 10 = 700$, $\because 700 - 500 = 200 =$ the Dividend; and $10 - 5 = 5 =$ the Divisor; $\because 200 \div 5 = 40 =$ the Number of Apples the Maid had at first, as may be easily proved.

516. *Scholium.* *Questions* of the Nature of the above may be solved independent of *Art.* 509, by working in a retrograde Order; which Method we will illustrate in a *Solution* of the last *Question.* To proceed then, by the *Question*, when she was got clear, she had 12 Apples left; therefore, before the last Boy returned her one, she had but 11; at which Time she had as many as the Boy, (because the Boy took $\frac{1}{2}$) and consequently, before she met with that Boy, she had $11 \times 2 = 22$ Apples: Hence, before the second Boy gave her back two, she could have but $22 - 2 = 20$; at which Time the *Question* says, she had $\frac{2}{3}$, and the Boy $\frac{1}{3}$; $\because$ as 2 Thirds : 20 :: 3 Thirds : $\frac{20 \times 3}{2} = 30 =$ the

 Num-

Number of Apples she had before she met with the second Boy; and ∴, before the first Boy returned 10, she had but $30 - 10 = 20$, at which Time by the *Question* the Boy had as many as she; consequently the Apples the Maid had at first were $20 \times 2 = 40$ Apples. *Q. E. I.*

517. It will be proper, before we conclude this Chapter, to shew what Kind of *Questions* are, and what Sort are not solvable by the *Rule* in *Art.* 509. First, then, when the *Question*, proposed to be solved, requires the Number sought to be augmented by the Addition of, or Multiplication by some given Number; or decreased by the Subtraction of, or Division by a given Number; there will be such * Analogy as

* The *Demonstration* of this may be conveniently parted into four *Cases*.

Case 1. When the Number sought is to be augmented by the Addition of the given Number.

Demonstration. Let $x =$ the Number sought, a and $b =$ the two Suppositions, $c =$ the Number to be added; then these Numbers by the Addition of c will become $x+c$, $a+c$, and $b+c$; ∴ the first Error is $\overline{x+c} \backsim \overline{a+c} = x \backsim a$, and the second Error is $\overline{x+c} \backsim \overline{b+c} = x \backsim b$. The Difference betwixt the Number sought and first Supposition is $x \backsim a$, and betwixt the Number sought and the second Supposition is $x \backsim b$. Consequently in this *Case* it is, as the first Error : the second Error :: the Difference betwixt the first Supposition and Number sought : the Difference betwixt the second Supposition and Number sought. *Q. E. D.*

Case 2. When the Number sought is to be multiplied by a given Number.

Demonstration. Let $x =$ the Number sought, a and $b =$ the two Suppositions, $c =$ the given Multiplier; then by multiplying by c we shall have xc, ac, bc; ∴ the first Error is $xc \backsim ac$, the second Error $xc \backsim bc$. But $\frac{xc \backsim ac}{xc \backsim bc} =$ (by dividing both the

* 184. Numerator and Denominator by c) $\frac{x \backsim a}{x \backsim b}$ ∴ as * $xc \backsim ac : xc \backsim bc :: x \backsim a : x \backsim b$. Which is the Analogy mentioned in *Art.* 510. *Q. E. D.*

Case 3.

as is mentioned in *Art.* 510, and, therefore, such *Questions* can be solved by that * *Rule.* * 510.

518. But, when, according to the Nature of the *Question*, a given Number is to be divided by the Number sought (or any Part thereof), or when the Number sought (or any Part of it) is to be squared, or cubed, &c. (or when some Parts of the Number sought are to be multiplied together) or, lastly, when the Square Root or Cube Root, &c. of the Number sought, or any Part thereof, is to be extracted; there will not * be such an Analogy as is mentioned in *Art.*

Case 3. When a given Number is to be subtracted from the Number sought.

Demonstration. Let $x =$ the Number sought, a and $b =$ the two Suppositions, $c =$ the Number to be subtracted; whence, by subtracting c, these Numbers will become $x - c$, $a - c$, and $b - c$; $\because$ the first Error is $\overline{x-c} \backsim \overline{a-c} = x \backsim a$; and the second Error is $\overline{x-c} \backsim \overline{b-c} = x \backsim b$; hence as in the Demonstration of *Case* 1. *Q. E. D.*

Case 4. When the Number sought is to be divided by a given Number.

Demonstration. Let $x =$ the Number sought, a and $b =$ the two Suppositions, $c =$ the Divisor; then we shall have $\frac{x}{c}$, $\frac{a}{c}$, $\frac{b}{c}$; $\because$ the first Error is $\left(\frac{x}{c} \backsim \frac{a}{c} = \right) \frac{x \backsim a}{c}$; the second Error $= \frac{x \backsim b}{c}$; but $\frac{x \backsim a}{c} \div \frac{x \backsim b}{c} = \frac{x \backsim a}{x \backsim b}$ (if this be not of itself sufficiently plain, see Division of Fractions.) $\because$ * $\frac{x \backsim a}{c} : \frac{x \backsim b}{c} :: x \backsim a : x \backsim b$. Which is the same Analogy as that in *Article* 510. *Q. E. D.* * 184.

* The *Demonstration* of this may be conveniently parted into three *Cases.*

Case 1. When a given Number is to be divided by the Number sought.

Let $x =$ the Number sought, a and $b =$ the two different Suppositions, $c =$ the given Number; then we have $\frac{c}{x}$, $\frac{c}{a}$, and $\frac{c}{b}$.

510, and, therefore, we are not to expect to solve such *Questions* by the *Rule* in *Art.* 509.

519.

$\frac{c}{b}$ ∴ the first Error is $\left(\frac{c}{x} \backsim \frac{c}{a} = \text{by Reduction of Fractions}\right)$ $\frac{ca \backsim cx}{xa}$, and the second Error is $\left(\frac{c}{x} \backsim \frac{c}{b} =\right)\frac{bc \backsim cx}{xb}$. Hence, if there is such an Analogy as is required in *Art.* 510, it will be, as $\frac{ca \backsim cx}{xa} : \frac{bc \backsim cx}{xb} :: a \backsim x : b \backsim x$; but, if these four Quantities are in direct Proportion, the Product of the Ex-

* 185. tremes must be * equal to the Product of the Means, that is, $\frac{b \backsim x \times ca \backsim cx}{xa} = \left(\frac{bc \backsim cx \times a \backsim x}{xb} =\right)\frac{b \backsim x \times ca \backsim cx}{xb}$; and, dividing both Sides of this Equation by $b \backsim x \times ca \backsim cx$, we

† 108. shall have $\frac{1}{xa}$ † $= \frac{1}{xb}$; whence it is evident that, according to this, b and a must be equal, which is contrary to the Supposition, for then they would not be two different, but only one Supposition; and ∴ there is not the required Analogy in this *Case*.

Case 2. When the Number sought is to be squared, cubed, &c.

We will demonstrate this in the second Power, and in the same Manner it may be done in the third or higher Power.

Let $x =$ the Number sought, and a, $b =$ the two Suppositions; then we have x^2, a^2, b^2; and ∴ the first Error is $x^2 \backsim a^2$, and the second Error $x^2 \backsim b^2$. Now let us suppose that there is such Analogy as is required in *Art.* 510, then it will be, as $x^2 \backsim a^2 : x^2 \backsim b^2 :: x \backsim a : x \backsim b$. But, when four Numbers are in direct Proportion, the Product of the

‡ 185. Means is equal to the ‡ Product of the Extremes, ∴ $x^2 \backsim a^2 \times x \backsim b = x^2 \backsim b^2 \times x \backsim a$; both Sides of this Equa-

‖ 108. tion, being divided by $x \backsim b$, give $x^2 \backsim a^2$ ‖ $= x + b \times x \backsim a$;

§ 108. and this divided by $x \backsim a$ will shew that $x + a$ § $= x + b$; and by subtracting x from both Sides of this Equation, we shall

¶ 36. have a ¶ $= b$, which is contrary to the Hypothesis, (for a and b were supposed two different Numbers) and therefore absurd. *Q. E. D.*

After the same Manner the Impossibility may be demonstrated, when the unknown Number is to be cubed, &c.

Case 3.

519. Though it appears by the laſt *Article*, that the following *Queſtions* cannot be ſolved by the *Rule* in *Article* 509, yet they may be transformed into others, which may be ſolved by that *Article*. The *Queſtions* which we ſhall here propoſe, are, *Queſtion* 1ſt.

What Number is that, by which if 213 be divided, the Quotient will be 71?

Solution. Though it appears by the laſt *Article*, that this *Queſtion* cannot be ſolved by *Art*. 510; yet it may be transformed into another, which is reſolvable by that *Rule*; for the Quotient multiplied by the Diviſor is equal to * the Dividend; and therefore the *Queſtion* may be transformed into this: What Number is that which multiplied by 71, the Product * 123.
will

Caſe 9. When the Square Root, Cube Root, &c. of the Number ſought, is to be extracted.

Demonſtration. This we will demonſtrate in the Square Root.

Let $x =$ the Number ſought, a and $b =$ the two Suppoſitions; then we have $\sqrt{x}$, $\sqrt{a}$, $\sqrt{b}$; ∵ the firſt Error is $\sqrt{x} : \backsim \sqrt{a}$; and the ſecond Error is $\sqrt{x} : \backsim \sqrt{b}$; hence, admitting that there is ſuch an Analogy as is required in *Article* 510, it will be, as $\sqrt{x} : \backsim \sqrt{a} : \sqrt{x} \backsim \sqrt{b} :: x \backsim a : x \backsim b$. Hence, making the Product of the Means * = the Product of the * 185.
Extremes, we ſhall have $\overline{\sqrt{x} \backsim \sqrt{a}} \times \overline{x \backsim b} = \overline{\sqrt{x} \backsim \sqrt{b}} \times \overline{x \backsim a}$; this divided by $\sqrt{x} \backsim \sqrt{a}$ gives $x \backsim b$ † $= \overline{\sqrt{x} \backsim \sqrt{b}} \times \overline{\sqrt{x} + \sqrt{a}}$; and dividing both Sides of this Equation by † 108.
$\sqrt{x} \backsim \sqrt{b}$ gives $\sqrt{x} : + \sqrt{b} :$ ‡ $= \sqrt{x} : + \sqrt{a}$; and, by ſub- ‡ 108.
tracting $\sqrt{x}$ from Side of this Equation, we ſhall have $\sqrt{b} = \sqrt{a}$; and by ſquaring each Side of this Equation we ſhall have (the Root × by the Root being = the Square) b ‖ $= a$; which is abſurd, for, if $a = b$, then there is but one ſuppoſed Number, whereas we have been diſcourſing of two; and conſequently theſe Kind of *Queſtions* do not admit of the Analogy mentioned in *Art*. 510, and ∵ we are not to expect to ſolve them by *Art*. 509. ‖ 452.

Note. After the ſame Manner the Impoſſibility may be ſhewn when the Cube Root, or the Root of any higher Power, is to be extracted.

will be 213? Now this may be solved by (single or double) *Position*, and the Answer will be 3. *Note*, it might have been found by dividing 213 by 71.

520. *Question* 2. If, at the Time of making this *Question*, *viz.* the 14th of *June* 1753, the Author's Age be multiplied by $\frac{1}{8}$ of his Age, the Product will be equal to 72 Years: *Quære* the Author's Age at that Time?

As this *Question* by *Art.* 518. cannot be solved by *Position*, proceed thus: By the *Question* it is manifest, that $\frac{1}{8}$ of the Square of the Number sought is $=72$; ∴ first find an Answer to this *Question*, What Number is that which divided by 8 is $=72$? Which may be found by (either single or double) *Position*, to be 576; or it may be found thus, $72 \times 8 = 576$. And it is evident, that as the Number 576, just now found, is = the Square of the Number sought; the Square Root of 576, *viz.* 24 Years, is = the Author's Age, which was required.

521. *Question* 3. What Number is that, to which if 2 is added, the Square Root of that Sum will be 7?

Solution. Here it is evident, that, before the Square Root is extracted, the Sum must be equal to the Square of 7, *viz.* $= 7 \times 7 = 49$; ∴ let us first find an Answer to this *Question*, What Number is that, to which if 2 be added, the Sum will be $= 49$? For the Answer to this, it is manifest, will be also the Answer to the proposed *Question*. And the Answer to this last *Question* may be found by *Art.* 509; but easier, by subtracting 2 from 49, which gives 47 for the Number sought.

522. But if, at any Time, we meet with a *Question* which cannot be solved by the common Rules of Arithmetic, nor by *Art.* 509; nor be transformed into one which may be solved by that *Art.* then we must have Recourse to Algebra, the Elements of which admirable Art will be treated of hereafter.

523. It may not be improper here to hint, that, in solving *Questions* by *Position*, the lesser the Numbers supposed are, the shorter will the Operation be.

524. We shall end this Chapter with the following Remark, *viz.* There is another Method of finding the Number sought, used by many Authors, and in Effect the same as that in *Art.* 509. It is this, say, as the Difference of the Errors, if alike, or Sum, if unlike, is to the Difference of the Suppositions, so is either Error to a fourth Number; which added to, or subtracted from, the proper Supposition, will give the Number sought. This may be demonstrated in nearly the same Manner as we have demonstrated *Art.* 509; for which Reason, and because we have already been much longer on this Chapter than was intended, we shall omit it in this Place.

CHAP. XL.

A METHOD *of finding* MULTIPLES, *or* NUMBERS, *which may be divided by given* NUMBERS, *without* REMAINDERS, *&c.*

525. IN *Position*, it is many Times necessary, in Order to avoid Fractions, to suppose such Number, or Numbers, as, being divided by given Divisors, shall leave no Remainder. And, as such Numbers do not always readily occur to the Mind, it may not be improper to insert this Chapter, which will remove that Obstacle, as well as solve several pleasant Questions, not reducible to any of the foregoing Heads of Arithmetic.

526. A Prime Number is such a Number as is only measured by Unity; or, in other Words, a Prime Number is such a Number as cannot be produced by the Multiplication of two, or more, Integers.

527. *Question* 1. Let it be required to find a Number, which can be divided by 2, 4, and 6, without a Remainder?

Solution.

Solution. It is evident, that $2 \times 4 \times 6 = 48$ may be divided, without a Remainder, by the given Divisors, and ∵ 48, $48 \times 2 = 96$, $48 \times 3 = 144$, &c. will all answer the *Question.* But the Method of finding the least Number that can be divided by given Divisors, without any Remainder, is explained in the next *Question.*

528. *Question* * 2. What is the least Number that can be divided by the nine Digits, without a Remainder?

Solution, First, then, $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 36280$ may be divided by the nine Digits. But, in Order to have the least Number, observe, that 4 is the Square of 2, 6 a Multiple of 2 and 3, and 8 is the Cube of 2, and 9 a Square of 3; and therefore, by omitting the composed Numbers, and putting the Roots of 4, 8, and 9, *viz.* 2, 2, and 3, for those Numbers respectively, the above continued Product will stand thus, $1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3 = 2520$ the least Number which will admit of the Conditions of the *Question.* And, having found the least Number, we may find other Numbers at Pleasure, by multiplying 2520 by 2, 3, 4, &c.

529. Hence may be deduced this general *Rule*, for finding the least Number that can be divided by given Divisors, without a Remainder: The Number required is equal to the continued Product of all the prime Numbers, and lowest Roots (of such of the Numbers as are integral Squares, Cubes, &c.) to the Height of the greatest given Divisor.

For the Reason of this *Rule* may be shewn, from the last *Article*, thus: It is evident that $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 = 36280$ may be divided by the nine Digits. But since 4 is the Square of 2, or equal to 2×2, if we only write one of the two's, instead of the 4, there will be two 2's, Multipliers, and ∵ $1 \times 2 \times 3 \times 2$, or, which is equal to it, $1 \times 3 \times 4$, can be divided by 4 without a Remainder; the next Number which is not a prime Number is 6, which is composed

* This was *Question* 73, in the *Lady's Diary*, 1719.

posed of 2 and 3, *viz.* $2 \times 3 = 6$; but, as we have already a 2 and 3 amongst the Multiplier, it is plain, we may omit the 6 entirely; for it is manifest, that $1 \times 2 \times 3 \times 2 \times 5$, being $= 1 \times 2 \times 5 \times 6$, can be divided by 6. The next Number which is not a Prime is 8, which is the Cube of 2, for $2 \times 2 \times 2 = 8$; but we have two 2's already in $1 \times 2 \times 3 \times 2 \times 5 \times 7$; and ∴, if we only put in one of these three 2's, we shall have three 2's Multipliers; and ∴ $1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2$, or, which is equal to it, $1 \times 3 \times 5 \times 7 \times 8$, can be divided by 8; lastly, the 9 is the Square of 3, or composed of 3 and 3, *viz.* $= 3 \times 3$; but we have one 3 a Multiplier already; and ∴, if we only put in one of these two 3's, we shall have $1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3$, which, being $= 1 \times 2 \times 2 \times 5 \times 7 \times 2 \times 9$, can be divided by 9; and consequently $1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3 = 2520$ can be divided by the nine Digits, without leaving any Remainder; or, in other Words, each of the nine Digits will measure it. And that it is the least Number capable of being so divided, is plain, by only considering that we have either omitted, or brought as low as possible, all Numbers which were not prime Numbers; that is, all such as admitted of being reduced.

530. *Question* *3.

A Country Girl to Town did go
Some Walnuts for to sell;
A Gentleman she chanc'd to meet,
And thus it her befel;
My pretty Maid, says he to her,
What Number have you here?
I can't tell, Sir, says she to him;
But this I'll make appear,
I told them o'er e're I came out
By Six's, Five's, Four's, Three's, Two's,
And, ev'ry Time I number'd them,
One remain'd Overplus;
I told them o'er by Sevens at last,
And there were no Remains;

If

* From a *Magazine* unanswered.

If you can find the Number out,
Pray take it for your Pains.

Solution. We must first find the least Number, that can be divided by 2, 3, 4, 5, 6, without a Remainder; which by *Art.* 529. may be found thus: First, $2 \times 3 \times 4 \times 5 \times 6$ it is evident can be divided by the given Divisors 2, 3, 4, 5, 6, without a Remainder; but 4 is a Square whose Root is 2; and 6 is composed of 2 and 3 or $2 \times 3 = 6$; $\therefore 2 \times 3 \times 2 \times 5 = 60$ is the least Number, that will admit of such Conditions; $\therefore$ if we add 1 to it, it is plain, that $60 + 1 = 61$, being divided by 2, 3, 4, 5, 6, will have 1 remain; but this is not the required Number, because, if divided by 7, there will be a Remainder, whereas by the *Question* there should not be any; therefore, we must seek other Numbers which may be divided by 2, 3, 4, 5, 6, without a Remainder; which may be found by multiplying 60 by 2, 3, 4, 5, *&c.* respectively, and to the Products add one; and the Numbers will be 121, 181, 241, 301, *&c.* but, by dividing them severally by 7, it will be found, that 301 is the first, or least Number, that admits of the Conditions of the *Question*; and therefore we may suppose the Maid had 301 Walnuts in her Basket. For the next Number that will admit of the required Conditions, will be too many for a common Basket to hold; for, if we multiply 60 by 6, 7, 8, 9, *&c.* respectively, and add 1 to the several Products, the next Number that can be divided by 7, will be found, after 7 Multiplications, to be $60 \times 12 + 1 = 721$ for the Number of Walnuts. Now, since the second Number is found after 7 Multiplications, $60 \times 7 = 420$ is the Difference, which, added successively to 301, will give as many Numbers, that may be divided according to the Conditions of the *Question*, as we please to have, as 301. 721. 1141. 1561. 1981. 2401. 2821. 3241. *&c. ad infinitum.*

531. *Question* * 4. To find the least Number of Guineas which, being divided by 6, 5, 4, 3, and 2 respectively, shall leave 5, 4, 3, 2, and 1 respectively remaining.

Solution.

* *Question* 296, in the *Lady's Diary*, 1748.

Solution. By the last *Article* it appears that 60 is the least Number that can be divided by 5, 6, 4, 3, and 2 without a Remainder; and, consequently, $60 - 1 = 59$ = the required Number.

532. *Question* * 5. Required the three least Numbers, which, divided by 20, shall leave 19 for a Remainder; but, if divided by 19, shall leave 18; if divided by 18, shall leave 17; and so on (always leaving one less than the Divisor) to Unity?

Solution. By the general *Rule**. Of the Divisors, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, the following 1, 2, 3, 5, 7, 11, 13, 17, and 19 are Primes. And 4 is a Square whose Root is 2; 8 a Cube, the Root 2; 9 a Square, the Root 3; 16 a fourth Power, the Root 2; (for $2 \times 2 \times 2 \times 2 = 16$); the other Divisors are composed Numbers, and ∴ omitted; hence by the general *Rule* $1 \times 2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3 \times 11 \times 13 \times 2 \times 17 \times 19 = 232792560$ the least Number that can be divided by the given Divisors, without a Remainder; and $232792560 \times 2 = 465585120$; also $232792560 \times 3 = 698377680$, being divided by the given Divisors, will leave no Remainders; and consequently, by deducting Unity, the three Numbers required are 232792559, 465585119, and 698377679. Agreeing with the ingenious Mr. ROBERT ROBINSON's Algebraic Process, in the *Gentleman's Diary*, 1748. * 529.

533. *Question* 6. Required the least Number that, being divided by 9, shall leave for a Remainder 6; if divided by 8, the Remainder will be 5; if divided by 7, the Remainder shall be 4; and so on each Time leaving for a Remainder 3 less than the Divisor, till, divided by 3, the Remainder will be nothing.

Solution. Here the Divisors are 3, 4, 5, 6, 7, 8, 9, but we must in finding the Number consider all the nine Digits; now by *Art*. 528. it appears, that 2520 is the least Number that can be divided without a Remainder; consequently $2520 - 3 = 2517$ is the Number sought.

534.

* This is *Question* 61, in the *Gentleman's Diary*, 1747.

534. *Scholium.* There are many *Questions* concerning Divisors which cannot be solved by this *Rule*; and, if the Learner should meet with any, which through the Irregularity of the Divisors or Remainders cannot be solved by this *Rule*, he ought to be contented till we come to Algebra; in which delightful Art, when we treat of unlimited Questions, or prime Numbers, we shall endeavour to explain a Method for solving all possible *Questions* concerning Divisors.

535. We will conclude this Chapter with the following useful TABLE.

A

A TABLE of the PRIME NUMBERS between 1 and 10000.

2	163	379	613	863	1117	1423	1663
3	167	383	617	877	1123	1427	1667
5	173	389	619	881	1129	1429	1669
7	179	397	631	883	1151	1433	1693
11	181	401	641	887	1153	1439	1697
13	191	409	643	907	1163	1447	1699
17	193	419	647	911	1171	1451	1709
19	197	421	653	919	1181	1453	1721
23	199	431	659	929	1187	1459	1723
29	211	433	661	937	1193	1471	1733
31	223	439	673	941	1201	1481	1741
37	227	443	677	947	1213	1483	1747
41	229	449	683	953	1217	1487	1753
43	233	457	691	967	1223	1489	1759
47	239	461	701	971	1229	1493	1777
53	241	463	709	677	1231	1499	1783
59	251	467	719	983	1237	1511	1787
61	257	479	727	991	1249	1523	1789
67	263	487	733	997	1259	1531	1801
71	269	491	739	1009	1277	1543	1811
73	271	499	743	1013	1279	1549	1823
79	277	503	751	1019	1283	1553	1831
83	281	509	757	1021	1289	1559	1847
89	283	521	761	1031	1291	1567	1861
97	293	523	769	1033	1297	1571	1867
101	307	541	773	1039	1301	1579	1871
103	311	547	787	1049	1303	1583	1873
107	313	557	797	1051	1307	1597	1877
109	317	563	811	1061	1319	1601	1879
113	331	569	821	1063	1321	1607	1889
127	337	571	823	1069	1327	1609	1901
131	347	577	827	1087	1361	1613	1907
137	349	587	829	1091	1367	1619	1913
139	353	593	839	1093	1373	1621	1931
149	359	599	853	1097	1381	1627	1933
151	367	601	857	1103	1399	1637	1949
157	373	607	859	1109	1409	1657	1951

1973	2269	2579	2857	3209	3529	3833	4153
1979	2274	2591	2861	3217	3533	3847	4157
1987	2281	2593	2879	3221	3539	3851	4159
1993	2287	2609	2887	3229	3541	3853	4177
1997	2293	2617	2897	3251	3547	3863	4201
1999	2297	2621	2903	3253	3557	3877	4211
2003	2309	2633	2909	3257	3559	3881	4217
2001	2311	2647	2917	3259	3571	3889	4219
2017	2333	2657	2927	3271	3581	3907	4229
2027	2339	2659	2939	3299	3583	3911	4231
2029	2341	2663	2953	3301	3593	3917	4241
2039	2347	2671	2957	3307	3607	3919	4243
2053	2351	2677	2963	3313	3613	3923	4253
2063	2357	2683	2969	3319	3617	3929	4259
2069	2371	2687	2971	3323	3623	3931	4261
2081	2377	2689	2999	3329	3631	3943	4271
2083	2381	2693	3001	3331	3637	3947	4273
2087	2383	2699	3011	3343	3643	3967	4283
2089	2389	2707	3019	3347	3659	3989	4289
2099	2393	2711	3023	3359	3671	4001	4297
2111	2399	2713	3037	3361	3673	4003	4327
2113	2411	2719	3041	3371	3677	4007	4337
2129	2417	2729	3049	3373	3691	4013	4339
2131	2423	2731	3061	3389	3697	4019	4349
2137	2437	2741	3067	3391	3701	4021	4357
2141	2441	2749	3079	3407	3709	4027	4363
2143	2447	2753	3083	3413	3719	4049	4373
2153	2459	2767	3089	3443	3727	4051	4391
2161	2467	2777	3109	3449	3733	4057	4397
2179	2473	2789	3119	3457	3739	4073	4409
2203	2477	2791	3121	3461	3761	4079	4421
2207	2503	2797	3137	3463	3767	4091	4423
2213	2521	2801	3163	3467	3769	4093	4441
2221	2531	2803	3167	3469	3779	4099	4447
2237	2539	2819	3169	3491	3793	4111	4451
2239	2543	2833	3181	3499	3797	4127	4457
2243	2549	2837	3187	3511	3803	4129	4463
2251	2551	2843	3191	3517	3821	4133	4481
2267	2557	2851	3203	3527	3823	4139	4483

4493	4817	5167	5503	5839	6197	6521	6857
4507	4831	5171	5507	5843	6199	6529	6863
4513	4861	5179	5519	5849	6203	6547	6869
4517	4871	5189	5521	5851	6211	6551	6871
4519	4877	5197	5527	5857	6217	6553	6883
4523	4889	5209	5531	5861	6221	6563	6899
4547	4903	5227	5557	5867	6229	6569	6907
4549	4909	5231	5563	5869	6247	6571	6911
4561	4919	5233	5569	5879	6257	6577	6917
4567	4931	5237	5573	5881	6261	6581	6947
4583	4933	5261	5581	5897	6269	6599	6949
4591	4937	5273	5591	5903	6271	6607	6959
4597	4943	5279	5623	5923	6277	6619	6961
4603	4951	5281	5639	5927	6287	6637	6967
4621	4957	5297	5641	5939	6299	6653	6971
4637	4967	5303	5647	5953	6301	6659	6977
4639	4969	5309	5651	5981	6311	6661	6983
4643	4973	5323	5653	5987	6313	6673	6991
4649	4987	5333	5657	6007	6323	6679	6997
4651	4993	5347	5659	6011	6329	6689	7001
4657	4999	5351	5669	6029	6337	6691	7013
4663	5003	4381	5683	6037	6343	6701	7019
4673	5009	5385	5689	6043	6351	6703	7027
4679	5011	5393	5693	6047	6353	6709	7039
4691	5021	5399	5701	6053	6359	6719	7043
4703	5023	5407	5711	6067	6361	6733	7057
4721	5039	5413	5717	6073	6367	6737	7069
4723	5051	5417	5737	6079	6373	6761	7103
4729	5059	5419	5741	6089	6379	6763	7109
4733	5077	5431	5743	6091	6389	6779	7121
4751	5081	5437	5749	6101	6397	6781	7127
4759	5087	5441	5779	6113	6421	6791	7129
4783	5099	5443	5783	6121	6427	6793	7151
4787	5101	5449	5791	6131	6449	6803	7159
4789	5107	5471	5801	6133	6451	6823	7177
4793	5113	5477	5807	6143	6469	6827	7187
4799	5119	5479	5813	6151	6473	6829	7193
4801	5147	5483	5821	6163	6481	6833	7207
4813	5153	5501	5827	6173	6491	6841	7211

7213	7573	7917	8287	8663	8999	9341	9679
7219	7577	7927	8291	8669	9001	9343	9689
7229	7583	7933	8293	8677	9007	9349	9697
7237	7589	7937	8297	8681	9011	9371	9719
7243	7591	7949	8311	8689	9013	9377	9721
7247	7603	7951	8317	8693	9029	9393	9733
7253	7607	7963	8329	8699	9041	9397	9739
7283	7621	7993	8353	8707	9043	9403	9743
7297	7639	8009	8363	8713	9049	9413	9749
7307	7643	8011	8369	8719	9059	9419	9767
7309	7649	8017	8377	8731	9069	9421	9769
7321	7669	8039	8387	8737	9091	9431	9781
7331	7673	8053	8389	8741	9103	9433	9787
7333	7681	8059	8419	8747	9109	9437	9791
7349	7687	8069	8423	8753	9127	9439	9791
7351	7691	8081	8429	8761	9133	9461	9803
7369	7699	8087	8431	8779	9137	9463	9811
7393	7703	8089	8443	8783	9151	9467	9817
7411	7717	8093	8447	8803	9157	9473	9829
7417	7723	8101	8461	8807	9161	9479	9833
7433	7727	8111	8467	8819	9173	9491	9839
7451	7741	8117	8501	8821	9181	9497	9851
7457	7753	8123	8513	8831	9187	9511	9857
7459	7757	8147	8521	8837	9199	9521	9859
7477	7759	8161	8527	8839	9203	9533	9871
7481	7789	8167	8537	8849	9209	9539	9883
7487	7793	8171	8539	8861	9221	9541	9887
7489	7817	8179	8543	8863	9227	9551	9901
7499	7823	8191	8563	8867	9239	9587	9907
7507	7829	8209	8573	8887	9241	9601	9923
7517	7841	8219	8581	8893	9257	9613	9929
7523	7853	8221	8597	8923	9277	9619	9931
7529	7867	8231	8599	8929	9281	9623	9941
7537	7873	8233	8609	8933	9283	9629	9949
7541	7877	8237	8623	8941	9293	9631	9967
7547	7879	8243	8627	8951	9311	9643	9973
7549	7883	8263	8629	8963	6319	9649	
7559	7901	8269	8641	8969	9323	9661	
7561	7907	8273	8647	8971	9337	9677	

CHAP. XLI.

Of Notation of Fractions.

536. HERE, we ſuppoſe any Number, or Magnitude, to be divided into any Number of equal Parts; then one or more of ſuch Parts is called a Fraction (*Fraction Fr.*) For Illuſtration, ſuppoſe a Line to be divided into ſix equal Parts, then if we would expreſs one of thoſe Parts, we write $\frac{1}{6}$, which is read one Sixth; if two Parts, it would be $\frac{2}{6}$, two Sixths; where it may be obſerved that the Number under the Daſh (-), here 6, ſhews how many Parts the whole Quantity is divided into, and is therefore called the *Denominator*; and the whole Quantity itſelf the *Integer*. We ought further to obſerve, that the Number over the Daſh expreſſes the Number of the Parts taken, and is therefore called the *Numerator*. Thus of any other Fraction, *viz.* $\frac{7}{9}$, 7 is the Numerator, and 9 the Denominator, and ſignifies, that, the whole Thing, whatever it be, being ſuppoſed to be divided into 9 equal Parts, what we would here ſignify is ſeven ſuch Parts of it.

537. Since the Denominator repreſents all the Parts of the Integer, and the Numerator the Number of Parts taken; it muſt follow, that, if the Numerator be leſs, equal to, or greater than the Denominator, the Quantity, expreſſed by that Fractional Number, is leſs, equal to, or greater than the Integer, reſpectively; for Inſtance, $\frac{2}{5}$ is leſs than the Integer, becauſe it expreſſes only two Parts, whereas the Integer is five ſuch Parts; and $\frac{5}{5}$ is equal to the Integer, as it repreſents all the five Parts; but $\frac{7}{5}$ is greater than the Integer, becauſe the Integer is ſuppoſed to be divided only into five equal Parts, whereas the Numerator expreſſes ſeven ſuch Parts; and it is evident that the Value of ſuch a Fraction is all the five Parts (or the whole Integer) and two ſuch Parts over, and there-

fore may be reduced to 1 and $\frac{2}{5}$; thus, if we suppose a Yard to be divided into five Parts, the Value of $\frac{7}{5}$ would be a Yard and $\frac{2}{5}$ of a Yard.

538. When the Numerator is less than the Denominator, the Fraction is called a proper Fraction.

539. *Scholium.* It is common to hear People say, that a proper Fraction is a Number less than Unity; but this is a vulgar Error; for Unity in its own Nature as Number is indivisible, for what Number of Things can there be less than one? This is considering it in its absolute Nature, or purely as a Number. But if we consider it, as applied to something as 1 Yard, 1 Pound, &c. we can conceive a Quantity less than 1 Yard, or 1 Pound, &c. and therefore they should express themselves, that a proper Fraction is less than the Integer, or relative Unit. How odly must this sound in our Eyes, that "the "Number placed below the Line is called the De-"nominator of the Fraction, because it denominates "the Fraction, or Number of Parts into which "*Unity* is broken or divided?" When the very Idea we
* 5. have of Unity is of * something considered as alone and undivided. Indeed many Authors do not seem to have considered the Difference there is betwixt Unity which is indivisible by the very Nature of the Thing, and the Integer, or relative Unit, or collective Unit, as we may properly express it, which we may consider to be made up of several lesser Things; for Instance, 1 Foot may be considered as an Integer, or collective Unit, being made up of 12 lesser Units, or Inches. The Reason of many Authors not attending to these Things is the Reason of their communicating inaccurate and absurd Ideas.

540. If the Numerator be greater than the Denominator, such Fraction is called an improper Fraction.

541. A compound Fraction is a Part of a Part of an Integer, having several Numerators and Denominators, with the Word *of* between them. Thus, for Illustration, suppose a Line *AB* divided into

three equal Parts; and that each of those Parts is divided into two Parts; and that each of these last Parts is divided into five equal Parts; then two of these last Parts may be expressed thus, $\frac{2}{5}$ of AC; but AC is $\frac{1}{2}$ of AD; and AD $\frac{1}{3}$ of AB; therefore, it is evident, $\frac{2}{5}$ of AC, with Respect to AB, may be thus expressed, $\frac{2}{5}$ of $\frac{1}{2}$ of $\frac{1}{3}$ of AB; and ∴ $\frac{2}{5}$ of $\frac{1}{2}$ of $\frac{1}{3}$ is called a compound Fraction.

$A \quad C \quad D \quad B$

542. A whole Number and a Fraction, as 2 ℔ $\frac{1}{3}$, is called a mixed Number.

543. From what has been said it plainly appears, that Fractions are relative Numbers; for, as Integral Numbers consider Things simply and absolutely in themselves, Fractional Numbers consider Things relatively as Parts of other Things; for which Reason, Integers and Fractions might have been justly distinguished by the Terms absolute and relative Numbers.

544. *Axiom.* The like Fractions of two equal Quantities are equal. For *Example*, if $a = b$, $\frac{1}{2}$ of $a = \frac{1}{2}$ of b, $\frac{1}{3}$ of $a = \frac{1}{3}$ of b, &c.

545. *Axiom.* In two equal Fractions, if one of their like Terms be equal, the other is also equal. Thus, if $\frac{a}{b} = \frac{a}{d}$, then $b = d$; and, if $\frac{a}{b} = \frac{c}{b}$, then $a = c$.

546. It ought to be here hinted, that all the common Axioms, already given in this Treatise, hold good also in Fractions.

547. The Numerator of any Fraction may be esteemed as the Dividend, and the Denominator as a Divisor*. Thus $\frac{2}{6}$ may be read 2 divided by 6, and the Quotient is $\frac{1}{3}$; (for, if any Thing is divided into

* The Reason of this may be shewn thus: Let $w =$ the whole Integer, $p =$ the required Part of it; then since p is to the Whole, as 6 to 2, in the above Illustration; it is, as $6 : 2 :: w : p$, ∴ $6p =$ * $2w$; and, dividing both Sides of the Equation by 6, it gives $p =$ † $\frac{2w}{6}$. But, since the Integer w may be represented by 1, we have $p = 2 \div 6$. * 185.
† 1[illegible].

6 equal Parts, two of such Parts are $\frac{1}{3}$ of the Whole;) for, if 2 be divided by 6, the Quotient must be $\frac{1}{3}$, for $\frac{1}{3} \times 6 = \frac{6}{3} = 2$ as it ought to be, because the Divi-
* 123. sor, multiplied by the Quotient, must be = * the Dividend.

548. *Corollary*. Hence, when one Number is to be divided by another, it may be considered as a Fraction; and a Fraction may be considered as the Numerator divided by the Denominator. Hence appears the Reason of the Method used by Algebraists to express Division; for, if n is to be divided by d, they express it by $\frac{n}{d}$, which is the very same as a Fraction whose Numerator is n, and Denominator d.

549. *Lemma* 1. If both the Numerator and Denominator of any Fraction be multiplied by one and the same Number, we shall have another Fraction of the same Value as the first.

For it is evident, that, as often as the Denominator is contained in the Numerator, so often twice the Denominator will be contained in the Numerator; and so often will three Times the Denominator be contained in three Times the Numerator; &c. Hence generally, if we call the Multiplier m, as often as the Denominator is contained in the Numerator, so often will m Times the Denominator be contained in m Times the Numerator*. And therefore the Fraction, so found, must be a like Part of the Integer, as the given Fraction is.

550.

* Or thus: Let $\frac{n}{d}$ (be the Fraction) $= p$; then, multiplying both Sides of the Equation by m, we have $\frac{nm}{d} =$
* 56. * pm; and dividing this by m (since multiplying the Denominator of a Fraction by m is making the Fraction m Times less, which is properly dividing by m) gives $\frac{nm}{dm}$
† 108. ‡ 23. $=$ † p; but $p = \frac{n}{d}$ by the above; ∵ $\frac{n}{d} =$ ‡ $\frac{nm}{dm}$. Q. E. D.

550. *Lemma* 2. If both the Numerator and Denominator of any Fraction be divided by one and the same Number, the Fraction, so obtained, will be of the same Value as the first.

For, as often as the Denominator is contained in the Numerator, so often, it is manifest, will $\frac{1}{2}$ of the Denominator be contained in $\frac{1}{2}$ the Numerator; and so often will $\frac{1}{3}$ of the Denominator be contained in $\frac{1}{3}$ of the Numerator; &c. Hence generally, if we call the Divisor D, as often as the Denominator is contained in the Numerator, so often is the Denominator, divided by D, contained in the Numerator divided by D. That is, if $\frac{n}{d}$ represent the given Fraction, $\frac{n}{d} = \frac{n \div D}{d \div D}$.* Q. E. D.

551. *Scholium*. These two *Lemma's*, in other Words, may be thus expressed: If both the Numerator, and Denominator of any Fraction be multiplied or divided by one and the same, or equal Numbers, the Numerator and Denominator of the Fraction, so obtained, will have the same Ratio to each other, as the Numerator of the first Fraction has to its Denominator; that is, as the Numerator of the First is to its Denominator, so is the Numerator of the Second to its Denominator. *Et contrà.*

552. *Lemma.* All Fractions, whose Numerators and Denominators are proportional, are equal to each other;

* Or the Truth of this may be thus shewn: If this *Lemma* be true, then the same Ratio, as the Denominator has to its Numerator, must the Denominator, divided by D, have to the Numerator divided by D; that is, $d : n :: \frac{d}{D} : \frac{n}{D}$. But the Product of the Extremes $=$ * the Product of the Means, that is, $d \times \frac{n}{D} = n \times \frac{d}{D}$; but $d \times \frac{n}{D} = \frac{dn}{D}$; and $n \times \frac{d}{D} = \frac{dn}{D}$ also; $\therefore d \times \frac{n}{D} =$ † $n \times \frac{d}{D}$ as it ought to be by the Supposition, $\therefore d : n :: \frac{d}{D} : \frac{n}{D}$. Q. E. D. * 185. † 23.

other; and on the contrary, if there are two equal Fractions, their Numerators and Denominators are proportional.

Demonstration. Let $\frac{n}{d}$, and $\frac{N}{D}$, be the two Fractions; then, if $n : d :: N : D$, we have $\frac{n}{d} = * \frac{N}{D}$.

* 184. *Q. E. D.* And, for demonstrating the second Part of this *Lemma*, by the Supposition, $\frac{n}{d} = \frac{N}{D}$, and therefore † $n : d :: N : D$. *Q. E. D.*

† 184.

553. A *Theorem.* Fractions, having the same Denominator, are in Proportion to each other as their Numerators.

The *Demonstration* is manifest; for, since the Denominator expresseth how many Parts the Integer is divided into, and as these Denominators are equal, the Value of each Fraction must be proportional to the Number of these Parts taken, that is, in Proportion to their Numerators.

554. A *Theorem.* Fractions, having equal Numerators, are in reciprocal Proportion to their Denominators.

The Truth of this may be easily shewn, thus: Since the Numerator expresses the Number of Parts of the Integer taken, and the Denominators the Number of Parts into which the whole Integer is divided, it is evident that, if the Numerators are supposed equal, and the Denominators are supposed to keep increasing, the Value of the succeeding Fractions must be decreasing, in the same Proportion as the Denominators increase; that is, they will be in reciprocal Proportion to their Denominators.

555. Since the Denominator of any Fraction expresseth the Number of equal Parts that the Integer is imagined to be divided into, and the Numerator the Number of such Parts taken, it must follow, that, as the Denominator is to the Integer, so is the Numerator to the Value of the Fraction. For

Exam-

Example, if the Fraction be $\frac{1}{4}$ of a £, then as 4 : 20*s*. :: 1 : $\frac{20 \times 1}{4} = 5s. = \frac{1}{4}$ of a £.

556. If two Fractions are equal, then, if each Numerator be multiplied by the other's Denominator, the Products will be equal; and on the contrary, if the Products of each Numerator into the other's Denominator are equal, the Fractions themselves are also equal.

For let $\frac{n}{d}$, and $\frac{N}{D}$ represent the two equal Fractions, then $\frac{n}{d} = \frac{N}{D}$ by the Supposition; and therefore * as $n : d :: N : D$; hence we get $n D =$ † $d N$. *Q. E. D.* * 552. † 185.

And, for the *Demonstration* of the second Part of this *Article*, let $\frac{n}{d}$ and $\frac{N}{D}$ represent two Fractions; then, by the Supposition, $n D = d N$; which, divided by D, will give $n =$ ‡ $\frac{d N}{D}$, and, dividing now by d, we ‡ 108.

shall have $\frac{n}{d} =$ ‖ $\frac{N}{D}$. *Q. E. D.* ‖ 108.

557. *Corollary*. Hence, if we have any Time a Mind to try, whether any two given Fractions are equal, we have only to multiply each Numerator into the other Denominator, and, if the Products are equal, the Fractions are so too; otherwise not.

558. This much being sufficient for the understanding the Nature of Fractions, we shall now proceed to apply them.

CHAP. XLII.

REDUCTION *of* FRACTIONS.

559. REDUCTION of Fractions is the Changing of Fractions into others of equal Value,

560. *Case* 1. Integers may be writ fractionally, by setting an Unit for the Denominator; and it is evident they will retain the same Value, because an Unit does not divide. Thus 3 may be writ $\frac{3}{1}$, and 5 may be writ $\frac{5}{1}$.

561. *Case* 2. To reduce a mixed Number into an improper Fraction. Multiply the whole Number by the Denominator of the Fraction, and add in the Numerator of the Fraction, the Product will be a Numerator; under which place the Denominator of the Fraction, and the Fraction, so formed, will be equal to the given mixed Number.

562. *Example.* Reduce 2 ℔ $\frac{1}{3}$ into a Fraction.

Solution. $2 \times 3 + 1 = 7$ for the Numerator; and hence the required Fraction is $\frac{7}{3}$.

The Reason of this is evident; for here we have multiplied by 3, and putting the 3 under the 7 is also representing that Product, as divided by 3; but, if any Number be multiplied and divided by one and the same Number, it is evident the Quotient must be the same as the Quantity first given.

563. *Case* 3. To reduce an improper Fraction to its equivalent, whole, or mixed Number. Divide the Numerator by the Denominator.

564. *Example*, Reduce $\frac{7}{3}$ to a mixed Number.

Solution. $7 \div 3 = 2\frac{1}{3}$. This is only the Reverse of the last *Case*.

565. *Case* 4. To reduce any Integer to a Fraction of a given Denominator.

Multiply the Integer by the Denominator, and the Product will be the required Numerator.

566. *Example.* Reduce 7 to a Fraction whose Denominator is 8.

Solution. $7 \times 8 = 56$; hence the Fraction is $\frac{56}{8}$. The Reason of this is manifest; for here we have both multiplied and divided by 8, and, consequently, the Quotient $\frac{56}{8}$ must be the same Value as before.

567. *Case* 5. To reduce a compound Fraction to its equal simple one. Observe, that the continued Product of all the Numerators will be the required Numerator,

merator; as the continued Product of all the Denominators will be the Denominator which was required.

568. *Example.* Bring $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{2}{5}$ in to a simple Fraction.

Solution. $1 \times 1 \times 2 = 2 =$ the Product of the Numerators; and $2 \times 4 \times 5 = 40 =$ the Product of the Denominators; $\therefore \frac{2}{40}$ is the required Fraction.

569. The Reason of this *Rule* may be shewn as as follows: Suppose a compound Fraction to be $\frac{2}{3}$ of $\frac{4}{7}$; here, if we take the $\frac{1}{7}$ of any Thing, and multiply that Seventh by 4, the Product, it is evident, must be equal to $\frac{4}{7}$: And, after the like Manner, the $\frac{1}{3}$ of this Part multiplied by 2 must be $= \frac{2}{3}$ of the $\frac{4}{7}$: Now it is plain, that we have here divided by both Denominators 7 and 3, and multiplied by both Numerators 2 and 4; and consequently, if we make the Product of both Numerators a Numerator and the Product of both Denominators a Denominator, we shall effect the same by one Multiplication, and one Division, as before, we did by two of each; and so of a Fraction compounded of any Number of simple Fractions; hence the Reason of the above *Rule* is clear: Or it may be illustrated by an *Example*, thus: Suppose it was required to take the $\frac{2}{3}$ of $\frac{4}{7}$ of 168£; then 168£. $\div$ 7 and $\times$ 4, or, which is the same, 168×4 and $\div 7 = 96$£; and $96 \div 3$ and $\times 2$, or 96×2 and $\div 3, = 64$£ $= \frac{2}{3}$ of $\frac{4}{7}$ of 168£. But $2 \times 4 = 8$, and $3 \times 7 = 21$, and so $\frac{8}{21} = \frac{2}{3}$ of $\frac{4}{7}$ by the *Rule.* Now $168 \div 21$ and $\times 8$; or 168×8 and $\div 21 = 64$£ as before.

570. *Corollary.* Hence if we are to take a Part of a Part or Parts of any Number, we may first bring the compound Fraction into a simple one, and then multiply by the Numerator of the simple Fraction, and divide the Product by its Denominator.

571. *Case* 6. To bring Fractions of different Denominators into one common Denominator.

The *Rule.* Multiply each Numerator into the Denominators of all the other Fractions continually for new Numerators, and the continued Product of all the Denominators will be the common Denominator.

572.

572. *Example*, Bring $\frac{1}{3}$ and $\frac{2}{5}$ and $\frac{3}{8}$ into one common Denominator.

Solution. $3 \times 5 \times 8 = 120 =$ the common Denominator; and $1 \times 5 \times 8 = 40 =$ a Numerator; also $2 \times 3 \times 8 = 48 =$ another Numerator; and $3 \times 5 \times 3 = 45 =$ the other Numerator; hence the three equivalent Fractions are $\frac{40}{120}$; $\frac{48}{120}$; and $\frac{45}{120}$; that is, $\frac{1}{3} = \frac{40}{120}$; $\frac{2}{5} = \frac{48}{120}$; and $\frac{3}{8} = \frac{45}{120}$.

The Reason of this may be thus shewn: We have, in the above Operation, multiplied the Numerator of the Fraction $\frac{1}{3}$ by 5 and by 8; also its Denominator by the same Numbers; and, therefore, the new Fraction
* 549. tion $\frac{40}{120}$ will be of the same Value * as $\frac{1}{3}$.

Again, the Numerator of the Fraction $\frac{2}{5}$ was multiplied by 3 and 8, and its Denominator by the same Numbers; and ∵ the new Fraction $\frac{48}{120}$ will still retain the same Value by *Art.* 549; and so of the other Fraction.

573. *Case* 7. To reduce a given Fraction to another equivalent to it, having a given Denominator, if possible.

The *Rule*. Multiply the Numerator by the Denominator of the Fraction sought, and divide the Product by the Denominator of the given Fraction; the Quotient, if there be no Remainder, will be the Numerator of the required equivalent Fraction *.

574. *Example*. Bring $\frac{2}{3}$ into an eqviualent Fraction whose Denominator is 6.

Solution. $2 \times 6 = 12$; and $12 \div 3 = 4$; ∵ the Fraction required is $\frac{4}{6}$.

575. *Lemma*. To find the greatest common Measure of any two Numbers, *i. e.* the greatest Number that the two given Numbers can be divided by, without leaving any Remainder. The

* *Demonstration*. Let $\frac{n}{d}$ be the given, and $\frac{N}{D}$ be the re-
* 557. quired Fraction; then, since $\frac{n}{d} = \frac{N}{D}$, we have $n\,d = {}^{*}\,d\,N$; whence, dividing by d, we get $\frac{N\,D}{d} = N$. *Q. E. D.*

The *Rule*. Divide the greater Number by the lesser, and, if there be a Remainder, divide the Divisor by that Remainder, and so continue dividing the next preceding Divisor by its Remainder, till there is no Remainder; then the last Divisor will be the Number sought*.

576. *Example*. What is the greatest Number, that will divide both 56 and 120, without a Remainder? See the Operation:

56) 120 ($2\frac{8}{56}$ Quotient	8) 56 (7
Remains 8	Remains 0

Here 8, the last Divisor, is the greatest common Measure.

577.

* We will demonstrate the Truth of this, when the greatest common Measure is found by 3 Divisions, and the same Method of Reasoning holds good in any other Number of Divisions. In Order to which, let a and b be the two Numbers, whose common Measure is to be found; let a divided by b give the integral Quote c, and Remainder d; then let b be supposed to be divided by d, and to give the integral Quote and Remainder f; lastly, suppose d divided by f, and to give the integral Quote g, and Remainder nothing: Then, as the Product of the integral Quote multiplied by the Divisor, *plus* the Remainder, is *equal to the Dividend, (in any Division) we get these three Equations: First, $bc+d=a$; Second, $de+f=b$; Third, $gf=d$. Now, as f divides d without a Remainder, it will also divide its Multiple de, in the second Equation, and also the other Part f, or itself; and will therefore divide the Whole $de+f$, or its equal b. And since, f divides b, it must also divide the Multiple bc, in the first Equation; and it was shewn from the third Equation, that f divides d; and, therefore, f divides the first Step $bc+d$, or its equal a. Hence we have shewn. that f is a common Measure, for we have shewn that it divides both a and b. Now, to shew that f is the greatest common Measure, let us, if possible, suppose that a greater Number m is the greatest common Measure. Then, by this Supposition, m divides both a and b, without a Remainder. Therefore, since m divides a, by the Supposition, it must divide its equal $bc+d$; but m divides b by the Supposition, ∴ it must divide its Multiple bc; and as it divides a, and one Part of a, *viz.* bc, it must also divide the other Part d; ∴ m divides d. And, since m divides b, it must divide its equal $de+f$; but we have just shewn that * 123.

577. *Case* 8. To reduce a Fraction to lower Terms; *i. e.* to find an equivalent Fraction expressed in lesser Numbers, if possible.

The *Rule.* Divide both the Numerator and Denominator by any Number that will leave no Remainder; and we shall have another Fraction* equal to the given one. And, in Order more readily to know what Numbers the Numerator and Denominator can be divided by, the following Observations will many Times be useful. (1.) If the Numbers are even, they can be divided by 2; that is, such Numbers as end in 2, 4, 6, 8, or 0. (2) If the Numerator and Denominator have both 5 in the Units Place, or one ends in 5, the other in 0, they are both divisible by 5. (3) If both the Numerator and Denominator have a Number of Cyphers (0,) on the Right Hand of the Figures, cut off an equal Number in each; for that is dividing by 10, or 100, or 1000, &c. (4.) But, if we cannot readily discover a common Divisor, then we must have Recourse to the *Lemma* in *Article* 575; and if, that *Lemma* will not discover a common Divisor, the Fraction is already in its lowest Terms.

578. *Example.* Abbreviate or reduce $\frac{75}{105}$ to lower Terms.

Solution. The Numerator and Denominator, having each 5 in the Units Place, are both divisible by 5; $\frac{1}{5}$ of $75 = 15$, and $\frac{1}{5}$ of $105 = 21$; ∵ we have reduced it to $\frac{15}{21}$; but this this may be further reduced, for $\frac{1}{3}$ of $15 = 5$, and $\frac{1}{3}$ of $21 = 7$. Hence $\frac{75}{105} = \frac{15}{21} = \frac{5}{7}$. But, if we find the greatest common Measure, the Operation will be thus:

$$75)\,\underline{105}\,(1 \qquad 30)\,\underline{75}\,(2 \qquad 15)\,\underline{30}\,(2$$
$$30 \qquad\qquad 15 \qquad\qquad 0$$

Hence

* 550.

that *m* divides *d*, ∵ it must divide its Multiple *de*; and since *m* divides the Whole *b*, and its Part *de*, it must divide the other Part *f* also; that is, *m* measures *f*, or *m* is contained in *f*; which is absurd, because *m* is greater than *f* by the Supposition. Hence *m* cannot be greater than *f*; consequently *f* is the greatest possible.

Hence the greateſt common Meaſure is 15; now $75 \div 15 = 5$, and $105 \div 15 = 7$; ∴, reduced to its loweſt Terms, the Fraction is $\frac{5}{7}$, as above.

579. We have hitherto in theſe Caſes been treating of *abſtract Fractions*, or Fractions related to the ſame Integer; we ſhall now proceed to thoſe Caſes which may be ſaid to belong to *applicate Fractions*, or Fractions related to different Integers.

Caſe 9. To bring a Fraction of a lower Integer into the Fraction of a higher Integer, the loweſt Integer having a known Relation to the higher. This is beſt explained by an *Example*.

580. *Example*. Bring $\frac{3}{4}$ of a Penny into the Fraction of a Pound. This may be ſolved thus: A Penny being $\frac{1}{12}$ of a Shilling, and a Shilling $\frac{1}{20}$ of a £; $\frac{3}{4}$ of a Penny, as a Fraction of a £, may be read $\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$ of a £; which, brought into a ſimple Fraction by *Caſe* the 5th, will be $\frac{3}{960}$. But I think the plaineſt Method is to bring a £ into 4ths of a Penny, or Farthings; thus $20 \times 12 \times 4 = 960$ Fourths of a Penny, or Farthings in a Pound. Conſequently, by Definition *Art.* 536, the Fraction is $\frac{3}{960}$ of a £, as above.

581. *Caſe* 10. To reduce a mixt Number, of a leſſer Name, into the Fraction of a greater.

The *Rule*. Reduce the mixt Number into an improper Fraction, by *Caſe* 2; then work as in the laſt *Caſe*.

582. *Example*. Bring 2 ℔ $\frac{1}{2}$ into the Fraction of a ℔.

℔

2 $\frac{1}{2}$ By the firſt Method ſay $\frac{5}{2}$ of $\frac{1}{112}$ of a C, ∴

2 the Anſwer will be $\frac{5}{224}$.

5

But, by the Method which I prefer, bring 112 ℔ into half Pounds, that is, $112 \times 2 = 224$ half Pounds in 1 C: And 2 ℔ $\frac{1}{2}$ in half Pounds is 5 half Pounds;

* 536. and consequently, by the Nature of Fractions *, $\frac{5}{124}$ is the required Fraction of a C.

583. *Case* 11. To reduce a Fraction of an Unit of a higher Value to the Fraction of an Unit of a lower Value. This is easily explained by an *Example*.

584. *Example*. Reduce $\frac{3}{7}$ of a £ into the Fraction of a Shilling.

Solution. First bring the higher Unit into Units of the lower, *viz*. 1 £ = 20 *s*; then, since our Fraction is $\frac{3}{7}$ of a £, it must be also $\frac{3}{7}$ of its Equal, *viz*. of 20 *s*; ∴, since $\frac{1}{7}$ of 20 may be represented $\frac{20}{7}$; $\frac{3}{7}$ may be expressed $\frac{20 \times 3}{7} = \frac{60}{7}$ of a Shilling. The same Method and Reason are good in any other Instance. And thus much for Reduction of Fractions; we shall now proceed to Valuation, which might have been added as a twelfth *Case*, but we shall refer it to the next Chapter.

CHAP. XLIII.

Of the VALUATION *of* FRACTIONS.

585. TO find out the Value of a Fraction of an higher Integer, in Integers of a lower Denomination.

The *Rule*. Multiply the Numerator of the given Fraction, by the Number of Species of the next lower Denomination that are equal to one of the given higher Integer, and divide the Product by the Denominator of the Fraction; and, if any Thing remains, multiply the Remainder by the next lower Denomination, and divide by the Denominator of the Fraction as before, until there be no Remainder, or we have brought it into the lowest Denomination.

586. *Example*. What is the Value of $\frac{1}{3}$ of a Pound Sterling?

1 £

1 £ = 20 Shillings.

× Numerator 3

5) 60 (12 Answer 12 Shillings.

0

587. Take another *Example*. What is the Value of $\frac{4}{7}$ of 1 C?

1 C. is = 4 Quarters

6

7) 24 (3

Remains 3

× 1 Quarter = 28 ℔ Quarters ℔

7) 84 (12 Answer 3 12

14

0

The Reason of this *Rule* may be seen in *Art.* 555.

CHAP. XLIV.

ADDITION of FRACTIONS.

588. ADDITION of Fractions is the Finding of a Fraction equal to the Sum of two, or more Fractions, taken together.

589. The *Rule*, If the Fractions are compound, bring them into simple ones by *Case* 5; and, if they have not the same Denominator, bring them into a common Denominator by *Case* 6. or 7. Then add their Numerators, and put the common Denominator underneath.

590. *Example* 1. Add $\frac{1}{3}$ and $\frac{1}{2}$ and $\frac{2}{5}$ together.

These, by *Case* 6, will be transformed to $\frac{10}{30}$, $\frac{15}{30}$, and $\frac{12}{30}$; ∵ $10 + 15 + 12 = 37$ = the Numerator; ∵ the required Sum = $\frac{37}{30}$ = (by *Case* 3. of Reduction) $1\frac{7}{30}$.

591. *Example* 2. Add $\frac{1}{2}$ of $\frac{1}{3}$ and $\frac{1}{4}$ of $\frac{1}{6}$ together.

By *Case* 5, $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$, and $\frac{1}{4}$ of $\frac{1}{6} = \frac{1}{24}$; and, by *Case* 7, $\frac{1}{6} = \frac{4}{24}$; $\because 1 + 4 = 5 =$ Numerator, and so the Answer $= \frac{5}{24}$. Or thus, by *Case* 6, $\frac{1}{6}$ and $\frac{1}{24}$, brought into one common Denominator, are $\frac{24}{144}$ and $\frac{6}{144}$; $\because 24 + 6 = 30$, and consequently the Sum $= \frac{30}{144}$; but this Abbreviation, by *Case* 8, will $= \frac{5}{24}$ as before.

592. *Example* 3. Add 2 ℔ $\frac{1}{3}$ and 1 ℔ $\frac{1}{5}$ together.

First, 2 ℔ + 1 ℔ = 3 ℔; then $\frac{1}{3}$ and $\frac{1}{5}$, by *Case* 6, may be changed to $\frac{5}{15}$ and $\frac{3}{15}$, and their Sum $= \frac{8}{15}$; $\therefore$ the whole Sum is 3 ℔ $\frac{8}{15}$. Or it may be solved thus, 2 ℔ $\frac{1}{3}$ and 1 ℔ $\frac{1}{5}$, by *Case* 2, may be changed to $\frac{7}{3}$ and $\frac{6}{5}$; and $\frac{7}{3}$ and $\frac{6}{5}$, by *Case* 6, are $= \frac{35}{15}$ and $\frac{18}{15}$; the Sum of these is $\frac{35 \times 18}{15} = \frac{53}{15}$ of a ℔, = (by *Case* 3.) 3℔ $\frac{8}{15}$ as before; but the first Method is shortest.

593. *Example* 4. Add 3, and $\frac{1}{3}$, and $\frac{1}{4}$ of $\frac{3}{7}$ and 2 $\frac{1}{5}$ together.

Solution. First, adding the Integers 3 and 2, we have their Sum = 5; then the compound Fraction $\frac{1}{4}$ of $\frac{3}{7}$, by *Case* the fifth, may be changed into the equivalent one $\frac{3}{28}$; next $\frac{1}{3}$ and $\frac{3}{28}$ and $\frac{1}{5}$, being reduced to one common Denominator, by *Case* the sixth, will be $\frac{140}{420}$ and $\frac{45}{420}$ and $\frac{84}{420}$; which we may now add together, and their Sum will be $\frac{140 \times 45 \times 84}{420} = \frac{269}{420}$, and $\therefore$ the Sum of all the given Fractions $= 5\frac{269}{420}$.

CHAP. XLV.

SUBTRACTION of FRACTIONS.

594. HAVING prepared the Fractions as directed in Addition, find the Difference of the two Numerators, and put the common Denominator under that Difference; and the Fraction, so found, will be equal to the Difference of the two given Fractions.

595. *Example* 1. From $\frac{3}{5}$ subtract $\frac{1}{6}$. These, by *Case* 6, are changed into their Equivalents $\frac{18}{30}$ and $\frac{5}{30}$; $\therefore \frac{18-5}{30} = \frac{13}{30} =$ the Difference sought.

596. *Example* 2. From $\frac{1}{2}$ subtract $\frac{1}{3}$ of $\frac{2}{5}$.

Here $\frac{1}{3}$ of $\frac{2}{5} = \frac{2}{15}$ by *Case* 5. Now $\frac{1}{2}$ and $\frac{2}{15}$, by *Case* 6, are equivalent to $\frac{15}{30}$ and $\frac{4}{30}$, $\because \frac{15-4}{30} = \frac{11}{30}$, = the required Difference.

597. *Example* 3. From $\frac{3}{4}$ of a £ subtract $\frac{2}{5}$ of a Shilling.

Solution. By *Case* the eleventh, $\frac{3}{4}$ of a £ = $\frac{60}{4}$ of a Shilling. Now $\frac{60}{4}$ and $\frac{2}{5}$ are by *Case* the sixth = $\frac{300}{20}$ and $\frac{4}{20}$; $\because$ the Difference is $\frac{300.-4}{20} = \frac{296}{20}$ = (by *Case* the eighth) $\frac{74}{5}$ of a Shilling =, by *Case* the third, 14 *s.* $\frac{4}{5}$.

598. *Example* 4. From 13 £ $\frac{1}{2}$ subtract 10£ $\frac{5}{6}$.

Solution. Questions of the Nature of this may be solved by putting the mixed Numbers into improper Fractions, but easier by the following Method: The Fractions $\frac{1}{2}$ and $\frac{5}{6}$, brought into one common Denominator by *Case* the sixth, are $\frac{6}{12}$ and $\frac{10}{12}$; $\because$ we are now to subtract 10£ $\frac{10}{12}$ from 13£ $\frac{6}{12}$, and consequently the Operation now will stand thus:

£	12ths
13	6
10	10
2	$\frac{8}{12}$

10 from 6 we cannot, and $\because$ (see above) 10 from 12, there remains 2, and the 6 makes 8 Twelfths of a £; now the 1 £ (*viz.* the 12 Twelfths that we borrowed to make the Subtraction) being carried to the Column of £s, we have 13 £ − 11 £ = 2 £; and $\because$ the required Difference = 2 $\frac{8}{12}$ £. = 2 $\frac{2}{3}$ £.

If the Reader understands Addition, these *Examples* are sufficient for Subtraction. We shall therefore only add under this Head, that Subtraction of Fractions is proved by adding the Minor and Remainder together, as in whole Numbers.

CHAP. XLVI.

MULTIPLICATION of FRACTIONS.

599. BY multiplying any Number, or Quantity, by a Fraction, is only meant the taking such Part, or Parts of it, as the Fraction expresses.

600. The *Rule* to perform Multiplication of Fractions is: If the Fractions are mixed, they must be first brought into improper Fractions; if compound, into single. Then multiply the Numerators together for a Numerator, and the Denominators for a Denominator.

601. *Example* 1. Multiply $\frac{3}{4}$ by $\frac{2}{3}$. Here $3 \times 2 = 6 =$ the Numerator; and $4 \times 3 = 12 =$ the Denominator; ∵ $\frac{6}{12} =$ the required Product; but this may be reduced to lower Terms, *viz.* $\frac{6}{12} = \frac{1}{2}$. See *Case* 8.

602. *Example* 2. Multiply $2\frac{1}{2}$ by $\frac{1}{8}$, and this Product by 2, and this again by $\frac{1}{3}$ of $\frac{5}{6}$.

Solution. First $2\frac{1}{2} = \frac{5}{2}$ by *Case* 2; and $2 = \frac{2}{1}$ by *Case* 1; and $\frac{1}{3}$ of $\frac{5}{6} = \frac{5}{18}$ by *Case* 5. Hence, we are now to multiply $\frac{5}{2}$ and $\frac{1}{8}$ and $\frac{2}{1}$ and $\frac{5}{18}$ together; ∵ by the *Rule* $5 \times 1 \times 2 \times 5 = 50 =$ the required Numerator, and $2 \times 8 \times 1 \times 18$ (the Product of the Denominators) $= 288 =$ the Denominator. Hence the required Product $= \frac{50}{288} =$ (by *Case* 8) $\frac{25}{144}$.

603. *Example* 3. Multiply 10 Yards, 2 Feet, 3 Inches, by $2\frac{1}{3}$ and $\frac{4}{12}$ of $\frac{1}{3}$. This is the same in Effect, as if it had been proposed to a Measurer to find how many Yards are contained in a Piece of Pavement 10 Yards, 2 Feet, 3 Inches long, and 2 Yards, 1 Foot, 4 Inches broad; for the *Rule*, observed by Measurers, is, to multiply the Length, taken as an applicate Number, by the Breadth considered as an abstract Number.

Solution. First, 1 Yard $= 1 \times 3 \times 12 = 36$ Inches, and 10 Yards, 2 Feet, 3 Inches $= 387$ Inches; ∵ the Length $= \frac{387}{36}$ of a Yard; and, bringing the Breadth into

into Inches, the Multiplier will be $\frac{88}{36}$. Hence we are to multiply $\frac{387}{36}$ of a Yard by $\frac{88}{36}$, or, by Abbreviation, by *Case* 8, it is the same to multiply $\frac{43}{4}$ of a Yard by $\frac{22}{9}$; $\because$ $43 \times 22 = 946 =$ the Numerator, and $4 \times 9 = 36 =$ the Denominator; and $\because$ the Answer $= \frac{946}{36}$ of a Yard, $= 26\frac{10}{36}$ Yards (by *Case* 3.) $= 26\frac{5}{18}$ Yards, by *Case* 8.

604. *Scholium*. In both whole Numbers, and Fractions, this Proportion holds good, *viz.* * as one is to the Multiplicand, so is the Multiplier to the Product.

605. Hence if the Multiplier be less than the Integer, that is a proper Fraction, the Product will be less than the Multiplicand.

606. The Reason of the *Rule* for Multiplication of Fractions may be shewn from the 1st *Example*. For, if it had been demanded to multiply $\frac{3}{4}$ by 2, it is evident at first Sight, that $\frac{3 \times 2}{4}$ would be the Answer; but it was required to multiply by $\frac{2}{3}$, that is, by the $\frac{1}{3}$ of 2; and $\because$ the required Answer must be $\frac{1}{3}$ of the Product $\frac{3 \times 2}{4}$, that is, $\frac{3 \times 2}{4 \times 3} = \frac{6}{12} = \frac{1}{2}$ †.

X 4 CHAP.

* Let $m =$ the Multiplicand, $f =$ the Multiplier, $p =$ the Product, then $mf = p$; and, dividing by m, we have $f =$ * $\frac{p}{m}$, and this divided by p gives $\frac{f}{p} =$ † $\frac{1}{m}$, $\because$ ‡ as $1 : m :: f : p$. *Q. E. D.* * 108. † 108. ‡ 184.

† Or the Truth of the *Rule* for Multiplication of Fractions may be shewn algebraically thus: Let $\frac{N}{D}$ and $\frac{n}{d}$ be the two Fractions, whose Product is required; let $\frac{N}{D} = a$, $\frac{n}{d} = b$; hence multiplying the first Equation by D, and the second by d, we shall get $N = Da$, and $n = db$ by *Art.* 123; $\because$ $Nn =$ * $Ddab$, and, dividing each Side of this Equation by Dd, we have $\frac{Nn}{Dd} =$ † ab; but $ab =$ the Product of $\frac{N}{D}$ by $\frac{n}{d}$, because $a = \frac{N}{D}$ and $b = \frac{n}{d}$ by the Supposition; $\because$ $\frac{Nn}{Dd} = \frac{N}{D} \times \frac{d}{n}$. *Q. E. D.* * 55.

CHAP. XLVII.

DIVISION of FRACTIONS.

607. AS Division of whole Numbers ſhews how often one Integer is contained in another Integer, ſo Diviſion of Fractions ſhews how often one Fraction is contained in another.

608. If the Numerator of any Fraction be made a Denominator, and the Denominator a Numerator, the Fraction, ſo made, is called the Reciprocal of the Former. Thus $\frac{2}{3}$ is the Reciprocal of $\frac{3}{2}$.

609. In Diviſion * as the Diviſor is to the Dividend, ſo is an Unit to the Quotient (both in whole Numbers and Fractions.)

610. To divide one Fraction by another, the *Rule* is: Having made the ſame Preparation as directed in Multiplication, multiply the Denominator of the Diviſor by the Numerator of the Dividend, for a Numerator; and the Denominator of the Dividend by the Numerator of the Diviſor, for a Denominator. Or, which is in Effect the ſame, change the Diviſor into its Reciprocal, and then work by Multiplication of Fractions.

611. *Example.* Divide $\frac{3}{4}$ by $\frac{2}{5}$.

Solution. By the *Rule*, $5 \times 3 = 15 =$ the Numerator; and $4 \times 2 = 8 =$ the Denominator; and ſo $\frac{15}{8} =$ the required Quotient, $= 1\frac{7}{8}$ by *Caſe* the 3d. Or thus, the Reciprocal of the Diviſor $\frac{2}{5}$ is $\frac{5}{2}$; and $\frac{3}{4} \times \frac{5}{2} =$ (by Multiplication of Fractions) $\frac{3 \times 5}{4 \times 2} = \frac{15}{8}$ as before.

612. The Reaſon of the firſt Method of Operation, in the laſt *Article*, may be eaſily ſhewn. For there

* Let $m =$ the Dividend, $d =$ the Diviſor, $q =$ the Quotient; then $\frac{m}{d} = q$; but this is the ſame as $\frac{m}{d} = \frac{q}{1}$ (becauſe * 184, an Unit does not divide) therefore * as $d : m :: 1 : q$. Q. E. D.

there it was required to divide $\frac{3}{4}$ by $\frac{2}{5}$. Now, if it had been required to divide $\frac{3}{4}$ by 2, it is manifest the Quotient would be $\frac{1}{2}$ of $\frac{3}{4}$, or $\frac{3}{4 \times 2}$; but since it was only required to divide by $\frac{2}{5}$, that is, $\frac{1}{5}$ of 2, it is evident, $\frac{1}{5}$ of 2, or $\frac{2}{5}$, must be contained 5 Times as often in $\frac{3}{4}$ as 2 is, that is, $\frac{3 \times 5}{4 \times 2}$ = the Answer, which is according to the *Rule* *.

613. As to the other Method of Operation, as it brings out $\frac{3 \times 5}{4 \times 2}$ as the first does, if the first be true, this last Method must also.

614. When the Fractions to be divided have both the same Denominator, it is sufficient to divide the Numerator of the Dividend by the Numerator of the Divisor, or, which is the same in Effect, to set them like a Fraction. For by the *Note* to *Art.* 612. it appears, that the Quotient of $\frac{N}{D}$ by $\frac{n}{D}$ would be $\frac{D}{n}$.

615. Divide $\frac{3}{5}$ by $\frac{4}{5}$. The Quotient $= \frac{3}{4}$.

616. By duly considering the direct contrary Effects of Multiplication and Division, we have this ge-

* There are many other Methods of shewing the Truth of the *Rule*, one of which is: Let $\frac{N}{D}$ be to be divided by $\frac{n}{d}$, these in one common Denominator are * $\frac{Nd}{Dd}$ and $\frac{nD}{Dd}$; $\because$ $\frac{N}{D} \div \frac{n}{D} =$ † $\frac{Nd}{Dd} \div \frac{nD}{Dd}$; but these last Fractions, having one common Denominator, are in Proportion to each other as their § Numerators, *viz.* as $Nd : nD :: \frac{Nd}{Dd} : \frac{nD}{Dd}$; $\because$ ¶ $\frac{Nd}{nD} =$ ‡ $\frac{Nd}{Da} \div \frac{nD}{Dd}$; but by the above, $\frac{N}{D} \div \frac{n}{d}$ is also $= \frac{Nd}{Dd} \div \frac{nD}{Dd}$ $\because$ $\frac{Nd}{nD} =$ ‖ $\frac{N}{D} \div \frac{n}{d}$. Q. E. D.

* 571.
† 108.
§ 553.
‡ 184.
‖ 23.

Corollary. ¶ Hence, if $\frac{N}{D}$ was to be divided by $\frac{n}{D}$, the Quotient would be = the Numerator of the Dividend, divided by the Numerator of the Divisor, $= \frac{N}{n}$.

general *Corollary*: That it is the ſame, in Effect, to multiply by any Number, whether integral or fractional, or to divide by its Reciprocal. For Inſtance, $3 \times 4 = 3 \div \frac{1}{4}$, or generally $a \times b = a \div \frac{1}{b}$, each being (by their reſpective Rules of Multiplication or Diviſion) $= ab$; alſo $a \times \frac{b}{c} = a \div \frac{c}{b} = \frac{ab}{c}$. Hence, any Thing that can be done by Multiplication, by taking the Reciprocal of the Multiplier, may be done by Diviſion; and, on the contrary, any Thing that can be done by Diviſion, may, by taking the Reciprocal of the Diviſor, be done by Multiplication.

617. We ſhall only add one Thing more under this Head, by Way of *Corollary*, and that is, that, if any Number, whether whole or fractional, be divided by a proper Fraction, the Quotient will be more than the Dividend; but, if the Diviſor be an improper Fraction, the Quotient will be leſs than the Dividend.

CHAP. XLVIII.

The RULE *of* THREE DIRECT *in* FRACTIONS.

618. HERE, as in whole Numbers, the ſecond and third Numbers muſt be multiplied together, and the Product divided by the firſt. But the Multiplication and Diviſion muſt be performed by the *Rules* for Multiplication and Diviſion of Fractions. Or the Anſwer, or fourth Number, may be found by this *Rule*, (which is the ſame in Effect:) Multiply the Denominator of the firſt by the Numerators of the ſecond and third Numbers, for the Numerator; and the Numerator of the firſt by the Denominators of the ſecond and third, for a Denominator.

619. *Example* 1. At 1 *l.* $\frac{1}{3}$ *per* C, what is that for 3 ℔ $\frac{1}{2}$? By Reduction of Fractions 1 *l.* $\frac{1}{3} = \frac{4}{3}$ of a £; and 3 $\frac{1}{2}$ ℔ $= \frac{7}{2}$ of a ℔; and 1 C in the raction of a ℔ $= \frac{112}{1}$. Hence the Stating would be, if $\frac{112}{1}$ ℔ : $\frac{4}{3}$*l.* :: $\frac{7}{2}$ ℔ : the Answer. By Multiplication of Fractions $\frac{4}{3} \times \frac{7}{2} = \frac{4 \times 7}{3 \times 2} = \frac{28}{6}$; and, by Division of Fractions, $\frac{28}{6} \div \frac{112}{1} = \frac{28}{6} \times \frac{1}{112} = \frac{28}{672} =$ by Abbreviation $\frac{7}{168}$ of a £. = by Valuation 10 Pence.

By the second Method above-mentioned, we have $112 \times 3 \times 2 = 672 =$ the Denominator, and $1 \times 4 \times 7 = 28 =$ the Numerator; and ∵ the Answer $= \frac{28}{672}$ as before. Hence plainly appears the Agreement of the two Methods.

620. *Example* 2. Admit a Dog is pursuing a Hare that is 50 Yards a Head of him; and that, for every Yard the Hare runs, the Dog runs 2 $\frac{1}{2}$ Yards: *Quære*, How many Yards the Hare will run, before the Dog gets up with her?

Solution. In the Time that the Dog runs 2 $\frac{1}{2}$ Yards, the Hare runs 1 Yard, by the *Question*; ∴, in the Time that the Hare runs 1 Yard, the Dog gains upon her 2 $\frac{1}{2}$. — $1 = 1 \frac{1}{2}$ Yard; hence the *Question* will now be to this Purpose, if, whilst the Dog gains on the Hare 1 $\frac{1}{2}$ Yard, the Hare runs 1 Yard, how many Yards will the Hare have run, when the Dog hath gained 50 Yards upon her, or, in other Words, hath caught her? Hence, 1 $\frac{1}{2}$ being $= \frac{3}{2}$, the Stating will be, as $\frac{3}{2} : \frac{1}{1} :: \frac{50}{1}$: the Answer, which is thus found, $2 \times 1 \times 50 = 100$ for the Numerator, and $3 \times 1 \times 1 = 3$ for the Denominator; and ∵ the Answer $= \frac{100}{3}$ of a Yard $= 33 \frac{1}{3}$ Yards. But, if it had been required to find how many Yards the Dog must run to overtake the Hare, the Stating would have been, as $\frac{3}{2} : \frac{5}{2} :: \frac{50}{1}$: the Yards the Dog must run; ∵ $2 \times 5 \times 50 = 500$ for the Numerator, and $3 \times 2 \times 1 = 6$ for the Denominator; and ∵ the Dog must run $\frac{500}{6}$ of a Yard $= 83 \frac{1}{3}$ Yards. And the Truth of these Operations may be easily proved thus: By the *Question*, the Dog must run 50 Yards more than the Hare, but

but $83\frac{1}{3}$ Yards $-$ $33\frac{1}{3}$ Yards $=$ 50 Yards, for Proof.

621. *Scholium.* In the *Rule* of Three, *&c.* in Order to avoid Fractions, as much as might be, we generally bring the middle Term into the loweſt Species; but, as the Learner is now ſuppoſed acquainted with Valuation of Fractions, he may many Times ſave ſome Trouble by putting it down in the Species given in the *Queſtion*, if it be but one; and, if more than one, by reducing no lower, than the leaſt Species mentioned in the *Queſtion*; and then, when we have found the Anſwer in that Species, if there remain any Fraction, we can by Valuation find its Value, in the inferior Species. Take an *Example.* In *Article* 198. it was required to find the Value of 2437 lb, at 13*s.* *per* C. This ſtated will be, if 112 lb : 13*s.* :: 2437 lb : $\frac{2437 \times 13}{112}$ *s.* $= \frac{31681}{112} =$ 282*s.* $\frac{97}{112}$ $=$ 14*l.* 2*s.* $\frac{97}{112}$, and by Valuation $\frac{97}{112}$ of a Shilling $=$ 10*d.* 1*qr.* $\frac{64}{112}$, and ∴ the Anſwer is 14*l.* 2*s.* 10*d.* 1*qr.* $\frac{64}{112}$, and this Fraction may be abbreviated to $\frac{4}{7}$.

CHAP. XLIX.

INVOLUTION of FRACTIONS.

622. INVOLVE the Numerator for a Numerator, and the Denominator for a Denominator. The Reaſon of this is evident, from Involution of whole Numbers, and Multiplication of Fractions.

623. *Example.* What is the Square of $\frac{2}{3}$?

Solution. $2 \times 2 = 4 =$ the Numerator, and $3 \times 3 = 9 =$ the Denominator, and ſo the Square of $\frac{2}{3} = \frac{4}{9}$.

CHAP. L.

EVOLUTION of FRACTIONS.

624. 1. IF the Fraction, whose Root is to be found, is an immediate Power of some Root, (formed as is shewn in Involution of Fractions) its Root may be found by extracting the Root of the Numerator for a Numerator, and the Root of the Denominator will be the Denominator, as is manifest.

2. But sometimes the Fraction proposed is not an immediate Power, but equal to such a Power of the Root; then we must reduce the given Fraction to its lowest Terms, and find the Root as above directed.

3. But if it should so happen, that the proposed Fraction, when reduced to its lowest Terms, cannot have the perfect Root of both its Numerator and Denominator found, then we may be assured, that it is neither an immediate Power, nor an Equivalent to one; and in such *Case* must turn the Fraction into a Decimal, and sometimes be contented with an Approximation of its Root; this we shall illustrate, when when we treat of Decimals.

625. *Example* 1. Extract the Square Root of $\frac{4}{9}$.

Solution. The Square Root of $4 = 2$, of $9 = 3$; $\because$ the required Root is $= \frac{2}{3}$.

626. *Example* 2. Extract the Cube Root of $\frac{80}{1250}$.

Solution. It does not admit of the true Root, as it stands here, but by Abbreviation it is $= \frac{8}{125}$; now the Cube Root of $8 = 2$; and of $125 = 5$; $\because$ the Root is $\frac{2}{5}$.

CHAP. LI.

Of POSITION *by* FRACTIONS.

627. IN Position, it may be obferved, we always fuppofed fuch Numbers as might avoid Fractions in the Operation; becaufe the Operation would be more fimple, and the Learner was not fuppofed at that Time to underftand the fundamental *Rules* of Fractions. But, as fometimes it may happen that fuch Numbers are not eafily thought on, perhaps it may not be ufelefs to give an *Example* folved by a fractional Operation.

628. *Example*. Let it be required to give a *Solution* to *Queftion* 2. *Art*. 506.

Solution. Here we put any Number, as 1 *d*, for a Share; then 4 Men muft have $1 \times 4 = 4$; the Captain $1\frac{1}{2}$, and the Boy $\frac{1}{3}$, the Sum of 4, $1\frac{1}{2}$, and $\frac{1}{3}$, $= 5\frac{1}{2} + \frac{1}{3} = 5\frac{5}{6} = \frac{35}{6}$. Now as $\frac{35}{6}$: 1 *d*. :: $\frac{51055}{1}$: $\frac{6 \times 1 \times 51055}{35 \times 1 \times 1} = 8752\,d.\ \frac{10}{35}$ as in *Art*. 506; and the remaining Part of the *Solution* is the fame as in that *Article*.

629. We think it needlefs to apply Fractions to any more *Rules* of Vulgar Arithmetic; becaufe, if the Reader rightly underftands what has been already laid down, he cannot, when Occafion requires, be at a Lofs to apply it to any other *Rule* in common Arithmetic.

MA-

Mathematical Essays.

ESSAY II.

Containing Decimal Arithmetic.

CHAP. I.

Notation *of* Decimals.

1. A Decimal Fraction (from *Decimus Lat.*) is a Fraction whose Denominator is 10, or 100, or 1000, or 10000, *&c.* For here we suppose any Integer to be divided into 10 Parts; and each of these into 10 Parts, making in the Whole 100 Parts; and each of these last Parts into 10 Parts, making in the Integer 1000 Parts; *&c.* at Pleasure; and any Number of these Parts are called Decimal Parts, and are the Numerator of the Fraction, by which we would express how many Parts we would signify; and the Number of Parts into which the Integer is divided, is the Denominator.

2. Hence these Parts may be expressed as in Vulgar Fractions; but, for the more ready Management, the Denominators are omitted, and only the Nume-

rators ſet down, with a Dot (.) or Comma (,) on the left Hand, to diſtinguiſh them from whole Numbers; and there is no Neceſſity of ſetting down the Denominators, becauſe they are always known by their Diſtance from the Decimal Point (.). For,

3. As it was neceſſary, for the better conveying of our Ideas in Writing, to fix on ſome Method, which ſhould be uſed and underſtood, by all Arithmeticians, for Writing of Decimals, the following is through Cuſtom become ſuch. Firſt make the Decimal Point, then, if the Number of Parts we would expreſs be Parts of 10, or Tenths, we write immediately on the right Hand of the Point (.); but, if we would expreſs Parts of 100, we put the Figure denoting the Number of Parts taken, in the ſecond Place from the Point (.), reckoning towards the right Hand; and, if there be no Figure already in the firſt Place, we ſupply the Vacancy with a Cypher (0); and ſo, if we would expreſs Parts of 1000, we write the Figure expreſſing them in the third Place, on the right Hand of the Decimal Point, ſupplying the Vacancies, if any, in the firſt and ſecond Decimal Places, with Cyphers (00). &c.

4. Hence, it may be obſerved here, as well as in whole Numbers, that 10 of any Decimal Place makes an Unit in the next Superior; and therefore, if the Number of Decimal Parts we would expreſs conſiſts of more than one Figure, we write that Figure whoſe real Value is the leaſt, on the right Hand of the reſt, in its proper Place, and the others in a ſucceſſive Order from the right Hand to the left; for *Example*, if it was required to write 154 Parts of a Thouſand, decimally, we firſt conſider that the 4 is 4 Parts of a Thouſand, and therefore by the laſt *Article* is to ſtand in the third Place of Decimals thus, .004, and then, putting the other two Figures in the Places of the Cyphers, the Number would be .154; for the 5, repreſenting 50 Parts of a Thouſand, is the ſame as 5 Parts of 100, and, therefore, muſt ſtand in the Place of hundredth Parts; as muſt the 1 in the

Place

Place of 10ths, because it signifies 100 Parts of 1000, or $\frac{1}{10}$.

5. Hence it will be no difficult Matter to read any Number of Decimal Parts, when written as above directed; for, from what has been just said, it plainly follows, that the Denominator is 1 with as many Cyphers on the right Hand, as there are Decimal Places in the Numerator; therefore, if we imagine the Decimal Point to be a 1, and the Figures in the Decimal Places to be all Cyphers (o's), the Number so, formed to the Imagination, will be the Denominator of the Decimal Fraction.

Thus .23 will be read $\frac{23}{100}$; as will .0125 be $\frac{125}{10000}$, $.125 = \frac{125}{1000}$, $1:25 = 1\frac{25}{100}$; and $12.5 = 12\frac{5}{10}$. Also $.5 = \frac{5}{10}$, $.05 = \frac{5}{100}$, $.005 = \frac{5}{1000}$, *&c.*

6. Hence it is evident, that each Cypher to the left Hand of any Decimal Part (though Cyphers in themselves signify nothing, yet as they remove the Figures further from the Point) decrease its Value 10 Times; and also that Cyphers, on the right Hand of such Parts, do not alter their Value, because they increase the Denominator, in the same Proportion as they do the Numerator; and therefore it is the same in Effect * as if they had not been so augmented. Essay 1 * Art. 552.
Thus $.5 = .50, = .500$, *&c.* because $\frac{5}{10} = \frac{50}{100} = \frac{500}{1000}$, *&c.*

7. We shall only remark further in this Chapter, that, since † the Value of Decimal Places increases, † 4.
or decreases, in a tenfold Proportion, as well as whole Numbers, it is manifest, that (finite) Decimals may be added, subtracted, multiplied, and divided, after the same Manner as whole Numbers.

☞ In our Marginal References, if there is 1 on the left of the *, it denotes the Reference to be the first Essay, and the Number on the right Hand the Article of that Essay; but, where there is no Number on the left of the *, the Reference is always to be understood of the present Essay.

CHAP. II.

ADDITION *of* DECIMALS.

8. HERE, Care muſt be taken to put each Figure of the ſeveral Numbers to be added, under thoſe of the ſame Name, as Tenths under Tenths, Hundredths under Hundredths, Thouſandths under Thouſandths, &c. (as in Vulgar Arithmetic we put Units under Units, Tens under Tens, &c.) and this is eaſily done, by placing the Decimal Points of the ſeveral Numbers directly under each other. Then find the Sum as in Addition of whole Numbers.

9. *Example.* Add 4.72, 87.123, .057, and 2.3 together. The Numbers, truly placed, appear thus:

$$\left\{\begin{array}{r}4.72\\87.123\\.057\\2.3\end{array}\right. \quad \text{or thus} \quad \left\{\begin{array}{r}4.720\\87.123\\.057\\2.300\end{array}\right.$$

$$\text{Sum} \quad \underline{94.200} \qquad \text{Sum} \quad \underline{94.200}$$

So that the whole Sum is 94 and (.200, or which is the ſame) .2 of another.

CHAP. III.

SUBTRACTION *of* DECIMALS.

10. HAVING placed the Numbers as directed in Addition of Decimals, ſubtract as in Subtraction of whole Numbers.

11. *Example.* From 17.3 ſubtract 2.857. Place the Numbers thus:

17.3

$$\begin{cases}17.3 \\ 2.857\end{cases} \quad \text{Or thus} \quad \begin{cases}17.300 \\ 2.857\end{cases}$$

Difference 14.443 Difference 14.443

CHAP. IV.

MULTIPLICATION *of* DECIMALS.

12. IN Multiplication of Decimals, we multiply as in whole Numbers. But from the Product we cut off, or separate, by the Decimal Point (.), as many Figures, counting from the Left towards the right Hand, as there are Decimal Places in both the Multiplicand and Multiplier. And, if it should happen that there are not as many Figures in the Product, as there are Places in both the Multiplier and Multiplicand, that Deficiency must be made up by placing Cyphers to the right Hand.

13. *Examples.* Multiply 3.14 by 2.5, 23.01 by 33.17, and .253 by 0.23. See the Operation.

```
 3.14         33.17        .253
 2.5          23.01        .023
 ----         -----        ----
 1570          3317         759
 628          99510        506
 -----       6634         -------
 7.850       --------     .005819
             763.2417
```

14. All that is necessary to be here shewn is the Truth of the above Method of separating the Decimal Places in the Product; and this may be done by observing, that both the Multiplicand and Multiplier may be considered as Vulgar Fractions, proper, or improper; thus, let any Multiplicand, taken as a whole Number, be denoted by N, and let $D =$ the Number of Decimal Places in the Multiplicand, then its true Value, represented as a Vulgar Fraction, is $\frac{N}{1 \text{ with } D, 0\text{'s}}$: Now let the Multiplier be represented

in the same Manner by $\frac{n}{1 \text{ with } d, \text{o's}}$; and, by Multiplication of Vulgar Fractions, the Product $=$
$\frac{N \times n}{1 \text{ with } D, \text{o's} \times 1 \text{ with } d, \text{o's}} = \frac{N \times n}{1 \text{ with } D + d, \text{o's}}$;
* 1 * 67 therefore, by the Nature of Decimal Notation, we must cut off $D + d$ Places for the Decimal Parts, in the Product $N \times n$.

15. As there are some useful Contractions in common Multiplication, so there are also in Multiplication of Decimals; and such are the following: 1. When it is required to multiply by 10, 100, 1000, *&c.* it is only to remove the Decimal Points as many Places to the right Hand as there are Cyphers in the Multiplier.

16. *Example.* Multiply 32.1341 by 1000.

Answer. By placing the Decimal Point 3 Places to the right Hand the Product is 32134.1; and the Reason is evident, for the removing the Point (.) 3 Places to the right Hand is decreasing the Denominator 1000 Times, by the Nature of Decimal Notation; and decreasing the Denominator is the same in Effect as multiplying the Numerator. Or the Reason will easily appear by comparing it with the Operation by the common Way in the Margin.

The common Way

	32.1341
x	1000
Prod.	32134.1000

17. *Case* 2. When large Decimal Numbers are to be multiplied by each other, it is many Times unnecessary to have all the Places of Decimals in the Product, that would arise from the whole Operation; because four or five Places are sufficiently exact, for most Purposes; for which Reason, it will be useful to explain a Method of shortening the Operation, by retaining, in the Product, so many Places only, as we shall at any Time think exact enough for our intended Design; and such is the following, *viz.* Put the Units Place of the Multiplier under that Figure of the Multiplicand, whose Place you are willing to keep in the Product; then write

write the Multiplier, in an inverted or retrograde Order; and, in multiplying, begin always with that Figure of the Multiplicand, which ſtands directly over that of the Multiplier which you are going to multiply by; remembering to add in the Increaſe, or Carriage, that would ariſe from the two next Figures, (were they to be multiplied) that are to the right Hand of that Figure, which you begin with in the Multiplicand; alſo remembering to let the firſt Figure of each particular Product ſtand directly one under the other.

18. *Example* 1. Let it be required to multiply 54.321711 by 3.12321, and have only four Places of Decimals in the Product. Here we put the Units Place of the Multiplier 3 under the fourth Place of the Multiplier 7:

	54.321711
Multiplier inverted	12321.3
543217 × 3 =	1629651
54321 × 1, + 1 =	54322
5432 × 2 =	10864
543 × 3, + 1 =	1630
54 × 2, + 1 =	109
5 × 1 =	5
	169.6581

In the ſecond Multiplication, becauſe 7 × 1 = 7 is nearer to carrying 1 than 0, we add in one; and, in like Manner, in the other Products:

	54.321711
	3.12321
5	4321711
108	643422
1629	65133
10864	3422
54321	711
1629651	33
169.6581	1101231

By comparing the foregoing Contraction with the Operation here worked at large, the Reaſon of that

 Method

Method will plainly appear; for the Figures to the right Hand of the Line are omitted in that short Method, and the Operation inverted, the last particular Product here being the first in that.

19. *Example* 2. Multiply 231.312 by 21.32, and have only 3 Places of Decimals in the Product.

The contracted Way		The common Way
	231.3121	231.3121
Multiplier inverted	23.12	21.32
	4626242	4626|242
	231312	69393|63
	69394	231312|1
	4626	4626242|
	4931.574	4931.573|972

20. *Example* 3. Multiply 432.12 by 0, 785, and have only the whole Numbers in the Product.

By Contraction		By the common Method
	0432.12	432.12
	587.0 The inverted Multiplier	0.785
	302	2|16060
	35	34|5696
The Carriage from the +	2	302|484
	339	339|.21420

21. Multiplication of Decimals may also be contracted, without inverting the Multiplier, by the following *Rule*, *viz.* The Multiplier and Multiplicand being placed as in their natural Order, from the Number of Decimal Places in both the Factors, deduct the Number of Decimal Places that you intend to keep in the Product; and then cut off as many Figures as remain from the Multiplicand, counting from the right Hand towards the Left; but, if there is not a sufficient Number of Figures in the *Multi-*

N. B. As some Figures of the Product are omitted in these Contractions, the Products cannot be proved by casting out the Nines.

Multiplicand, cut off the Defect from the Multiplier. Then multiply the Figures to the left of the Line of Separation, by the first Figure (to the right Hand) of the Multiplier, remembering to add the Carriage that would arise from the Multiplication of two Figures to the right Hand (which must be also done in the Multiplication by the other Figures) and set down the Product.

Secondly, in Multiplying by the second Figure take in one Figure more of the Multiplicand; and in this Manner proceed, each Time taking in one Figure more of the Multiplicand, and writing the Units Place of each particular Product directly under the Units Place of the superior or preceding Product.

Note. In Order to prevent forgetting what Figures of the Multiplicand and Multiplier we are at any Time come to, it may be useful to dot as we proceed: An *Example*, or two, will make this intelligible.

22. *Example* 1. Multiply 54.321711 by 3.12321, and have only four Places of Decimals in the Product.

Here are 11 Decimal Places in both the Multiplicand and Multiplier, and we are only to have 4 in the Product, ∴ we have $11 - 4 = 7$ Places to be cut off in the Multiplicand.

$$
\begin{array}{rl}
5|4.321711 & \\
3.12321 & \\
\hline
5 & = 5 \times 1 \\
109 & = 54 + 2, + 1 \text{ Carriage} \\
1630 & = 543 \times 3, + 1 \text{ carried} \\
10864 & = 5432 \times 2 \\
54322 & = 54321 \times 1, + 1 \text{ for nearest Car.} \\
1629651 & = 543217 \times 3 \\
\hline
169.6581 &
\end{array}
$$

23. *Example* 2. Multiply 43.212 by 0.785, and have only the Integers in the Product.

 Here

Here being 6 Places to be cut off, and only 5 Figures in the Multiplicand, all the five must be cut off, and one Figure from the Multiplier.

	0|43.21
	.78
0 x 8 + 3 carried =	3
4 x 7, + 2 carried =	30
	33

Whoever compares this Method of contracting Decimal Multiplications with that before delivered, will plainly see, that they are in Effect the same. We have given both Methods, that the Learner make Use of that which he likes best.

CHAP. V.

DIVISION *of* DECIMALS.

24. IN Division of Decimals, take the Divisor and Dividend as whole Numbers, and divide as has been already taught in Division of whole Numbers; but, if there are not so many Decimal Places in the Dividend as there are in the Divisor, that Defect is first to be supplied by annexing o's to the right Hand of the Dividend; in like Manner, if the Divisor, considered as a whole Number, cannot be taken once out of the Dividend, also considered as a whole Number, we must first add as many Cyphers (o's) to the right Hand of the Dividend, as will make the Dividend, taken as a Whole, greater than (or at least equal to) the Divisor. And the annexing these o's does not alter the Value of the Dividend, as appears by Notation of Decimals; but only prepares it for the Operation.

Having found how many Times the Divisor is contained in the Dividend, both considered as whole Numbers, we must now see, how many of the Places in the Quotient must be Decimal Parts; and this we do by the following *Rule*, *viz.* Mark off in the Quotient as many Places of Figures for the Decimal Part, as

as there are Decimal Places in the Dividend more than in the Divisor. The Reason of which will easily appear; for, by Multiplication of Decimals, the Decimal Places in the Multiplicand *plus* the Decimal Places in the Multiplier, are equal to the Decimal Places in the Product; and, by the Proof of Division, the Divisor, multiplied by the Quotient, is equal to the Dividend; therefore, the Decimal Places in the Divisor *plus* the Decimal Places in the Quotient are equal to the Decimal Places in the Dividend; hence, by subtracting the Decimal Places in the Divisor from both Sides of this Equation, we have * the Decimal Places in the Quotient equal to the Decimal Places in the Dividend *minus* the Number of Decimal Places in the Divisor. * 36.

N. B. When it happens that there are not so many Places of Figures in the Quotient, found by the Division, as there must be Decimal Places in the Quotient, that Deficiency must be supplied by placing Cyphers to the right Hand of the Figures.

Note. If, after all the Figures in the Dividend have been taken down in the Operation, there be a Remainder, we may continue the Division by adding Cyphers, each Cypher added giving one Decimal Place more in the Quotient. A few *Examples* will better explain this, than more Words.

25. *Example* 1. Divide 763.2417 by 33.17.

$$\begin{array}{r|l}
33.17)\,763.2417\,(& 23.01 \\
\underline{6634\ldots\;\;} & \\
9984 & \\
\underline{9951} & \\
3317 & \\
\underline{3317} & \\
\ldots\ldots &
\end{array}$$

26. *Example* 2. Divide .005819 by .253.

.253).005819(.023
506
———
759
759
———

In this *Example*, 5819 divided by 253 is 23; but, since by the above *Rule* there must be three Decimal Places in the Quotient, we add a Cypher (0), on the left Hand of 23, and then, putting the decimal Point, the Quotient is .023. These two *Examples* are the Reverse of two in Multiplication of Decimals.

27. *Example* 3. Divide 15.73 by 3172.

Here, because we cannot take 3172 out of 1573 once, place a Cypher on the right Hand; then the Operation will stand thus:

3172)15.730(.00495
12688
———
30420
28548
———
18720
15860
———
2860

Here we may continue on the Division at Pleasure, by annexing an 0 each Time; but, first, we must take Notice, that the 4 in the Quotient must be .004; because, at that Time, there were three Places to be cut off.

28. *Example* 4. Divide 1 by 3.

3)1.0(.333
9
——
10
9
——
10
9
——
1

&c. *ad infinitum.* Hence it is evident, that sometimes it will happen, that we cannot get the exact Quotient by Division of Decimals. However, the Division may be continued, till the imperfect Quotient may differ in Value from the Truth, less than any assignable Quantity.

29. *Example* 5. Divide 3.4172 by .34172.

```
.34172 )3.41720( 10 Answer 10, a whole Num.
        3 4172
        ------
            0
```

That 3.4172 ÷ .34172 = 10, is alſo plain from Notation; for it is evident from that *Rule*, that 3.4172 is 10 Times .34172.

30. *Example* 6. Divide 22 by .54.

```
.54 )22.00(40.7407, &c. continually 740
     216                ad infinitum.
     ---
      400
      378
      ---
       220
       216
       ---
         400
         378
         ---
          22
```

We have here made Uſe of the long Method of Diviſion, becauſe we would be underſtood by ſuch Perſons as know no other Method.

31. We ſhall now proceed to the moſt uſeful Contraction in Diviſion of Decimals, *viz.* When it is required to divide by 10, or 100, or 1000, &c. it may be done by only removing the Decimal Point as many Places to the left Hand as the Diviſor contains Cyphers.

32. *Example.* 543.17 Divided by 100 is = 5.4317, found by only removing the Decimal Point 2 Places to the left Hand. The Reaſon is evident, from the Nature of Notation of Decimals.

33. This may alſo be applied to dividing by any Number of Tens, Hundreds, or Thouſands, &c. *Example.* Divide 316.4 by 50: Firſt 316.4 divided by 10, by removing the Point, is 31.64; but, ſince we are to divide by 5 Tens, the Quotient muſt be $\frac{1}{5}$ of 31.64 = 6.328 = the Quotient required.

CHAP.

CHAP. VI.

Of the REDUCTION of VULGAR FRACTIONS to DECIMAL ones, et contrà.

34. *Case* 1. TO reduce a Vulgar Fraction to its equivalent Decimal Fraction, (or near it. Divide the Numerator by the Denominator, by Division of Decimals*.

35. *Example* 1. Change $\frac{3}{12}$ into a Decimal Fraction.

$$\begin{array}{r} 12)3.0(.25 \\ \underline{2\ 4} \\ 60 \\ 60 \end{array} \qquad \textit{Answer } \tfrac{3}{12} = .25.$$

36. *Example* 2, Change $\frac{1}{3}$ into a Decimal Fraction.

$$\begin{array}{r} 3)1.0(.33 \\ \underline{9} \\ 10 \\ \underline{9} \end{array}$$

Here it is evident there will always be 1 remaining, and ∵ the exact Value of $\frac{1}{3}$ cannot be expressed by a Decimal Fraction, but may be expressed nearer than any assigned Quantity, for $\frac{1}{3} = .33333333$, &c. *ad infinitum.*

In Decimal Operations we omit taking Notice of the Remainders, as inconsiderable.

37. *Example* 3. Put 15*s*. 6*d*. $\frac{1}{4}$ into the Decimal of a lb.

Here

* This *Rule* may be demonstrated Algebraically thus: Let $\frac{a}{m}$ be any Vulgar Fraction, d = the Decimal Numerator required, then, *per* Notation of Decimals, $\frac{a}{m} = \frac{d}{1000, \&c.}$ ∵ multiplying by m, we get $a = * \frac{dm}{1000, \&c.}$ this multiplied by 1000, &c. gives $a \times 1000$, &c. $= \dagger\ dm$, which, divided by m, gives $\frac{a \times 1000, \&c.}{m} = \ddagger\ d$. Q. E. D.

* 1 * 56.
† 1 * 56.
‡ 1 * 108.

Here follows the Operation:

$$\begin{array}{ll} 1\,l. = 20s. & 15s.\ 6d.\ \tfrac{1}{4} \\ \times\ 12 & \times\ 12 \\ \hline 240 & 186 \\ \times\ 4 & 4 \\ \hline 1\,l. = 960 \text{ Farthings.} & 745 \\ \hline \end{array}$$

= the Farthings in 15*s.* 6*d.* $\frac{1}{4}$; ∵ 15*s.* 6*d.* $\frac{1}{4}$, in the Fraction of a *l*, is $\frac{745}{960}$, which we must now put into a Decimal Fraction thus:

$$\begin{array}{l} 96|0)745.0(.776041 \\ \quad 672 \\ \quad \overline{730} \\ \quad672 \\ \quad \overline{580} \\ \quad576 \\ \quad \overline{400} \\ \quad384 \\ \quad \overline{160} \\ \quad96 \\ \quad \overline{64} \end{array}$$

Answer .776041 very near the Truth, differing from Truth only $\frac{64}{96000000}$, for $.776041 + \frac{64}{96000000}$ is the exact Quotient; whence it follows, that if, when we have carried on the Division so far as to produce 5 or 6 Places of Decimals, there is still a Remainder, that Remainder may be omitted, as too minute to cause any Error great enough to make it regarded, in the common Purposes of Life.

38. The Reason of this *Case* may be easily shewn, independently of Algebra; for our adding Cyphers to the Numerator, and dividing by the Denominator, is in Effect the same, as multiplying the given Numerator by 10, or 100, or 1000, *&c*; and ∵ the Quotient will be 10, or 100, or 1000, *&c.* Times

as

as much as it would otherwise have been, and ∴ the Numerator will be increased in the same Proportion as the Denominator, that is, as the given Denominator is to its Numerator, so is the Decimal Denominator to its respective Numerator; and, consequently, the Decimal Fraction thus found (if there be
* 552. no Remainder) will be equal to * the Vulgar Fraction given. *Q. E. D.*

39. *Case* 2. To change a Decimal Fraction into a Vulgar Fraction. This is done by only writing the Decimal as a Vulgar Fraction, by writing down its Denominator; which Fraction, so written, may be abbreviated, if it is not in its lowest Terms.

40. *Example.* Change .25 into a Vulgar Fraction.

First, .25, written as a Vulgar Fraction, is $\frac{25}{100}$ = by Abbreviation $\frac{1}{4}$, found by dividing both 25 and 100 by 25, the Quotients being 1 and 4 respectively; which, it is manifest, is the true Answer, for, as 1 is a Fourth of 4, so is 25 a Fourth of 100.

41. *Case* 3. To find the Value of a Decimal Fraction. Multiply by the Number of Units in the next lower Denomination.

42. *Example* 1. What is the Value of .776041?

.776041
1 *l.* = 20 Shillings

Shillings 15 | .520820
1 Shilling = 12 Pence

Pence 6 | .249840
1 *d.* = 4 Farthings

Farthings 0 | .999360

Answer 15 *s.* 6 *d.* $\frac{1}{4}$ very near, being but $\frac{64}{100000}$ of a Farthing too much.

43. *Example* 2. What is the Value of ,931913 of a C?

```
           ·931913
 1 C. =          4 Quarters
       ------------
 Qrs. 3|·727652
 1 Qr. | =    28 lb
       ------------
        5821216
       1455304
       ------------
 lb 20|·374256
 1 lb |  =    16
       ------------
        2245536
        374256
       ------------
 oz.  5|·988096
       |      16
       ------------
        5928576
        988096
       ------------
 Drams 15|·809536
```

Answer 3 *qrs.* 20 lb. 5 *oz.* 15 *drs.* .809536 or 3 *qrs.* 20 lb. 6 *oz.* nearly.

We need say nothing concerning the Reason of these Operations, it being manifest.

44. *Case* 4. To reduce a Vulgar Fraction into a Decimal one, so that the Decimal found, though it be not exact, may yet want less than any assigned Fraction.—Reduce the Vulgar Fraction to a Decimal Fraction by *Case* 1, carrying on the Division, till there are as many Decimal Places in the Quotient, as the Denominator of the assigned Fraction has Figures.

45. *Example*. Let it be required to reduce $\frac{2}{3}$ to a Decimal Fraction, so that the Decimal Fraction may not want $\frac{2}{998}$ of the true Quotient.

$$\begin{array}{r|l} 3 \;) \; 2.0 & \cdot 666 \\ 18 & \\ \hline 20 & \\ 18 & \\ \hline 20 & \\ 18 & \\ \hline 2 & \end{array}$$

The Denominator of the assigned Fraction has three Figures, and ∴ ·666 is the required Answer, according to the above *Rule*; and it is evident, that ·666 does not want $\frac{2}{998}$ of the Truth; for it wants only $\frac{2}{3}$ of an Unit in the last Place of Decimals, *viz.* $\frac{2}{3}$ of $\frac{1}{1000} = \frac{2}{3000}$, which it is manifest is less than $\frac{2}{998}$.

46. The Reason of this *Rule* will easily appear thus: Let $a =$ the Number of Figures in the Denominator of the assigned Fraction; $d =$ the Divisor, and $n =$ the Remainder; then it is plain, that $\frac{n}{d}$ is a (proper) Fraction of an Unit in the last Decimal Place, which Unit is in its real Value $\frac{1}{1 \text{ with } a, \text{ o's}}$, which is less than the assigned Fraction; (because this Denominator is greater than that of the assigned Fraction, and the Numerator of the assigned Fraction cannot possibly be less than that of this, which is 1.) Consequently $\frac{n}{d}$ of $\frac{1}{1 \text{ with } a, \text{ o's}}$ must be less also, for it is less than $\frac{1}{1 \text{ with } a, \text{ o's}}$.

47. Reduction of Decimals admitting of some Compendiums, we shall now proceed to give the most useful; and first, by the Help of Decimal Tables, such as the following, the Reduction of Money, Weights, and Measures will be much facilitated: As to the Method of constructing these Tables, whoever is acquainted with what has been already treated of, cannot be ignorant of it; for which Reason, we shall omit giving needless Directions.

The

Table I. of Money, 1 *l.* the Integer.

Farthings	
1	·0010416
2	·0020833
3	·003125
Pence	
1	·0041666
2	·0083333
3	·0125
4	·0166666
5	·0208333
6	·025
7	·0291666
8	·0333333
9	·0375
10	·0416666
11	·0458333
Shillings	
1	·05
2	·1
3	·15
4	·2
5	·25
6	·3
7	·35
8	·4
9	·45
10	·5
11	·55
12	·6
13	·65
14	·7
15	·75
16	·8
17	·85
18	·9
19	·95

Table II. Troy Weight, 1 Ounce the Integer.

Grains	
1	·0020833
2	·0041666
3	·00625
4	·0083333
5	·0104166
6	·0125
7	·0145833
8	·0166666
9	·01875
10	·0208333
11	·0229166
12	·025
13	·0270833
14	·0291666
15	·03125
16	·0333333
17	·0354166
18	·0375
19	·0395833
20	·0416666
21	·04375
22	·0458333
23	·0479166

The Table of Penny-weights is the ſame as the Shillings in the Money Table.

Table III. Avoirdupois Weight, 1 ℔ the Integer.

Qrs. of a Dram.	
1	·00097656
2	·00195313
3	·0029296
Drams	
1	·00390625
2	·0078125
3	·01171875
4	·015625
5	·01953125
6	·0234375
7	·02734375
8	·03125
9	·03515625
10	·0390625
11	·04296875
12	·046875
13	·05078115
14	·0546875
15	·05859375
Ounces	
1	·0625
2	·125
3	·1875
4	·25
5	·3125
6	·375
7	·4375
8	·5
9	·5625
10	·625
11	·6875
12	·75
13	·8125
14	·875
15	·9375

Table IV. Avoirdupois Weight, 1 C the Integer.

Ounces	
1	·0005580
2	·0011160
3	·0016741
4	·0022321
5	·0027901
6	·0033482
7	·0039062
8	·0044642
9	·0050223
10	·0055803
11	·0061383
12	·0066964
13	·0072544
14	·0078125
15	·0083705
Pounds	
1	·0089285
2	·0178571
3	·0267857
4	·0357142
5	·0446428
6	·0535714
7	·0625 exact.
8	·0714285
9	·0803571
10	·0892857
11	·0982142
12	·1071428
13	·1160714
14	·125 exact.
15	·1339285
16	·1428571
17	·1517857
18	·1607142
19	·1696428
20	·1785714
21	·1875 ex.

Pounds.	
22	·1964285
23	·2053571
24	·2142857
25	·2232142
26	·2321428
27	·2410714
Quarters	
1	·25
2	·5
3	·75

Table V. Long Measure, 1 Foot the Integer.

Inches	
1	·0833333
2	·1666666
3	·25
4	·3333333
5	·4166666
6	·5
7	·5833333
8	·6666666
9	·75
10	·8333333
11	·9166666

Table VI. Cloth Measure, 1 Yard the Integer.

Nails	
1	·0625
2	·125
3	·1875
Qrs. of a Yard.	
1	·25
2	·5
3	·75

Table VII. Liquid Measure, 1 Gallon the Integer.

Qrs. of a Pint.	
1	·03125
2	·0625
3	·09375
Pints.	
1	·125
2	·25
3	·375
4	·5
5	·625
6	·75
7	·875

Table VIII. Dry Measure, 1 Qr. the Integer.

Pints	
1	·001953
2	·003906
3	·005859
Qrs. of a Peck.	
1	·0078125
2	·015625
3	·0234375

Pecks and Bushels, the same as Quarters of Pints, and Pints in Liquid Measure.

Table IX. Of Time, 1 Year (of 365 Days) the Integer.

Days	
1	·002739726
2	·005479452
3	·008219178

Days	
4	·010958904
5	·01369863
6	·016438356
7	·019178062
8	·021917808
9	·024657534
10	·02739726
20	·05479452
30	·08219178
40	·10958904
50	·13698630
60	·16438356
70	·19178062
80	·21917808
90	·24657534
100	·27397260
200	·54794520
300	·82191780

Table X. Of Time, 1 Hour the Integer, or for Minutes of Motion, &c. 1 Degree the Integer.

Minutes	
1	·0166666
2	·0333333
3	·05
4	·0666666
5	·0833333
6	·1
7	·1166666
8	·1333333
9	·15
10	·1666666
20	·3333333
30	·5
40	·6666666
50	·8333333

Seconds	
1	·0002777
2	·0005555
3	·0008333
4	·0011111
5	·0013888
6	·0016666
7	·0019444
8	·0022222
9	·0025
10	·0027777
20	·0055555
30	·0083333
40	·0111111
50	·0138888

A TABLE, ſhewing the Number of Days, from any Day in any Month to the ſame Day of any other Month.

From	January	February	March	April	May	June
	Feb. 31	March 28	April 31	May 30	June 31	July 30
	March 59	April 59	May 61	June 61	July 61	Aug. 61
	April 90	May 89	June 92	July 91	Aug. 92	Sept. 92
	May 120	June 120	July 122	Aug. 122	Sept. 123	Oct. 122
	June 151	July 150	Aug. 153	Sept. 153	Oct. 153	Nov. 153
To	July 181	Aug. 181	Sept. 184	Oct. 183	Nov. 184	Dec. 183
	Aug. 212	Sept. 212	Oct. 214	Nov. 214	Dec. 214	Jan. 214
	Sept. 243	Oct. 242	Nov. 245	Dec. 244	Jan. 245	Feb. 245
	Oct. 273	Nov. 273	Dec. 275	Jan. 275	Feb. 276	March 273
	Nov. 304	Dec. 303	Jan. 306	Feb. 306	March 304	April 304
	Dec. 334	Jan. 334	Feb. 337	March 334	April 335	May 334
	Jan. 365	Feb. 365	March 365	April 365	May 365	June 365
From	**July**	**Auguſt**	**Septem.**	**October**	**Novem.**	**Decem.**
	Aug. 31	Sept. 31	Oct. 31	Nov. 31	Dec. 30	Jan. 31
	Sept. 62	Oct. 61	Nov. 61	Dec. 61	Jan. 61	Feb. 62
	Oct. 92	Nov. 92	Dec. 91	Jan. 92	Feb. 92	March 90
	Nov. 123	Dec. 122	Jan. 122	Feb. 123	March 120	April 121
	Dec. 153	Jan. 153	Feb. 153	March 151	April 151	May 151
To	Jan. 184	Feb. 184	March 181	April 182	May 181	June 182
	Feb. 215	March 212	April 212	May 212	June 212	July 212
	March 243	April 243	May 242	June 243	July 242	Aug. 243
	April 274	May 273	June 273	July 273	Aug. 273	Sept. 274
	May 304	June 304	July 303	Aug. 304	Sept. 304	Oct. 304
	June 335	July 334	Aug. 334	Sept. 335	Oct. 334	Nov. 335
	July 365	Aug. 365	Sept. 365	Oct. 365	Nov. 365	Dec. 365

Though this *Table* is not a Decimal One, yet its Uſe is ſufficient to apologiſe for our inſerting it here.

N. B. As this *Table* is made for ſuch Years as have 365 Days, when it is Leap-Year, and the Month of *February* comes in, we muſt add 1 to the Number of Days found by this *Table*.

48. *Case* 5. To put Money, Weights, and Measures, into Decimals by the Help of Tables. This is best illustrated by an *Example*. Let it be required to put 15*s*. 6*d*. $\frac{1}{4}$ into the Decimal of a £?

By Table 1st.	15 Shillings	=	.75
	6 Pence	=	.025
	1 Farthing	=	.0010416
Hence 15*s*. 6*d*. 1*qr*.		=	.7760416 of a £.

See *Art*. 37.

49. *Case* 6. To find the Value of a Decimal Part of Money, Weight, or Measure, by the Help of Decimal *Tables*. An *Example* will shew the Method better than a Multitude of Words. *Example*. What is the Value of .931913 of a C?

	3 *qrs*.	=	.931913
			.75
		Remains	.181913
By Table 4th	20 lb.	=	.1785714
		Remains	.0033416
	6 oz.	=	.0033482

Hence it appears, that .931913 of 1 C. is very nearly = 3 *qrs*. 20 lb. 6 oz.

50. *Case* 7. To find the Value of any Decimal of a *l*. without out either *Tables* or Pen.

Here we only consider the first three Figures after the Decimal Point, rejecting the others as inconsiderable in common Affairs. We mentally multiply the Figure which stands in the first Place after the Point by 2, and the Product will be the Shillings, if the Figure in the second Place is not 5 or greater; but, if it be 5 or greater, then 1 is to be added to the Number of Shillings already found, for the Number of Shillings required. Then take the Excess of the Figure in the second Place above 5, or the Figure itself, if it be not 5; and this Figure, considered as so many Tens as it contains Units, together with the Fi-

Figure in the third Place, considered as so many Units, will express the Value of the remaining Part in Farthings, if it be not above 24; but if the Number is, or exceeds 25, one Farthing must be deducted. And thus we shall obtain the Value of the Decimal, true to a Farthing; which is sufficient for most Purposes of Life.

51. *Example* 1. What is the Value of .776 of a *l*?

Here, the Figure in the first Place $7 \times 2 = 14$, and $14 + 1$ (because the Figure in the second Place is greater than 5) is $= 15$ Shillings. Now, the Figure in the second Place being above 5, its Excess is $7 - 5 = 2$, which, being placed on the left Hand of the Figure in the third Place, is 26; but, this being above 25, we deduct 1, *viz.* $26 - 1 = 25$ Farthings $= 6d\ \frac{1}{4}$, and so the whole Value is 15*s.* 6*d.* $\frac{1}{4}$ very near.

52. *Example* 2. What is the Value of .545833 of a *l*?

Here, taking only the three first Figures, we have .545; and 5, the first Figure, $\times 2 = 10$ Shillings; and the two last Figures are 45, from which deducting 1, we have $45 - 1 = 44$ Farthings $= 11\,d$, and so the given Decimal $= 10s.\ 11d.$ *ferè.*

53. The Reason of this *Rule* may be shewn as follows: First, a Pound Sterling being $= 20$ Shillings, $\frac{1}{10}$ of a £ $= 2$ Shillings; ∴ twice the Figure in the first Place (or Place of Tenths) is Shillings.

Secondly, $\frac{5}{100}$ is by Abbreviation $= \frac{1}{20} = 1$ Shilling, ∴ 5 in the second Place (or Place of hundredth Parts) is $= 1$ Shilling; and therefore, if the Figure in the second Place is so great as 5, one Shilling must be added to those found from the first Figure. Thirdly, since 5 in the second Place is $= 1$ Shilling $= 48$ Farthings, it follows that 1 in the second Place is $= 9\frac{3}{5}$ Farthings; (for as $5 : 48 :: 1 : 9\frac{3}{5}$,) which, to avoid Fractions, we call 10; (but, 10 being $\frac{2}{5}$ of a Farthing upon 10 too great, the *Rule* will require a Correction, and, what the Correction is, shall be shewn presently), and therefore, the Figure in the second Place, if less than 5, or the Excess above 5, if greater than 5, is

considered as so many ten Farthings as it contains Units.

Fourthly, one in the third Place, by Notation, being $= \frac{1}{1000}$ of a £, and 1 Farthing being $= \frac{1}{960}$ of a £, the Figure in the third Place will be nearly = so many Farthings, but something less, *viz.* $\frac{1}{960} - \frac{1}{1000} =$ (by Subtraction of Vulgar Fractions) $\frac{40}{960000} =$, by Abbreviation, $\frac{1}{24000}$ of a Farthing too little; and, since the Figure in the third Place cannot be greater than 9, the Error, occasioned by taking the Figure in the third Place as Farthings, can never exceed $\frac{9}{24000}$ of a Farthing, and therefore may very justly be omitted, as not worth Correction.

Lastly, we now come to the Correction abovementioned, where we have shewn, in the third Part of this *Article*, that, though we considered the Figure in the second Place as so many ten Farthings, yet each Unit in that Place was but $9\frac{3}{5}$ Farthings; and so make it $\frac{2}{5}$ of a Farthing upon 10 too great, as $\frac{2}{5}$ of a Farthing : 10 :: 1 Farthing : 25 Farthings; hence, if the Farthings amount to 25, the Error would be a Farthing, for which Reason, we have directed in *Article* 50. to deduct 1 Farthing, when their Number is, or exceeds 25. And now we have shewn the Reason of all the Parts of that compendious *Rule*.

54. *Case* 8. To put Shillings, Pence, and Farthings into the Decimal of a Pound Sterling, without either *Tables* or *Pen*, true to three Places of Decimals.

The *Rule*. Imagine a Nought (0) on the right Hand of the Shillings, and then take the Half, which Half, if but one Figure, is 10ths; if of 2, is 100ths; and the Decimal thus found will be the Value of the Shillings in the Decimal of a £. This being done, turn the Pence and Farthings into Farthings, and take them as Thousandths of a £; (remembering, if they amount to 24, to add 1) and the Decimal of the Shillings, and of the Pence and Farthings, being collected together, will be the Decimal required.

55.

55. *Example* 1. Find the Value of 15*s.* 6*d* $\frac{1}{4}$ in the Decimal of a £, to three Places of Decimals.

First, 15 Shillings, with an o on its right Hand, is 150, Half of which call .75. And 6*d* $\frac{1}{4}$=25 Farthings, which being above 24, we add one, calling it 26 Farthings, or .026; and .75 + .026 = .776 for the Decimal required.

56. *Example* 2. Put 10*s.* 11*d.* into the Decimal of a £.

Here, 10*s*, with an o on the Right, becomes 100, Half of which is 50, which is 10*s* = .50 of a £. And 11*d* being = 44 Farthings, and 44 + 1 (because above 24 is) = 45, which is 11*d* = .045 of a £, and consequently 10*s* 11*d* = .50 + .045 = .545 of a £, true to three Places of Decimals.

The Reader, by comparing these *Examples* with the *Examples* in *Case* 7, will easily see the Reason of these Operations; this *Case* being only the Reverse of that.

57. Having now given the most useful *Cases* of Reduction, such Readers as are well acquainted with them, will find very little, if any Difficulty, in applying Decimals to any *Rule* of Arithmetic. However, for *Example* Sake, we shall proceed to apply Decimals to a few of the most useful *Rules*.

CHAP. VII.

Of the APPLICATION *of* DECIMALS *to the* RULE *of* THREE DIRECT.

58. A Few *Examples* will plainly shew the Method of applying Decimals to this *Rule*, without the Help of any formal Precepts.

Example 1. What comes 6C. 1 Qr. 14 to, at 2£ 16*s* *per* C? *Solu-*

Solution. By Reduction of Decimals 1 Qr. 14 lb, in the Decimal of a C, is = .375 of a C. very near, and so 6 C. 1 Qr. 14 lb = 6.375 C; and 16 *s*, in the Decimal of a £, = .8, and ∵ 2 £ 16 *s* = 2.8 £: Hence the Stating will be,

```
      C.    £      C.
   If 1 ÷ 2.8 :: 6.375
                    2.8
                 ------
                  51000
                 12750
                 ------
Answer £ 17.8500 = (by Reduction, Case 3,
                     or 6 or 7) 17 £ 17 s.
```

See this worked by common Arithmetic, *Article* 191. of the first Essay.

59. *Example* 2. What come 7 Yards of Linnen to, at 2 *s* 1 *d* $\frac{1}{2}$ *per* Ell?

An Ell is 1 $\frac{1}{4}$ Yard, which in Decimals is 1.25; and 2 *s* 1 *d* $\frac{1}{2}$, in the Decimal of a £, is = .106249, whence the Stating is, If 1.25 Yds : 0.106249 :: 7 Yds

```
                       7
                 ---------
         1.25 ) .743743 ( .5949
                 625...
                 ------
                 1187
                 1125
                 ----
                  624
                  500
                  ----
                 1243
                 1125
                 ----
                  118
```

Answer, .5949 of a £ = (by Reduction of Decimals) 10 *s* 10 *d* $\frac{3}{4}$. See this solved by common Arithmetic in *Art.* 192. in the first Essay.

60. *Example* * 3.

A May-pole there was, whose Height I would know;
The Sun shining clear straight to work I did go:
The Length of the Shadow, upon level Ground,
Just sixty-five Feet, when measur'd, I found:

A

* This *Question* I believe was first proposed in one of the *Monthly Entertainments* for the Year 1711.

A Staff I had there, just five Feet in Length;
The Length of its Shadow was four Feet one Tenth:
How high was the May-pole, I gladly would know?
And it is the Thing you're desir'd to shew.

Solution. It is evident, that if there are two Poles standing upright on the Ground, and their Heights be as 2 to 1, their Shadows must also be as 2 to 1; for a Pole, being twice as high as another, must certainly cast a Shadow twice as long: And, if the Ratio of their Heights be as 3 to 1, that of their Shadows will be as 3 to 1, or in the same Ratio with their Heights, for the above Reason, *&c.* Consequently, as the Length of the Shadow of any Thing is to its Height, so is the Length of the Shadow of any other Thing to its Height.

Hence the above *Question* may be stated thus:

Shadow Feet	Height Feet.		Shadow Feet
If 4.1	: 5	::	65

```
            65
             5
     4.1)325.0(79.2683 very
         287          near
          380
          369
           110
            82
           280
           246
            340
            328
             120
             123
```

Answer. The Height of the May-Pole was 79.2683 Feet, which is 79 Feet 3 Inches and a little more.

CHAP.

CHAP. VIII.

Fellowship.

61. FELLOWSHIP shews how to divide a Number into any Number of Parts proportional to other given Numbers. In Vulgar Arithmetic, we gave a Definition not so general as this.

62. As the first and second Terms continue the same in all the Statings, it will be many Times the shortest Method of *Solution* to divide the second Term by the first, and reserve the Quotient as a common Number which, being separately multiplied by the third Number of each Stating, will give the Answers for each respective Stating*.

63. *Example.* Suppose 4 Men, *A*, *B*, *C*, and *D*, trade in Company; *A* put in 50£; *B*, 16£; *C*, 25£; and *D*, 18£. 10*s*; they gained 20£. 15*s*: What was each Man's Part?

Solution. First $50+16+25+18.5=109.5l=$ the whole Stock put in; hence the Statings would stand thus:

	l.	*l.*	*l.*	
As	109.5 :	20.75 ::	50 :	*A's* Share.
	109.5 :	20.75 ::	16 :	*B's* Share.
	109.5 :	20.75 ::	25 :	*C's* Share.
	109.5 :	20.75 ::	18.5 :	*D's* Share.

Now, $20.75 \div 109.5 = .189497 =$ the common Number. Hence, $.189497 \times 50 = 9.47485 =$ 9£. 9*s* 5*d* ¾ = *A's* Share. And $.189497 \times 16 =$

3.

* The Reason of this will easily appear, for, if four Quantities, *a*, *b*, *c*, *d*, are proportional, that is, as $a : b :: c : d$, then $d = {}^{*}\frac{bc}{a}$, but $\frac{bc}{a} \dagger \frac{b}{a} \times c$; $\because \frac{b}{a} \times c = \ddagger\, d$.

* 1.186.
† 1.600.
‡ 1.23.

£. £. s. d.
3.031952 = 3 0 7 ¼ B's Share; and .189497 × 18.5
£. £. s. d.
= 3.5056945 = 3 10 1 ¼ nearly = D's Share; and
l. l. s. d.
.189497 × 25 = 4.737425 = 4 14 8 ¾ = C's Share.

And, for Proof, 9.474850 + 3.031952 + 4.737425 + 3.5056945 = 20.749921, which is very nearly = 20.75 the whole Gain.

CHAP. IX.

SIMPLE INTEREST.

64. TO find the Interest. Multiply the Principle by the Time, and that again by the Rate; the last Product will be the Interest required. A Year being the Integer for the Time, and one Pound the Integer for the Money*.

65. By the Rate, in the last *Article*, we would be understood to mean a hundredth Part of the Rate *per Cent. per Annum*; or, which is the same Thing, the Interest of 1 *l per Annum*. Thus, if the Interest of 100*l* for 1 Year be 5 *l*, that of 1 *l* for the same Time is .05 of a Year; and, if the Rate *per Cent. per Annum* be 6 *l*, that of 1 *l* for a Year is .06, &c. for as

* The Reason of this *Rule* will plainly appear thus: Let r = the Interest of 100£ for 1 Year; t = the Time in Years, p = the Principal; then by five Numbers (in common Arithmetic) the Numbers will stand thus:

$$\left.\begin{array}{lll} 100£ & 1\ \text{Year} & r \\ p & t & \end{array}\right\}$$ Here the Blank falls under the third Place, and ∵ $\frac{ptr}{100}$ = the Interest = $pt \times \frac{r}{100}$ (for by Multiplication of Fractions $\frac{pt}{1} \times \frac{r}{100} = \frac{ptr}{100}$;) which is the same as the above *Rule*. *Q. E. D.*

as $100l : 5l :: 1l : 0.05l$; and as $100l : 6l :: 1l : 0.06l$. &c.

66. *Example*. What is the Interest $410l$ $10s$ for 1 Year, and 40 Days, at $5l$ *per Cent. per Annum*?

Solution. Here, the Interest of $100l$ for a Year being $1l$, the Rate is .05 of a l; and, the Decimal of 40 Days being .109589, the Time expressed Decimally is 1 Year 1.109589: Hence by the above *Rule*, $1.109589 \times 410.5 \times .05 = 22.7743$ *ferè* $= 22l$ $15s$ $5d\frac{1}{4}$. See this worked by common Arithmetic in the first Essay, Chap. 21.

CHAP. X.

COMPOUND INTEREST.

67. RAISE the Amount of $1l$ for one Year (at the given Rate of Interest) to the Power whose Index is expressed by the Time; and this Power, multiplied by the Principal, will give the required Amount*.

68. *Example*. What will $210l$. $7s$ $6d$ amount to in 3 Years, at 5 *per Cent. per Annum*?

Solution. The Rate *per Cent. per Annum* being $5l$, the Amount of $1l$ in one Year is $1.05l$; (if it had been

* To shew the Reason of this *Rule*: Let a = the Amount of 1£ for 1 Year. It is evident, that as 1£ : its Amount in 1 Year :: any other Principal : its Amount in the same Time : Hence, since a may be considered as the Principal for the second Year, it will be, as $1 : a :: a : a^2$ = the Amount in two Years; and now a^2 becomes the Principal for the third Year, ∴ as $1 : a :: a^2 : a^3$ = the Amount of 1£ in 3 Years, &c. hence the Amount of 1£ in t Years will be expressed by a^t; and, putting p = the given Principal, it is plain, that as 1£ : the Amount of 1£ in any Time, :: any other Principal : its Amount in the same Time :: $1 : a^t :: p : a^t \times p$ = the Amount required. *Q. E. D.*

been 4 *l per Cent per Annum*, the Amount of 1 *l* would have been 1.04 *l*; if at 6 *l*, the Amount would be 1.06 *l*; *&c.*) Hence, by the above *Rule*, 1.05 *l* must be raised to the third Power, (the Time being 3 Years.) Now 1.05 x 1.05 x 1.05 = 1.157625 = 1.05 raised to the third Power = the Amount of 1 *l* in three Years; ∴ 1.157625 x 210.375 (the Principal expressed Decimally) = 243.5353 *l* = 243 *l* 10 *s* 8 *d* $\frac{1}{4}$ = the Amount which was to be found. See this worked by Vulgar Arithmetic, in the 22d Chap. of the first Essay.

Note, We intend to treat more largely of Interest hereafter.

CHAP. XI.

EVOLUTION.

69. THOUGH it frequently happens, that the Number whose Root we are to extract, does not admit of an integral Root, yet the Root may be found by Decimals, either exactly or approximated, so as to differ from the true Root, less than any given Quantity.

In extracting the Square Root, every Pair of Cyphers that we annex, or Decimals that are annexed to the whole Number, will give one Decimal in the Root; (because it is known from Evolution in common Arithmetic, that there will be as many Figures in the Root, as there are Dots (.) over the given Number; and, by the 36th Chap. of the first Essay, in the Square Root the Dots are over every other Figure beginning with, and counting from the Units Place.) And, in extracting the Cube Root, every three Cyphers that we place, or every three Decimals that are annexed to the given Number, will give one Decimal Place in the Root, *&c.* This will be better

explained by a few *Examples*, than by a great Number of *Rules*.

70. *Examples*. Let it be required to extract the Square Root of 50, and of 1.1025; and the Cube Root of 1.157625, of 30, and of .010648?

The bare Operations will be sufficient Explanations.

50) 7.071, &c.
49

1407) 10000
9849

14141) 15100
14141

959

1.157625 | 1.05
1

157 Resolvend
300 Triple Square
30 Triple Quotient
330 Divisor

1576.25 New Resolvend
30000 New Triple Square
300 New Triple Quotient
303.00 New Divisor

See *Art.* 482
Essay 1st

1 25 = 5 × 5
1500 = 300 × 5
30000 = Triple Square
31525
5
157625 Ablatitium
0

1.1025 (1.05
1

205) 1025
1025

30 | 3.1
27

3000 Resolvend

2700 Triple Square
90 Triple Quotient

2790 Divisor

1 = 1 x 1
90 = Triple Quotient x 1
2700 = Triple Square

2791
x1

2791 Ablatitium.

209000 = New Resolvend, and in this Manner we we may proceed at Pleasure.

0.010648 (0.22 exact.
.008

2648 Resolvend

1200 Triple Square
60 Triple Quotient

1260 Divisor

4 = 2 x 2
120 = 60 x 2
1200 = Triple Square

1324
x 2

2648

0

CHAP. XII.

EQUATION *of* PAYMENTS *the* TRUE WAY.

71. THE Reader may remember, that in Equation of Payments, in the first Essay, we engaged to give a true and new Theorem for solving this *Rule*, which we shall now do in this Place: But first we must observe, that, in a just *Solution* of this *Rule*, there must be an Equality of Gain and Loss; and it is manifest, that the Gains of the Payer must be occasioned by his keeping the Debts after they are due, and so must be equal to the Interest of such Debts, for the Time they are kept in his Hand after they are due; and, on the contrary, his Loss must be occasioned by paying some Debts before they are due, which Loss must be the Discount of those Debts, for the Time they are paid before due: And therefore, when these Gains and Losses are equal, there cannot, in strict Propriety of Speech, be said to be either Gain or Loss, occasioned in paying the Sum all the Debts, at such equated Time.

72. To find such equated Time, for the Payment of two Debts, the Theorem is, * Multiply the Debt first payable by the Rate of Interest, and divide the Sum

* The Investigation of this *Theorem* is as follows: Let $a =$ the Debt first payable, $m =$ its Time when due; $d =$ the other Debt, and $t =$ its Time; $r =$ the Rate of Interest, or the Interest of 1£ for 1 Year; $s = t - m$; $x =$ the
* 1.48. equated Time before the last Payment; then $\overline{s - x} =$ * the Time from the first Payment. Hence, $ar \times \overline{s - x} = ars$
† 64. $- arx =$ † the Interest of a Debt for $\overline{s - x}$ Time; and as
‡ 1.186. $1 : r :: x :$ ‡ $rx =$ the Interest of 1£ for x Time, ∵ 1£ in x Time would amount to $1 + rx$; and so the present Value of $1 + rx$ paid, x Time before due, would be 1£; or, which is the same, the Discount upon $1 + rx$ Pounds
|| 1.186. would be rx; ∵ as $1 + rx : rx$ (Discount) $:: d :$ || $\dfrac{drx}{1+rx}$ = the

Sum of both Debts by this Product; and from the Quotient subtract the Time betwixt the two Payments, and take Half of the Remainder, which Half we call the reserved Number. Having proceeded thus far, divide the Distance of Time betwixt the two Payments by the Rate of Interest; and to the Quotient add the Square of the reserved Number, and extract the Square Root of the Sum; from which Root subtract the reserved Number, and the Remainder will be the equated Time of Payment, before the last Debt would have been payable. *Note*, The Integer for the Time is one Year.

Aa 2 73.

= the Discount on d, for the Time x, the Time it is to be paid before it is due; $\because ars - arx = {}^{*}\frac{drx}{1+rx}$, and, mul- * 71.
tiplying both Sides by $1+rx$, we get $\overline{ars-arx}\times\overline{1+rx}$
$= \dagger\, drx$; but $\overline{ars-arx}\times\overline{1+rx}$ (by the Operation † 1. 56
in the Margin, is) $= ars - arx + ar^2sx - ar^2x^2$, $\because ars - arx + ar^2sx - ar^2x^2 = \ddagger\, drx$, ‡ 1. 23.
and, by adding $arx - ar^2sx + ar^2x^2$ to both Sides of this Equation, we have $ars = \|\, ar^2x^2 +$ ‖ 1. 22.

$$\begin{array}{r} ars - arx \\ 1 + rx \\ \hline ars - arx \\ + ar^2sx - ar^2x^2 \\ \hline ars - arx + ar^2sx - ar^2x^2 \end{array}$$

$drx + arx - ar^2sx$; which divided by r gives $as = \S\, arx^2$ § 1.108.
$+ dx + ax - arsx$; but $arx^2 + dx + ax - arsx$ may be expressed thus, $arx^2 + \overline{d+a-ars}\times x$, and $\because as = \P\, arx^2$ ¶ 1.23.
$+ \overline{d+-ars}\times x$, and, dividing by ar, we shall have
$\frac{as}{ar} = {}^{**}x^2 + \frac{d+a-ars}{ar}\times x$; which may be otherwise ex- ** 1.108.
pressed thus, $\frac{s}{r} = x^2 + \overline{\frac{d+a}{ar} - s}\times x$, (because $\frac{as}{ar} = \frac{s}{r}$
and $\frac{ars}{ar} = s$;) now, to have a more simple Expression, let
$2b = \frac{d+a}{ar} - s$, or $b =$ Half of $\frac{d+a}{ar} - s$; (this we call the reserved Number, above;) then, by writing $2b$ for $\frac{c+a}{ar} - s$, we have $\frac{s}{r} = x^2 + 2bx$; and, by adding b^2 to each Side of this Equation, we shall have $\frac{s}{r} + b^2 = {}^{\dagger\dagger}x^2 +$ †† 1. 22.

2 b —

73. *Example.* Let it be required to solve the *Question* in Equation of Payments, in the first Essay, by this Method?

The Operation will stand thus: The Sum of both Debts is 200*l.* + 205*l.* = 405*l.*; and the Debt first due 200*l.* x (by the Rate of Interest) .05 = 10, and 405 ÷ 10 = 40.5; and 40.5 — 1 (the Time betwixt the two Payments) = 39.5; and 39.5 ÷ 2 = 19.75 = the reserved Number. Now (the Distance of Time betwixt the two Payments) 1 ÷ .05 (the Rate of Interest) = 20; and 19.75 x 19.75 = 390.0625 the Square of the reserved Number; and 20 + 390.0625 = 410.0625, the Square Root of which is 20.55; and 20.55 — 19.75 (the reserved Number) = 0.5 of a Year, for the equated Time from the last Payment; 2 Years — .5 of a Year = 1.5 Year from the Beginning, or the equated Time, is in the Middle of the Times of the two Payments; so that the Debt first payable, *viz.* 200*l*, will not be paid according to this *Solution* until ½ a Year after due, and, therefore, the Payer, by keeping it half a Year longer in his Hands, gains ½ a Year's Interest of 200*l.* = 5*l*; and, by paying the second Payment 205*l*, ½ a Year before due, he loses the Discount of 205*l.* for a half Year = 5*l.* and, hence, his Gains would be equal to his Loss, and, therefore, this *Solution* is true; and, consequently, that given in Vulgar Arithmetic, and all others differing from this, are false, as is there hinted.

CHAP.

$2bx + b^2$; but $\overline{x+b} \times \overline{x+b} = x^2 + 2bx + b^2$, see

* 1. 452. the Margin; $\because \sqrt{\frac{s}{r} + b^2} = {}^* x + b$;

and, by subtracting b from each Side of this Equation, we have the required *Theorem* $\sqrt{\frac{s}{r} + b^2} : - b = x$. (*Q. E. I.* and *Q. E. D.*) the Half of $\frac{d+a}{ar} - s$.

$$\begin{array}{l} x + b \\ x + b \\ \hline x^2 + xb \\ \quad + xb + b^2 \\ \hline x^2 + 2xb + b^2 \\ \hline \end{array}$$

b, being =

CHAP. XIII.

REBATE or DISCOUNT.

74. TO find the present Worth, or Money that, being paid in Hand, would satisfy the Debt, this is the *Rule* *: Divide the given Debt, by the Product of the Rate into the Time, *plus* 1.

Note, By the Rate, we mean the Interest of 1 *l.* for 1 Year; and the Integer of the Time is 1 Year.

75. *Example.* What ready Money will pay off a Debt of 210*l.* 10*s.* due at the End of 2 Years, Rebate being made at the Rate of 5 *l per Cent. per Annum?*

Solution. Here the Rate is .05, which, multiplied by 2 the Time, gives .1 for the Product; and $.1 + 1 = 1.1$ for a Divisor: Now the Debt $210.5 l. \div 1.1 = 191.363 l. = 191 l.\ 7 s.\ 3 d. \frac{1}{4}$ = the present Worth, or Money, which, being immediately paid, will satisfy the Debt. And, if the Discount be required, then $210 l.\ 10 s. - 191 l.\ 7 s.\ 3 d \frac{1}{4} = 19 l.\ 2 s.\ 8 d \frac{3}{4}$ = the Discount. See this worked by common Arithmetic, *Art.* 351. of the first Essay.

76. Hence it appears, that Decimals are of great Service, even in common Business; though, for the most Part, *Questions* in common Business, as in Buying and Selling, are more readily worked by common Arithmetic; and no *Rules* can be laid down to direct,

 when

* Let p the present Worth, which may be considered as a Principal put out to Interest; $r =$ the Rate, $t =$ the Time; $d =$ the Debt, which may be considered as the Amount. Then $prt =$ the Interest of p, and $\because prt + p = d$; and, dividing by $rt + 1$, we have $p = $ * $\frac{d}{rt+1}$.

* 1.108.

when it is beſt to uſe the common Methods, and when Decimals; for it requires much Practice to make a proper Choice; but the chief Value of Decimals is not in its Application to common Arithmetic, but to Menſuration, and other Geometrical Purpoſes, &c. of which, in the proper Places.—We ſhall now put an End to this Eſſay, believing, that, if the Reader underſtands what has been here ſaid, he cannot be at a Loſs to apply it to any other Part of Arithmetic; and in our Opinion thoſe Authors who ſpin out their Works to a great Length, by a Detail of Things, do not only miſpend their own Time, but alſo that of their Readers.

AN APPENDIX;

CONTAINING

Some Miscellaneous Questions.

FOR the Learner's further Exercise, we have thought proper to add the following *Questions*, promiscuously proposed.

Question 1. Three Men, *A*, *B*, and *C*, enter into Partnership; *A* puts in 24£, *B* puts in 32£, but what *C* puts in is forgot; however, it is remembered that the whole Gain was 12£, of which *C* had for his Share 5£; whence, *C*'s Stock in Trade and the Gain of *A* and *B* may be easily found: Required the Method of *Solution*?

Solution. First, $12£ - 5l = 7£ =$ the Sum of the Gains of *A* and *B*; and $24£ + 32£ = 56£ =$ the Sum of the Stocks of *A* and *B*; hence this Stating, as *A* and *B*'s Gain 7£ : *A* and *B*'s Stock 56£ :: *C*'s Gain 5£ : *C*'s Stock 40£. Again, as the Sum of *A* and *B*'s Stocks 56£ : the Sum of their Gains 7£ :: *A*'s Stock 24£ : his Gain 3£; and :: *B*'s Stock 32£ : *B*'s Gain 4£; or *B*'s Gain may be otherwise found, being $= 7l - 3l = 4l$.

Question 2. Suppose two Men trade in Company, whose Names are *John* and *William*; and that *John* put in 12*l*. Sterling, and *William* 192 *French* Livres;

they gained 10*l*; of which *John* had for his Share 6*l*: *Quære* the Rate of Exchange at that Time *per* Livre?

Solution. The whole Gain 10*l.* — *John*'s Share 6*l.* = 4*l.* = *William*'s Share; hence as *John*'s Share of the Gain 6*l* : *John*'s Stock 12*l* :: *William*'s Gain 4*l* : *William*'s Stock 8£; hence 192 *French* Livres = 8£ Sterling; ∵, as 1.92 Livres : (8£ or) 1920 Pence :: 1 Livre : 10 Pence; that is, the Rate of Exchange is 10*d* Sterling *per* Livre, or, which is the same Thing, 30*d per* 1 Crown of 3 Livres.

Question 3. Suppose 3 Men trade together, and that *A* and *B* put into the common Stock 56£; *B* and *C* 72£, and *A* and *C* 64; and gained in all 12£: It is required to find their respective Stocks and Gains?

Solution. The Sum of 56*l*, 72*l*, 64£ = 192£ = twice the whole Stock, because each Partner's Stock is twice included in this Sum; consequently 192£ ÷ 2 = 96£ = the whole Stock; and the whole Stock 96£ — 56£ (the Sum of *A* and *B*'s Stocks) = 40£ = *C*'s Stock; and 96£ — 64£ (the Sum of *A* and *C*'s Stocks) = 32£ = *B*'s Stock; also, the whole Stock 96£ — 72£ (the Sum of *B* and *C*'s Stocks) = 24£ = *A*'s Stock. Now, their respective Stocks being known, each Person's Share of the Gain will be found, as in common Fellowship, to be 3£, 4£, and 5£ respectively.

Question 4. A Merchant insured 200£ on a Ship from *Virginia*, the Rate of Insurance at 5£ *per Cent*; but, the Vessel being lost in her Voyage, it is required to find what the Insurers must pay the Merchant, an Abatement being made as customary, in Case of a Loss, of 2 $\frac{1}{2}$ *per Cent.* and also what the Merchant loses on the Whole?

Solution. Was there no Abatement to be made, it is manifest the Merchant must receive from the Insurers the whole 200£; but, there is, by Custom, an Abatement of 2 $\frac{1}{2}$ *per Cent* to be made, that is, 5£ on the 200£; and ∵ the Merchant will receive of the Insurers 200£ — 5£ = 195£. As to his Loss, the Abatement being 5£, and the Premium of In-

surance

ſurance 10£ for the ſaid 200£, his whole Loſs will be = 5£ + 10£ = 15£.

Queſtion 5. Admit a Merchant ſhipped, on Board a Veſſel bound to *Virginia*, Goods to the Value of 200 £. Now the Premium for Inſurance being at 5£ *per Cent*; and, in Caſe of the Goods being loſt, the Merchant being to allow the Inſurer 2 $\frac{1}{2}$ *per Cent*; he is deſirous of knowing what Sum he muſt inſure for, ſo that, in Caſe of a total Loſs of the Goods, he may recover of the Inſurers 200£ (the full Value of the ſaid Goods) free of all Deductions.

Solution. The Premium of Inſurance being 5£ *per Cent*, and the Allowance in Caſe of a Loſs 2£ $\frac{1}{2}$ *per Cent*, the whole Allowance to the Inſurers, *&c.* is 5£ + 2£ $\frac{1}{2}$ = 7 $\frac{1}{2}$£. Hence for every 100£ the Merchant will recover, free of all Deductions, only 100£ — 7£ $\frac{1}{2}$ = 92.5£; ∵ the *Queſtion* becomes, If, to recover 92.5£, he muſt inſure 100£, what muſt he inſure, to receive 200£; and conſequently the Stating will be, as 92.5*l* : 100*l* :: 200*l* : 216.216£ = 216£ 4*s* 4*d fere*, the Anſwer which was required.

Queſtion 6. When upon the Arrival of a Ship it appears by the Invoice, that the Merchant has inſured more than his real Intereſt on Board, it is the Cuſtom of Inſurers to return the Premium of what is over inſured after this Manner: They firſt compute (as ſhewn in *Queſtion* the 5th) what the Merchant ought to have inſured, in Order to have received (in Caſe of a Loſs) of the Inſurers, free from all Deductions, the full Coſt of his Goods *per* Invoice; and then what this Sum is leſs than that inſured, which is ſaid to be ſo much over inſured, and, for that Reaſon, the Premium on this Difference is to be returned to the Merchant. This being premiſed, let it it be required to find, what muſt be returned a Merchant who had inſured 200£, on a Ship from St. *Chriſtopher's*, at the Rate of 8£ *per Cent*, it appearing, on the Arrival of the Ship, that the Value of the Goods on Board, as *per* Invoice, was only 150£?

Solution.

Solution. By the Method of *Solution* ſhewn in the laſt *Queſtion* we have, firſt, 8£ + 2.5£ = 10.5£; and 100£ — 10.5£ = 89.5£; and then, as 89.5£ : 100£ :: 150£ : 167.597£ = what he ought to have inſured, and ∴ 200£ — 167.597£ = 32.403£ = what he over inſured: Now to find the Premium of this Sum we ſay, as 100£ : 8£ :: 32.403£ : $\frac{32.403 \times 8}{100}$ = $\frac{259.224}{100}$ = 2.59224£ = 2£ 11*s* 10*d* nearly the Premium to be returned on the 200£, or its Half = 1£ 5*s* 11*d per Cent.*

Queſtion 7. Admit two Perſons *A* and *B* trade in Company; that *A* put in 30£ for 6 Months, and *B* 20*l*, but for what Time is forgot; however, it is remembered that each Perſon's Share of the Gain was equal, from whence may be found *B*'s Time?

Solution. In Fellowſhip with Time it is ſhewn, that the Gain of each Partner muſt be in Proportion to the Product of the Time and Stock, and ∴, as the Gains are in this *Queſtion* equal, *A*'s Stock, multiplied by his Time, muſt be equal to *B*'s Stock into his Time, ∴ *B*'s Stock x his Time = 30 x 6 = 180; and ∴ *B*'s Time = 180 ÷ 20 = 9 Months.

Queſtion 8. Having ſold 50 Yards of Cloth for 13*l* 4*s*, I gained at the Rate of 10*l. per Cent*; from hence I would know what the Cloth coſt me *per* Yard?

Solution. As the Perſon gained 10*l per Cent*, he received 110*l* for what coſt him but 100*l*, ∴ the Stating is, firſt, as 110*l* : 100 :: 13*l* 4*s* : 12*l* = the prime Coſt of the 50 Yards, and ∴ if 50 Yards : 12*l* :: 1 Yard : 4*s* 9*d* 2 *qrs.* $\frac{2}{5}$ = the prime Coſt of 1 Yd.

Queſtion. 9. A *Mancheſter* Chapman, going to a Fair, ſold Fuſtians for 11*s*. 6*d*. the End, wherein was gained 15*l*. *per Cent*; and, ſeeing no other Chapman had ſo good, raiſeth them at the latter End of the Fair to 12*s*; I demand what he gained *per Cent.* by this laſt Fair?

Solution. This *Queſtion* is from Mr. HILL's Arithmetic, where it is ſtated thus, If 11.5*s* : 15*l* :: 12*s* : 15.652*l*. gained by the laſt Sale; which is a wrong Stating,

Stating, the true *Solution* being thus: The Gain at the first Sale being 15 *l* *per Cent.* for every 100 *l* worth (prime Cost) he received 100 *l* + 15 *l* = 115 *l*; ∵ we say first, as 115 : 100 :: 11 *s* 6 *d* : 10 *s* the prime Cost of one End; and then, as each End cost him 10 *s*, and he sold them at the second Sale for 12 *s* each, he gained by that Sale 12 *s* — 10 *s* = 2 *s* upon each End, or 2 *s* upon 10 *s*; ∵ we have, as 10 : 2 :: 100 *l* : 20 *l* = the Gain *per Cent.* by the last Sale. Or it may be solved by one Stating, *viz.* As 11 *s* 6 *d* : 115 *l.* :: 12 *s* : 120 *l.* = the Sum of the prime Cost and Gain, on 100 *l* worth at prime Cost, ∵ 120 *l* — 100 *l* = 20 *l.* as before.

Question 10. Bought 5 Dozen of Books, at 3 *l* *per* Dozen; by selling them for ready Money I propose to get 20 *l.* *per Cent*; but, if on Trust, 10 *l.* *per Cent.* *per Annum* more for the Forbearance: Hence, if I sell at 6 Months Credit, what must I have *per* Dozen?

Solution. First 100 *l.* prime Cost, with 20 *l.* *per Cent.* Profit, amounts to 120 *l*; and ∵ as 100 *l* : 120 *l* :: 3 *l* : 3 *l.* 12 *s.* the ready Money Price of one Dozen; which may also be found thus, 20 *l* being $\frac{1}{5}$ of 100 £, the Profit on 3 £ must be $\frac{1}{5}$ of 3 *l* = 12 *s*; and ∵ a Dozen of Books, sold for ready Money, must come to 3 £ 12 *s.* But by the *Question*, when the Person sells on Time, he is to have 10 £ *per Cent.* *per Annum* more on this 3 £. 12 *s*; and ∵ we are to find what will be the Gain, or Interest, of 3 *l.* 12 *s.* for 6 Months at the Rate of 10 *l.* *per Cent.* *per Annum*; which is most compendiously found by taking $\frac{1}{10}$ (10 *l* being $\frac{1}{10}$ of 100 *l*) and this gives 7 *s.* 2 *d.* 1 *qr.* $\frac{6}{10}$ for 1 Year, or its Half = 3 *s.* 7 *d.* 0 *qrs.* $\frac{8}{10}$ for 6 Months; and the required Answer is = 3 *l.* 12 *s.* + 3 *s.* 7 *d.* 0 *qrs.* $\frac{8}{10}$ = 3 *l* 15 *s* 7 *d.* 0 *qrs.* $\frac{4}{5}$.

Question 11. A Merchant would exchange 200 *l* Sterling for Dollars or Crowns: He is offered Dollars at 4 *s* 6 *d* which are worth but 4 *s* 3 *d*, or Crowns at 5 *s* worth but 4 *s* 8 *d*. Which of them shall he take to lose the least, and how many will he receive?

Solution. This *Question* is from the learned Mr. MALCOLM's Arithmetic, and the Method of *Solution*, given by that ingenious Gentleman, is: "Find how "many Dollars at 4*s* 6*d*, and Crowns at 5*s*, he would "get for 200*l*; then find the Value of that Number "of Dollars at 4*s* 3*d*, and that Number of Crowns "at 4*s*. 8*d*; the Comparison will shew which is of the "greatest Value; and the Value of that which is "the greatest, compared with 200 *l*, shews what he "loses". But this may be more compendiously solved thus: By taking Dollars at 4*s* 6*d* — each, which are worth but 4*s* 3*d*, he loses 4*s* 6*d* — 4*s* 3*d* = 3*d* upon each Dollar, ∵ say, as 4*s* 3*d* : 3*d* :: 4*s* 8*d* : 3*d* $\frac{15}{51}$ = what he would lose upon 4*s* 8*d* by taking Dollars: Now, by taking Crowns, he loses, by the *Question*, 5*s* — 4*s* 8*d* = 4*d* upon 4*s* 8*d*, which is a greater Loss than that by exchanging for Dollars; ∵ he must receive Dollars, and their Number is easily found by saying, If 4*s* 6*d* : 1 Dollar :: 200 *l* : 888 $\frac{8}{9}$ Dollars. By this last Method we make but two Statings, whereas the first Method requires four.

Question 12.* A Father and his Son upon a Time
Were laden with some Bottles of *French* Wine;
The Son unto the Father did complain,
That th' Weight of them his Arms did sorely pain;
The Father said, if one of yours I take,
My Number double unto yours will make;
But, if I one of mine to you do give,
As many as you have in all I still shall have:
How many Bottles of this Wine
Had each of them, I pray define?

Solution. Suppose the Son had 3 Bottles, then since, if the Father had one from the Son, he would have double what the Son would then have, that is, 4, it is plain according to this Supposition the Father had 3 Bottles; but, if we take one from the Father and give to the Son, the Son will have 4, and the Father but 2, whereas the *Question* says they would then have equal; and consequently the first Error is = 2 too little. Again, suppose the Son had 4, then the Father

* From the *Monthly Entertainments*, for the Year 1711.

Father would have 5; now, if we take one from the Father and give to the Son, the Son will have 5, and the Father but 4, and ∴ the ſecond Error = 1 too little. Hence, by double Poſition, we have $4 \times 2 - 3 \times 1 = 8 - 3 = 5 =$ the Dividend, and, the Diviſor being $2 - 1 = 1$, the Quotient is alſo $= 5 =$ the Number of Bottles the Son had, and conſequently the Father had 7.

Queſtion 13. Admit there is an Iſland whoſe Circumference is 40 Miles; and that two Men *A* and *B* ſet out at the ſame Time from a certain Place in the Circumference, to travel the ſame Way round it till they meet again; *A* travels each Day 10 Miles, *B* 12; it is required to find in how many Days they will meet, and how many Times each will have gone round it?

Solution. With a little Conſideration it will be evident, that (ſince the Circumference of the Iſland is 40 Miles) when *B* has got 40 Miles a-head of *A*, or, which is the ſame Thing, has travelled 40 Miles more than *A*, that then *A* and *B* muſt be both at one Place; ∴, becauſe *B* travels 2 Miles each Day more than *A*, the Stating will be, if 2 Miles : 1 Day :: 40 Miles : 20 Days, the Time of their Meeting; in which Time *A* will have travelled $10 \times 20 = 200$ Miles, and $B = 12 \times 20 = 240$ Miles; conſequently *A* will have been round the Iſland $200 \div 40 = 5$ Times, and $B = 240 \div 40 = 6$ Times.

Queſtion 14.

* Suppoſe a round Ball for to move in the Air,
In a certain Proportion which I ſhall declare;
Let the firſt Hour be 12 Miles, the next to move ten,
And ſo in Proportion from whence it began,
As 12 is to 10; now try, if you can
Tell the Miles it will move, ſuppoſe it to be
Continu'd in Motion to Eternity?

Solution. By the Note to *Article* 13th of the third Eſſay, we have this *Rule* to find the Sum of a Geometrical Progreſſion decreaſing *ad infinitum*. viz. multiply

* From the *Monthly Entertainments*, for the Year 1711.

tiply the first or greatest Term by the common Divisor, and divide the Product by one less than the common Divisor, the Quotient will be the Sum of the whole Progression. Now in this *Question*, the Ratio being as 12 to 10, the common Divisor is $\frac{12}{10} = 1.2$; hence, by this *Rule*, the Sum of the whole Progression is $= \frac{12 \times 1.2}{1.2 - 1} = \frac{14.4}{.2} = 72$ Miles.

Question 15. At 12 the Hour and Minute Hands of a Clock are in Conjunction; it is required to find the Time of their Opposition?

Solution. The Dial-Plate of a Clock is divided into 12 equal Parts called Hours; of which Parts the Minute Hand passes over 12 in one Hour, but in the same Time the Hour Hand only moves over 1; ∵ the Minute Hand separates from the Hour Hand 11 of these Parts in one Hour; but, when the Hands are in Opposition, the Minute Hand must be 6 of these Parts a-head of the Hour Hand; hence say, if 11 such Parts : 60 Minutes :: 6 : $32\frac{8}{11}$ Minutes; hence the Time is $32\frac{8}{11}$ Minutes after 12 o'Clock. But, if it had been required to find the Time of the next Conjunction, it would have been, if 11 : 60 :: 12 : $65\frac{5}{11}$ Minutes = 1 Hour $5'\frac{5}{11}$.

Question 16.

* Walking the other Day to take the Air,
(Bright shone the Sun, the Weather very fair)
At Distance I a dismal Cloud did spy,
Which (as methought) against the Wind did fly.
While I upon my Watch did look to see,
How Time did pass away; lo instantly
A dreadful Flash of Lightning pierc'd the Cloud;
Just fourteen Seconds after which aloud
The Thunder roar'd: Now I inform'd would be,
How many Feet the Cloud did burst from me?

Solution. In *Article* 189. of the first Essay, we have already observed, that Sound moves 1142 Feet in a Second, and ∵ the Answer is $= 1142 \times 14 = 15988$ Feet.

Question 17. Being at so large a Distance from the Dial-Plate of a great Clock, that I could not distinguish

* From the *Monthly Entertainments*, for the Year 1711.

guiſh the Figures; but, as the Hour and Minute Hands were very bright and glaring, I could perceive, that the Minute Hand pointed upwards to the right Hand, at the ſame Time the Hour Index pointed downward to the left, ſo as both were in a right Line, or diametrically oppoſite, and in ſuch Poſition, as that the Elevation (I gueſſed) was ſome few Degrees more than fifty above the Horizon: *Quære* * the Hour and Minute of the Day?

Solution. When a Circle is divided into 360 equal Parts, thoſe Parts are called Degrees; hence, from 12 to 3, or from 9 to 6 o'Clock, &c. is $\frac{1}{4}$ of a Circle, or 90 Degrees; and as by 50 Degrees above the Horizon is meant 50 Degrees above 3 o'Clock, and conſequently the Hour Hand is more than 50 Degrees below 9 o'Clock; ∴, to find the neareſt Hour to the required Time, ſay, if 90 Degrees : 3 Hours :: 50 Degrees : 1 Hour $\frac{2}{3}$; hence, by the *Queſtion*, it wants more than 1 Hour $\frac{2}{3}$ of 9 o'Clock, ∴ the neareſt Hour to the Time required is 7 o'Clock; and ∴ it is evident, that the firſt Time the Hands are oppoſite after 6 o'Clock is the Time required. But we have ſhewn in *Queſtion* 15th, that from one Conjunction to another is 1 Hour $5' \frac{5}{11}$; but it is evident that the Time from one Oppoſition to another is the ſame as from one Conjunction to another; for in both *Caſes* the Minute Hand muſt gain a whole Round on the Hour Hand; and ∴ the Time required is 6 Hours + 1 Hour $5' \frac{5}{11}$ = 7 Hours $5' \frac{5}{11}$.

† *Queſtion* 18.

One Day for Diverſion (or Paſtime and Pleaſure)
An exact *Engliſh* Mile on the Ground I did meaſure;
The Place being level, I concluded (in fine)
That, along on the ſame, I would ſtretch out a Line:
This done, then, kind Reader, my Paſtime to crown,
At every Yard a ſmall Stone I laid down:

Now

* From the *Ladies Diary* 1744, which, in that for 1745, was anſwered by Algebra.

† From the *Monthly Entertainments*, for the Year 1711.

Now each of theſe Stones I'd have brought, one by one,
To a Baſket that ſtands one Yard from the firſt Stone.
A Footman came by, and, to gain himſelf Praiſe,
He wager'd to take them all up in ſix Days.
When the Time was expir'd, the Stones he did count,
And ſeven hundred ſixty-nine was the Amount.
And now (Sir) leſt any ſhould call him a Drone,
He would gladly know how many Miles he had gone:
And alſo how many remain ſtill behind,
Suppoſe that to take up the reſt he'd a Mind?

Solution. A Mile being = 1760 Yards, there were 1760 Stones, ſo that we have given the Number of Places = 1760, the firſt Number 1 Yard, and common Difference 1, to find the total Sum of an Arithmetical Progreſſion, which, by the *Rule* of Arithmetical Progreſſion in the firſt Eſſay, will be found to be = 154968 Yards = 880 $\frac{1}{2}$ Miles; but, ſince the Man muſt go as much backward as forward, in Order to take up all the Stones, he muſt go 880 $\frac{1}{2}$ × 2 = 1761 Miles. But, to find the Number of Miles which he did travel, we have given the Number of Places = 769, the firſt Number 1 Yard, and the common Difference 1, by which the Sum of the Progreſſion will be found to be = 296065 Yards = 168 Miles 385 Yards; but, as he went as much backward as forward, he went in all 168 Miles 385 Yards × 2 = 336 Miles 770 Yards; and ∴ he has 1761 Miles — 336 Miles 770 Yards = 1424 Miles 990 Yards more to go, if he would take up the Stones remaining.

* *Queſtion* 19.

I walked forth to take the Air,
The Heav'ns and Nature ſmiling were;
The Morning bluſh'd with *Phœbus*'s Ray,
And every Tree was green and gay;
Each roſy Field ſweet Odours ſpread,
And a delightful Proſpect made;

Here

* From the *Monthly Entertainments*, for the Year 1711.

Here Flocks of Sheep fed on the Plains,
And Shepherds sung their rural Strains.
A grave old Shepherd there I spy'd,
Close by a Crystal Fountain's Side;
Under a Tree the Shepherd sat,
And seem'd well pleased with his Fate:
He tun'd his Pipe with wond'rous Art,
Which pleasant Music did impart;
Straight unto him I thus did say,
Resolve me one Thing (Sir) I pray:
What is the Number of the Sheep,
Which in these verdant Plains you keep?
The Shepherd soon reply'd to me,
I'll tell you what their Numbers be:
One Half, one Fourth, one Fifth of these,
One Eighth, one Tenth, add, if you please,
One Twentieth, one Fortieth too,
These being added up by you,
The total Sum it will agree
With my own Age, as you will see,
Exactly as Fifteen to Three.
What is your Age (good Sir) said I?
To which the Shepherd made Reply,
One Half, one Fourth, one Fifth do take,
One Tenth, one Twentieth, they will make,
If added, Fivescore and ten more,
And now my Age, Sir, I implore.
Being in a Rage, I flung away,
And would no longer with him stay:
And yet methinks his Age I'd know,
Which I must beg of you to show,
Likewise the Number of the Sheep,
Which this crabb'd Shepherd there did keep.

Solution. The first Thing to be done is to find the Shepherd's Age, which may be found thus: Suppose

the Shepherd's Age 20 As $22 : 20 :: 110 : 100$ Year

$\frac{1}{2}$ 10 $=$ the Shepherd's Age.

$\frac{1}{4}$ 5

$\frac{1}{5}$ 4

$\frac{1}{10}$ 2

$\frac{1}{20}$ 1

22

Now, to find the Number of Sheep, we make another Suppoſition, as under. Suppoſe the Number of Sheep $=$ 40

$\frac{1}{2}$ 20

$\frac{1}{4}$ 10

$\frac{1}{5}$ 8

$\frac{1}{8}$ 5

$\frac{1}{10}$ 4

$\frac{1}{20}$ 2

$\frac{1}{40}$ 1

50

then, ſince the Sum of theſe Parts ſhould be in Proportion to the Shepherd's Age as 15 to 3, we have as $3 : 15 :: 100 : 500 =$ the Sum which theſe Parts ſhould amount to, whence, as $50 : 40 :: 500 : 400$ $=$ the Number of Sheep which was required.

* *Queſtion* 20.

Once as I walk'd upon the Banks of *Rye*,
To ſee the purling Streams glide gently by,
And hear the pretty Birds to chirp and ſing,
Making the Groves with Melody to ring;
I in the Meads three beauteous Nymphs did ſpy,
That for their Pleaſure came as well as I;
And unto me their Steps they did direct,
Saluting me with moſt benign Reſpect;
Saying, 'Well met, we've Buſineſs to impart,
'Which we cannot decide without your Art.
'Our Grannum's dead, and left a Legacy
'Which is to be divided 'mongſt us three.
'In Pounds it is two hundred twenty-nine;
'Alſo a good Mark, being Sterling Coin:'

Then

* Queſtion 55, in the *Ladies Diary*, 1717.

Then ſpake the eldeſt of the lovely Three,
'I'll tell you how it muſt divided be;
'Likewiſe our Names I unto you will tell;
'Mine is *Moll*, the others *Anne* and *Nell*;
'As oft as I five and five Ninths do take,
'*Anne* takes four and three Sevenths her Part to make;
'As oft as *Anne* four and one Ninth does tell,
'Three and two Thirds muſt be took up by *Nell*.

Solution. In Order to avoid the Trouble of Vulgar Fractions, we ſhall ſolve this by Decimals firſt, then 229*l.* 13*s.* 4*d.* = 229.666666; $5\frac{5}{9}$ = 5.555555, $4\frac{3}{7}$ = 4.428571; and $4\frac{1}{9}$ = 4.111111; hence, to find the Sum which *Nell* muſt take as often as *Anne* takes $4\frac{3}{7}$, we have this Stating, as 4.111111 : $3\frac{2}{3}$ :: 4.428571 : 3.949808 = what *Nell* muſt take as often as *Anne* takes $4\frac{3}{7}$.

Hence as often as *Moll* takes	5.555555
Anne takes	4.428571
And *Nell* muſt take	3.949808
	13.933934

And now, to find each Perſon's Share of the Whole, we ſtate as in Fellowſhip:

		£.	
As 13.933934 :	$5\frac{5}{9}$:	91.569684	= *Moll*'s Share
229.666666 ::	$4\frac{3}{7}$:	72.994061	= *Anne*'s Share
	3.949808 :	65.102890	= *Nell*'s Share

		£.	s.	d.	
Whence this Anſwer:	*Moll*'s Share =	91	11	$4\frac{3}{4}$	very near the Truth.
	Anne's =	72	19	$10\frac{3}{4}$	
	Nell's =	65	2	$0\frac{1}{2}$	
	Proof	229	13	4	

The END *of the* ESSAYS *on Vulgar and Decimal* ARITHMETIC.

Though the Par *of Exchange, in Chap.* XXIX. *of the* 1*st*, *Essay, was given on the Authority of the most noted Authors; yet the great Difference between the* Par *and* Course *of Exchange, in some Places, made the Author doubt the Truth of their Determinations; and that Mistrust has occasioned his making the following Remarks.*

I. On *France.*

By conversing with some *Frenchmen*, it appears they have no such Coin as a *five Livre Piece*; but their *six Livre Piece* is of the same Weight as the *five Livre Piece* in Sir *Isaac Newton*'s Table; whence it follows, that the 5 *Livre Piece* is now raised to six Livres; hence the 3 *Livre Piece*, on which they exchange, must be $= \frac{60.39}{2} = 30.2$ Pence nearly.

II. Of *Italy.*

By Sir *Isaac*'s Table, the Ducat of *Florence* and *Leghorn*, or Piece of 7 Livres, is 64.62*d.* Hence the Dollar or 6 Livres must be 55.3 Pence nearly: And, by the same Table, the Croisat of *Genoa*, or Piece of $7\frac{1}{2}$ Livres, is 78.74*d*; consequently the Dollar or 5 Livres = 52.5*d.* nearly.

III. Of *Portugal.*

The *Doppia Mœda*, or Piece of 4 Mill 800 Rees (*i. e.* 4800 Rees, or as we call it Moidore) is by the abovesaid Table 26*s.* 10*d.* 4: Whence the Mill-Ree, or 1000 Rees, is = 5*s.* 7*d.* nearly.

IV. Of *Spain.*

By the same Table, the *New Seville* Piece of Eight 43'. 11 d is 43 11*d.* Hence appears what little Care has been taken by Writers to adjust the *Par* of Exchange. We wish we had proper *Data* to proceed further on this Head.

THE

THE

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